AE 429 - Aircraft Performance and Flight Mechanics

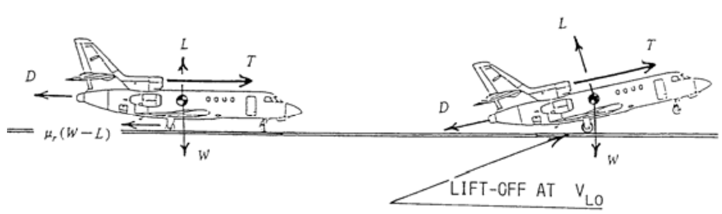
Take off and Landing

Takeoff Roll

- Consider the average forces acting during a takeoff

$$F_{eff} = T - [D + \mu_r (W - L)]_{average} \approx CONSTANT$$

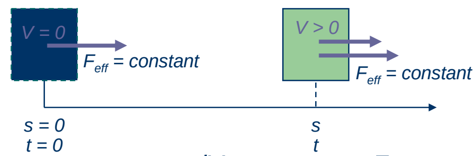
  - The first question we seek to answer: how far do we roll before L = W
  - When L = W, the airplane lifts off



  - Why consider “average” force during the takeoff roll?

Takeoff Roll

- We are no longer considering a “Statics” problem
  - Finite (even large) accelerations are present
  - If we apply Newton’s second law to any body initially at rest,



$$F = ma = m\frac{dV}{dt} \quad \Rightarrow \quad dV = \frac{F}{m}dt$$

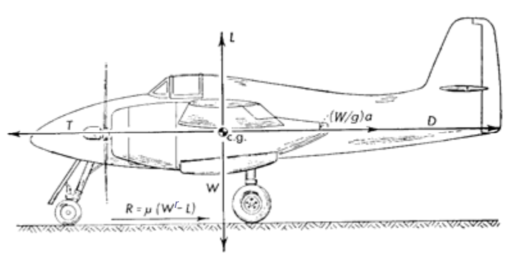
  - Integrating

$$\int_0^V dV = \frac{F_{eff}}{m}\int_0^t dt \quad \Rightarrow \quad V = \frac{F_{eff}}{m}t \quad OR \quad t = \frac{Vm}{F_{eff}}$$

Takeoff Roll

- Still working on ground roll distance

$$\int_0^s ds = \int_0^t Vdt = \int_0^t \frac{F}{m}tdt = \frac{F_{eff}}{m}\int_0^t tdt \quad \Rightarrow \quad s = \frac{F_{eff}}{m}\frac{t^2}{2}$$



  - Summing the forces during takeoff roll

$$F = T - D - R = T - D - \mu_r(W - L) = m\frac{dV}{dt} \quad \Rightarrow \quad WHERE \quad R \equiv \mu_r(W - L)$$

Takeoff Roll

- This equation gives instantaneous forces during the acceleration

$$F = T - D - R = T - D - \mu_r (W - L) = m\frac{dV}{dt} \quad \Rightarrow \quad WHERE \quad R \equiv \mu_r (W - L)$$

  - T is fairly constant (especially for a jet airplane) during takeoff roll
  - W is also approximately constant
  - Both L and D vary with V

$$L = \frac{1}{2}\rho_\infty V_\infty^2 S C_L \qquad D = \frac{1}{2}\rho_\infty V_\infty^2 S\left(C_{D,0} + \phi\frac{C_L^2}{\pi e AR}\right)$$

  - Where ϕ accounts for ground effect

$$L = \frac{1}{2}\rho_\infty V_\infty^2 S C_L \qquad \phi = \frac{(16h/b)^2}{1 + (16h/b)^2}$$

Takeoff Roll

- Lift-off distance for a jet airplane
  - Using the average Force we have postulated
    - The sum

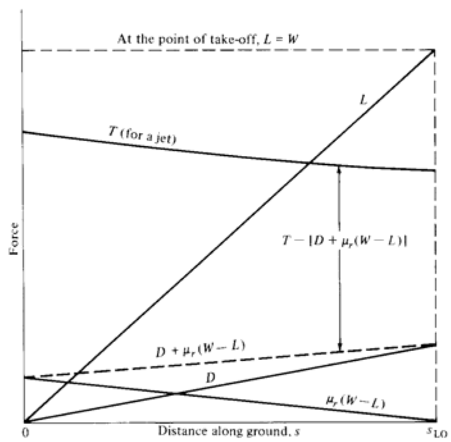
$$F_{eff} = T - [D + \mu_r (W - L)]_{average}$$

