

AE 429 - Aircraft Performance and Flight Mechanics

Range and Endurance

Range and Endurance

- Definitions

- Range

- Total ground distance traversed on a full tank of fuel -- Anderson book

- Distance an airplane can fly on a given amount of fuel -- alternative definition

- Endurance

- Total time an airplane stays in the air on a full tank of fuel -- Anderson book

- Time an airplane can fly on a given amount of fuel -- alternative definition

- Specific fuel consumption

- Weight of fuel consumed per unit power per unit time

Range and Endurance

Propeller-Driven Airplane

- Generalized endurance parameters

- Specific fuel consumption (SFC or c) $\frac{LBS\ FUEL}{BHP - HR}$
- Weight (W) lbs
- Time (t) secs or hrs
- Engine power (P) hp
- Airplane gross weight (W_0) lbs
includes fuel and payload
- Airplane empty weight (W_1) lbs
weight of airplane without fuel

$$W = W_1 + W_f$$

- Endurance calculation

- Fuel flow rate $\dot{W}_f = -c P$ (lbs fuel/unit time) = $\frac{dW_f}{dt}$ $c = -\frac{\dot{W}_f}{P}$
- Differential fuel burned $dW_f = dW = -c P dt$
- Solving for time and integrating

$$E = \int_0^E dt = \int_{W_0}^{W_1} dW_f = \int_{W_1}^{W_0} \frac{dW}{cP}$$

Range and Endurance

Propeller-Driven Airplane

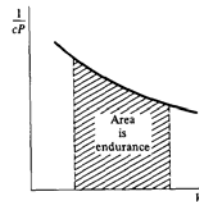
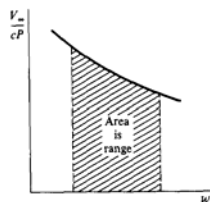
- Generalized Range Equation

- Multiply V_∞ by $dt = -\frac{dW}{cP}$ to get differential air distance covered

$$ds = V_\infty dt = -V_\infty \frac{dW}{cP}$$

- The total distance covered during a range flight is the integral of this expression

$$R = \int_0^R ds = -\int_{W_0}^{W_1} V_\infty \frac{dW}{cP} = \int_{W_1}^{W_0} \frac{V_\infty dW}{cP}$$



Range and Endurance

Propeller-Driven Airplane

- Breguet Range Equation

- In level flight cruise (no acceleration), the throttle is set to maintain constant airspeed

- $P_A = P_R = DV_\infty$
- Shaft HP = $P = \frac{P_A}{\eta} = \frac{DV_\infty}{\eta}$

- Substituting into the general range equation for propeller airplanes

$$R = \int_{W_1}^{W_0} \frac{V_\infty dW}{cP} = \int_{W_1}^{W_0} \frac{V_\infty \eta dW}{cDV_\infty} = \int_{W_1}^{W_0} \frac{\eta dW}{cD}$$

- Multiplying by w/w and using $L=W$ for equilibrium cruise flight (level and unaccelerated)

$$R = \int_{W_1}^{W_0} \frac{\eta}{c} \frac{W}{W} \frac{dW}{D} = \int_{W_1}^{W_0} \frac{\eta}{c} \frac{L}{D} \frac{dW}{W}$$

Range and Endurance

Propeller-Driven Airplane

- Breguet Range Equation (continued)

Assume

h is constant throughout flight

L/D is constant throughout flight

c is constant throughout flight

Then, we can integrate

$$R = \frac{\eta}{c} \frac{L}{D} \int_{W_1}^{W_0} \frac{dW}{W} \Rightarrow R = \frac{\eta}{c} \frac{L}{D} \ln \frac{W_0}{W_1}$$

- Because of the assumptions listed, the Breguet approximation is valid only for small dW

- Angle of attack changes over a weight change as large as the fuel load
- If weight changes significantly, L/D changes

Range and Endurance

Propeller-Driven Airplane

- Breguet Endurance Equation

Using the same assumptions ($L = W$ and $P_A = P_R = DV_\infty$)

$$E = \int_{W_1}^{W_0} \frac{dW}{cP} = \int_{W_1}^{W_0} \frac{\eta}{c} \frac{dW}{DV_\infty} = \int_{W_1}^{W_0} \frac{\eta}{c} \frac{L}{DV_\infty} \frac{dW}{W}$$

Since

$$L = W = \frac{1}{2} \rho_\infty V_\infty^2 S C_L, \quad V_\infty = \sqrt{\frac{2W}{\rho_\infty S C_L}}$$

and

$$E = \int_{W_1}^{W_0} \frac{\eta}{c} \frac{C_L}{C_D} \sqrt{\frac{\rho_\infty S C_L}{2}} \frac{dW}{W^{3/2}}$$

