

AE 429 - Aircraft Performance and Flight Mechanics

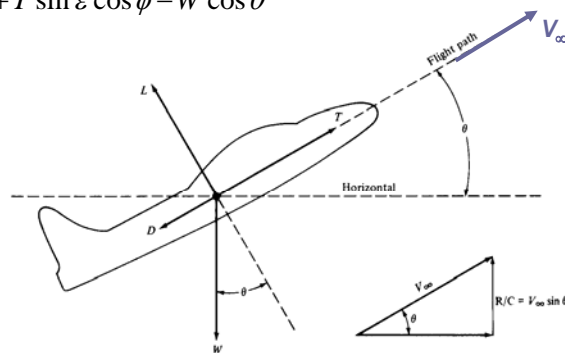
Rate of Climb
Time to Climb

Rate of Climb R/C

- Now let's analyze a steady climb
 - Forces include a gravity component now

$$m \frac{dV_{\infty}}{dt} = T \cos \varepsilon - D - W \sin \theta \quad \Rightarrow \quad T = D + W \sin \theta$$
$$L = W \cos \theta$$

$$m \frac{V_{\infty}^2}{r_1} = L \cos \phi + T \sin \varepsilon \cos \phi - W \cos \theta$$



Rate of Climb R/C

- For low climb angles (up to about 20°)

- We can assume $\cos \theta \approx 1$ to calculate C_L
- So we work on the drag equation, multiplying by V_∞

$$TV_\infty = DV_\infty + WV_\infty \sin \theta$$

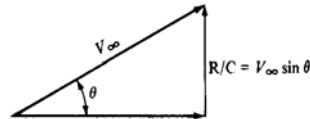
$$\frac{TV_\infty - DV_\infty}{W} = \frac{(T-D)V_\infty}{W} = V_\infty \sin \theta$$

← Excess Power

- In climb, the rate of climb is the vertical component of velocity

$$R/C = V_\infty \sin \theta$$

- Maximum R/C
 - Maximize excess power
 - Minimize weight
- Maximum angle of climb
 - Maximize excess thrust
 - Minimize weight



$$R/C = \frac{(T-D)V_\infty}{W}$$

$$\theta = \sin^{-1} \frac{(T-D)}{W} \approx \frac{(T-D)}{W}$$

Rate of Climb R/C

$$C_L = \frac{L}{q_\infty S} = \frac{W \cos \theta}{q_\infty S}$$

$$D = q_\infty S C_D = q_\infty S \left(C_{D,0} + K C_L^2 \right) = q_\infty S \left(C_{D,0} + K \left(\frac{W \cos \theta}{q_\infty S} \right)^2 \right) =$$

$$= q_\infty S C_{D,0} + \frac{KW^2 \cos^2 \theta}{q_\infty S}$$

$$R/C = V_\infty \sin \theta = \frac{(T-D)V_\infty}{W} = V_\infty \left(\frac{T}{W} - \frac{D}{W} \right)$$

$$R/C = V_\infty \sin \theta = V_\infty \left[\frac{T}{W} - \frac{1}{2} \rho_\infty V_\infty^2 \left(\frac{W}{S} \right)^{-1} C_{D,0} - \frac{2K}{\rho_\infty V_\infty^2} \frac{W}{S} \cos^2 \theta \right]$$

Preliminary design $\cos \theta \approx 1$ OK for $\theta \leq 50^\circ$

Maximum Climb Angle

θ_{\max}

$$\sin \theta = \frac{T}{W} - \frac{D}{W} = \frac{T}{W} - \frac{D}{L/\cos \theta} = \frac{T}{W} - \frac{1}{L/D}$$

$$\sin \theta_{\max} = \frac{T}{W} - \frac{1}{(L/D)_{\max}} = \frac{T}{W} - \sqrt{4C_{D,0}K}$$

$$L = W \cos \theta = \frac{1}{2} \rho_{\infty} V_{\infty}^2 S C_L \quad C_L = \sqrt{\frac{C_{D,0}}{K}}$$

$$L = W \cos \theta_{\max} = \frac{1}{2} \rho_{\infty} V_{\theta_{\max}}^2 S \sqrt{\frac{C_{D,0}}{K}}$$

Speed at max θ for jet propelled airplane

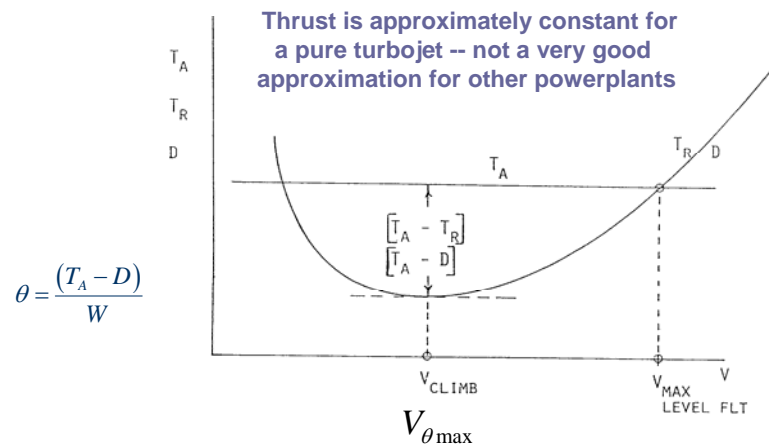
$$V_{\theta_{\max}}^2 = \frac{2}{\rho_{\infty}} \sqrt{\frac{K}{C_{D,0}}} \frac{W}{S} \cos \theta_{\max}$$