    - Is fairly constant
    - Thrust is also
  - Substituting into

$$D + \mu_r (W - L)$$

$$s = \frac{V^2 m}{2F_{eff}} \quad \Rightarrow \quad s_{LO} = \frac{V_{LO}^2 W}{2gF_{eff}}$$

$$s_{LO} = \frac{V_{LO}^2 W}{2g\{T - [D - \mu_r (W - L)]_{average}\}}$$



Takeoff Roll

- Lift-off distance (continued)
  - Generally we use a safety margin at lift-off by 20%

$$V_{LO} = 1.2V_{stall} = 1.2\sqrt{\frac{2W}{\rho_\infty S C_{L_{max}}}}$$

  - Substituting into the expression for S_{LO}

$$s_{LO} = \frac{1.44W^2}{g\rho_\infty S C_{L_{max}}\left\{T - [D + \mu_r(W - L)]_{average}\right\}}$$

$$[D + \mu_r(W - L)]_{average} \approx [D + \mu_r(W - L)]_{0.7V_{LO}}$$

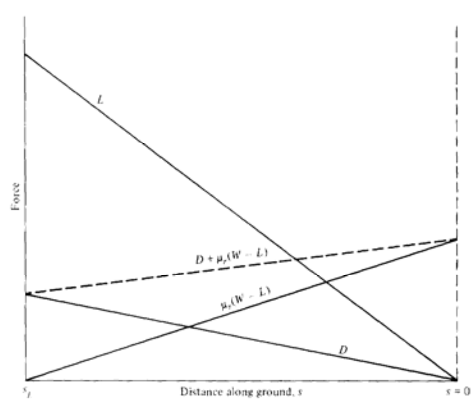
  - One method of estimating quickly the average drag and rolling resistance force is to use $0.7V_{LO}$ in calculating the aerodynamic forces

Landing Roll

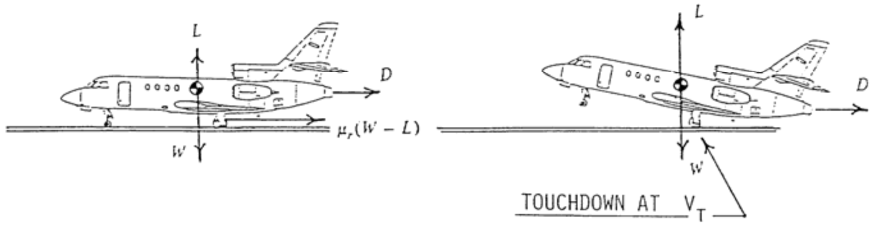
- The landing roll is very similar to the takeoff ground roll, except:
  - Thrust = 0 (or near zero)
  - The sign of the acceleration is negative

$$-[D + \mu_r(W - L)]_{average} = m\frac{dV}{dt}$$

  - We seek an approximate expression like S_{TO} for landing using an average Force again



Landing Roll



- Following this logic

$$F \approx -[D + \mu_r(W-L)]_{average} \approx -[D + \mu_r(W-L)]_{0.7V_T}$$

  - Notice that this assumption is less accurate for landing than for takeoff
  - Nonetheless, let's do the integration again

$$\int_{s_L}^{0} ds = \frac{F}{m}\int_0^t dt \quad \Rightarrow \quad s_L = -\frac{F}{m}\frac{t^2}{2} \; OR \; s_L = -\frac{V^2 m}{2F}$$

Landing Roll

- Still working on landing roll distance
  - Combining the previous expression for s_L and using the approximately constant retarding force

$$s_L \approx \frac{V_T^2 \, W/g}{2[D + \mu_r(W-L)]_{0.7V_T}}$$

  - again, adding a safety factor (30% in this case) to touch down at a speed above the stall speed

$$V_T = 1.3V_{stall} = 1.3\sqrt{\frac{2W}{\rho_\infty S C_{L_{max}}}}$$

  - Substituting into the expression above for s_L

$$s_L \approx \frac{1.69W^2}{g\rho_\infty S C_{L_{max}}[D + \mu_r(W-L)]_{0.7V_T}}$$

Landing Roll

- Landing roll can be reduced if a thrust reverser is installed

$$-T_R - D - \mu_r (W - L) = m\frac{dV}{dt} \Rightarrow \text{where } T_R = \text{Reverse Trust}$$

– If the thrust reverser produces constant thrust,

$$s_L \approx \frac{1.69W^2}{g\rho_\infty SC_{L_{max}} \left\{ T + \left[D + \mu_r (W - L) \right]_{0.7V_T} \right\}}$$

– Lift and drag forces are calculated, accounting for ground effect as we did for takeoff roll

$$L = \frac{1}{2}\rho_\infty V_\infty^2 SC_L \qquad D = \frac{1}{2}\rho_\infty V_\infty^2 S\left(C_{D,0} + \phi \frac{C_L^2}{\pi eAR} \right)$$

Where $$\phi = \frac{(16h/b)^2}{1 + (16h/b)^2}$$