If we assume C_L , C_D , η , c , and ρ_∞ are all constant

$$E = \frac{\eta}{c} \frac{C_L^{3/2}}{C_D} \sqrt{2\rho_\infty S} \left(\frac{1}{\sqrt{W_1}} - \frac{1}{\sqrt{W_0}} \right)$$

in using this equation, E must be in seconds and c must be in ft⁻¹

Importance of the Ratios C_L/C_D , $C_L^{3/2}/C_D$, $C_L^{1/2}/C_D$

$$\left. \frac{C_L}{C_D} \right|_{\max} \Rightarrow \text{Max Range for reciprocating engine/propeller airplanes}$$

$$\left. \frac{C_L^{3/2}}{C_D} \right|_{\max} \Rightarrow \text{Max Endurance for reciprocating engine/propeller}$$

Range and Endurance

Jet-powered airplane

- **General endurance equation**

- Specific fuel consumption is based on thrust, not power output

$$TSFC \equiv c_t \Rightarrow \frac{\text{lbs of fuel consumed}}{\text{lb of thrust produced} - \text{sec}}$$

- Again, let dW be the differential change in aircraft weight as fuel is burned in dt

$$c_t = -\frac{\dot{W}_f}{T_A} \quad dW = -c_t T_A dt \Rightarrow dt = -\frac{dW}{c_t T_A}$$

- Integrating: $E = \int_0^E dt = -\int_{W_0}^{W_1} \frac{dW}{c_t T_A} = \int_{W_1}^{W_0} \frac{dW}{c_t T_A}$

- Recalling that $L = W$ and $T_A = T_R = D$

$$E = \int_{W_1}^{W_0} \frac{1}{c_t} \frac{L}{D} \frac{dW}{W}$$

- Assuming constant c_t and L/D

- The Breguet approximation is:

$$E \cong \frac{1}{c_t} \frac{L}{D} \ln \frac{W_0}{W_1}$$

Range and Endurance

Jet-powered airplane

- **To maximize endurance**

- Reduce thrust specific fuel consumption, TSFC
 - Use a turbofan engine
 - Fly at an altitude where the engine is efficient (high, but not too high)
- increase fuel fraction, W_f / W_0 , to increase W_1 / W_0
 - Carry less payload
 - Use drop tanks
- maintain L/D_{max}
 - Loiter at velocity for L/D_{max} (where $C_{D,0} = C_{Di}$)
 - Cruise climb
 - Notice that a jet airplane has best endurance at this velocity for L/D_{max} , while a propeller-driven airplane gets best range at this true airspeed

Range and Endurance

Jet-powered airplane

- **general range equation**

- going back to the differential time expression and multiplying by V

$$ds = V_\infty dt = -\frac{V_\infty dW}{c_t T_A}$$

- of course, ds is the differential distance traveled by the jet during time dt

- s = 0 when w = w₀

- s = R when w = w₁

- integrating to get range

$$R = \int_0^R ds = -\int_{W_0}^{W_1} \frac{V_\infty dW}{c_t T_A}$$

- for steady, unaccelerated level flight: $T_A = T_R = \frac{W}{L/D}$

$$R = \int_{W_1}^{W_0} \frac{V_\infty C_L}{c_t C_D} \frac{dW}{W}$$

Range and Endurance

Jet-powered airplane

- **general range equation (continued)**

- since

$$V_\infty = \sqrt{\frac{2W}{\rho_\infty S C_L}}$$

, R becomes

$$R = \int_{W_1}^{W_0} \sqrt{\frac{2}{\rho_\infty S}} \frac{1}{c_t} \frac{\sqrt{C_L}}{C_D} \frac{dW}{\sqrt{W}}$$

- **Breguet approximation**

- if we assume constant c_t, C_L, C_D, and ρ

$$R = 2 \sqrt{\frac{2}{\rho_\infty S}} \frac{1}{c_t} \frac{\sqrt{C_L}}{C_D} (\sqrt{W_0} - \sqrt{W_1})$$

- for maximum range

- reduce TSFC, c_t

- fly at low density (high altitude)