Rate of climb at max θ

$$(R/C)_{\theta_{\max}} = V_{\theta_{\max}} \sin \theta_{\max}$$

Max Angle to Climb

- If thrust available is constant with V
 - Maximum climb angle occurs at minimum drag
 - This speed also gives best acceleration in level flight



Rate of Climb R/C

- Typically, climbs are not flown at constant V_∞

$$T = D + W \sin \theta + \frac{W}{g} \frac{dV_\infty}{dt} = D + W \sin \theta + \frac{W}{g} \frac{dV_\infty}{dh} \frac{dh}{dt}$$

- Recognizing

$$\frac{dh}{dt} = R/C = V_\infty \sin \theta$$

- and rearranging

$$\frac{T-D}{W} = \sin \theta + \frac{1}{g} \frac{dV_\infty}{dh} V_\infty \sin \theta \Rightarrow \sin \theta = \frac{\frac{T-D}{W}}{1 + \frac{V_\infty}{g} \frac{dV_\infty}{dh}} = \frac{T-D}{W} \frac{1}{k}$$

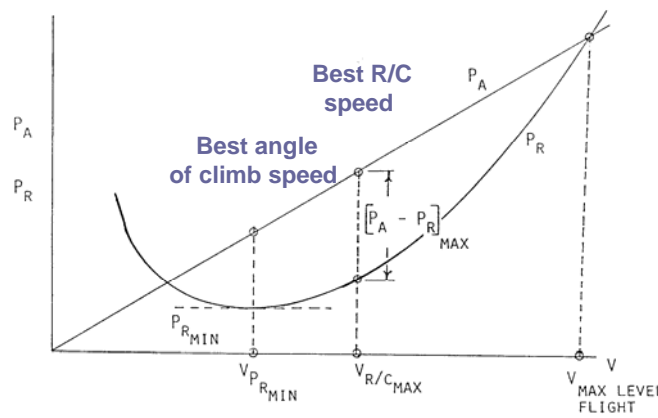
$$k = 1 + \frac{V_\infty}{g} \frac{dV_\infty}{dh}$$

Rate of Climb R/C

- For our idealized jet airplane, best rate of climb does not occur at minimum power required
 - Maximum rate of climb occurs at the velocity where excess power is greatest
 - The velocity for maximum rate of climb is determined for any aircraft by
 - Plotting the power required versus true airspeed
 - Overplotting the power available versus true airspeed
 - Choosing the velocity where the distance between the two curves is greatest

Rate of Climb R/C

- The chart below illustrates this procedure



Rate of Climb R/C

Max Rate of Climb

$$R/C = V_{\infty} \left[\frac{T}{W} - \frac{1}{2} \rho_{\infty} V_{\infty}^2 \left(\frac{W}{S} \right)^{-1} C_{D,0} - \frac{2K}{\rho_{\infty} V_{\infty}^2} \frac{W}{S} \right]$$

$$\frac{d(R/C)}{dV_{\infty}} = \left[\frac{T}{W} - \frac{3}{2} \rho_{\infty} V_{\infty}^2 \left(\frac{W}{S} \right)^{-1} C_{D,0} + \frac{2K}{\rho_{\infty} V_{\infty}^2} \frac{W}{S} \right] = 0$$

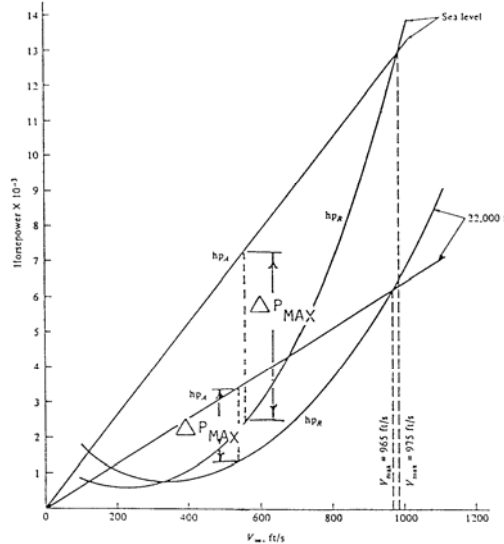
$$V_{(R/C)_{\max}} = \left\{ \frac{(T/W)(W/S)}{3\rho_{\infty} C_{D,0}} \left[1 + \sqrt{1 + \frac{3}{(L/D)_{\max}^2 (T/W)^2}} \right] \right\}^{1/2}$$