- use a true airspeed to maximize $\Rightarrow \frac{C_L^{1/2}}{C_D}$

Range and Endurance

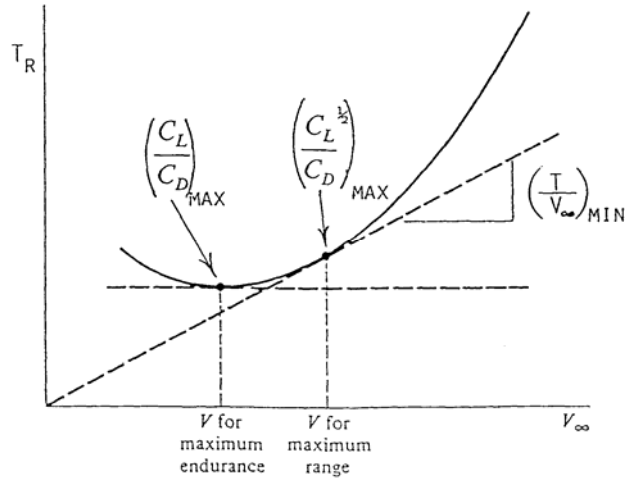
Jet-powered airplane

- Cruise for Max range

- Fly at

$$\left(\frac{\sqrt{C_L}}{C_D} \right)_{max}$$

- The true air-speed for This optimum Speed gives $C_{D,0} = 3C_{di}$



Importance of the Ratios C_L/C_D , $C_L^{3/2}/C_D$, $C_L^{1/2}/C_D$

$$\left. \frac{C_L}{C_D} \right|_{max} \Rightarrow \text{Max Endurance for jet-propelled airplanes}$$

$$\left. \frac{C_L^{1/2}}{C_D} \right|_{max} \Rightarrow \text{Max Range for jet-propelled airplanes}$$

Range and Endurance

Summary

- Relationship of $C_{D,0}$ and $C_{D,i}$

- At L/D_{\max}

- Using $C_D = C_{D,0} + \frac{C_L^2}{\pi eAR} \implies \frac{C_L}{C_D} = \frac{C_L}{C_{D,0} + \frac{C_L^2}{\pi eAR}}$

- Differentiating with respect to C_L and setting to zero

$$\frac{d\left(\frac{C_L}{C_D}\right)}{dC_L} = \frac{C_{D,0} + \frac{C_L^2}{\pi eAR} - C_L \left(2 \frac{C_L}{\pi eAR}\right)}{\left(C_{D,0} + \frac{C_L^2}{\pi eAR}\right)^2} = 0$$

$$C_{D,0} + \frac{C_L^2}{\pi eAR} - 2 \left(\frac{C_L^2}{\pi eAR}\right) = 0 \implies C_{D,0} = \frac{C_L^2}{\pi eAR}$$

$$C_{D,0} = C_{D,i} \quad \text{FOR} \quad \left(\frac{C_L}{C_D}\right)_{\max}$$

Range and Endurance

Summary

- Relationship of $C_{D,0}$ and $C_{D,i}$

- Similar developments show:

$$C_{D,0} = 3C_{D,i} \quad \text{FOR} \quad \left(\frac{\sqrt{C_L}}{C_D}\right)_{\max}$$

- Applies to maximum range for a jet airplane

$$3C_{D,0} = C_{D,i} \quad \text{FOR} \quad \left(\frac{\sqrt{C_L^3}}{C_D}\right)_{\max}$$

- Applies to maximum endurance for a propeller-driven airplane

RANGE

PROPELLER/RECIPR. ENGINE
 $c = -\dot{W}_f / P_{\text{shaft}}$

JET-PROPELLED AIRPLANE
 $c_t = -\dot{W}_f / T_A$

Since
 $T_A = \frac{P_A \rho R}{V_\infty}$; NOTE $P_A = T_A V_\infty$

$V_\infty = \frac{ds}{dt} \Rightarrow ds = V_\infty dt$

$c_t = -\frac{dW_f/dt}{T}$ ↓

$dt = -\frac{dW_f}{c_t T} \Rightarrow ds = -\frac{V_\infty}{c_t T} dW_f$

$dW_f = dW$ → $ds = -\frac{V_\infty}{c_t T} dW = -\frac{V_\infty \bar{w}}{c_t T} \frac{dW}{W}$

$ds = -\frac{V_\infty L}{c_t D} \frac{dW}{W}$

$R = \int_0^R ds = -\int_{W_0}^{W_1} \frac{V_\infty L}{c_t D} \frac{dW}{W}$

$R = \int_{W_1}^{W_0} \frac{V_\infty L}{c_t D} \frac{dW}{W}$ RANGE

ENDURANCE

$$\frac{dW_f}{dt} = -c_t T$$

$$dt = -\frac{dW_f}{c_t T} = -\frac{dW_f}{c_t D} = -\frac{dW_f}{W} \frac{L}{D} \frac{1}{c_t}$$

\uparrow
 $L=W$
 $D=T$

$$E = -\int_{W_0}^{W_1} \frac{1}{c_t} \frac{L}{D} \frac{dW_f}{W} = \int_{W_1}^{W_0} \frac{1}{c_t} \frac{L}{D} \frac{dW}{W}$$