$$(R/C)_{\max} = \left[\frac{(W/S)Z}{3\rho_{\infty} C_{D,0}} \right]^{1/2} \left(\frac{T}{W} \right)^{3/2} \left[1 - \frac{Z}{6} - \frac{3}{2(T/W)^2 (L/D)_{\max}^2} Z \right]$$

$$Z \equiv 1 + \sqrt{1 + \frac{3}{(L/D)_{\max}^2 (T/W)^2}}$$

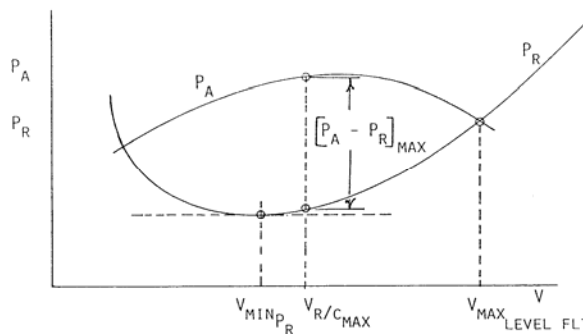
Rate of Climb R/C: Effect of the Altitude

- This chart shows the effect of altitude
 - At higher true airspeeds, P_R decreases with altitude
 - However, P_A falls off faster than P_R
 - The best climb speed usually decreases slightly with altitude



Rate of Climb R/C

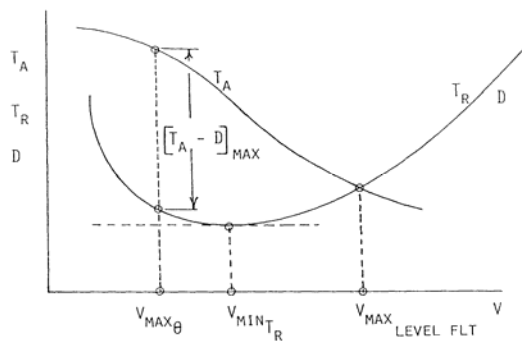
- For a propeller aircraft
 - Maximum rate of climb occurs at the V_∞ where maximum excess power occurs



- Maximum R/C does not occur at V_{minPR}
- DV_∞ is approximately the power required in the climb

Rate of Climb R/C

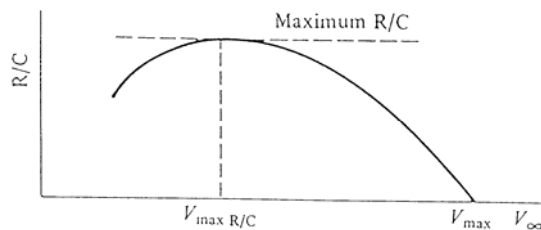
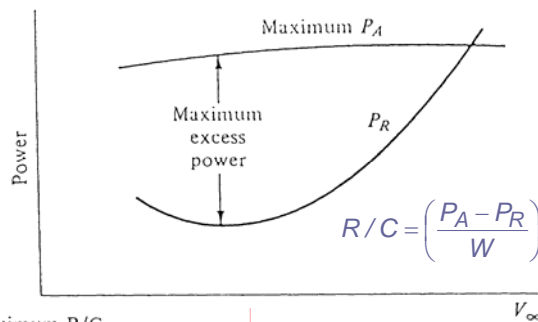
- For propeller aircraft
 - Maximum angle of climb occurs at the V_{∞} for which maximum excess thrust occurs



- Maximum climb angle (which is used to clear obstacles on takeoff) occurs at a velocity $< V_{minTR}$

Rate of Climb R/C

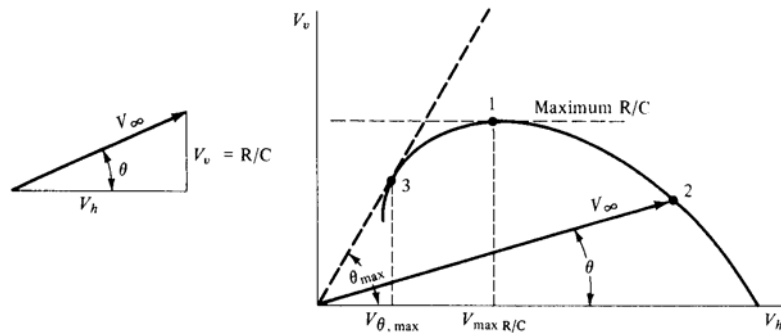
- For a given altitude
 - For any type of airplane, excess power determines R/C
 - Induced drag changes P_R



- R/C changes with velocity
- Plots like the one on left allow determination of best R/C speed

Rate of Climb R/C

- Climb performance hodograph
 - Vertical velocity versus horizontal velocity



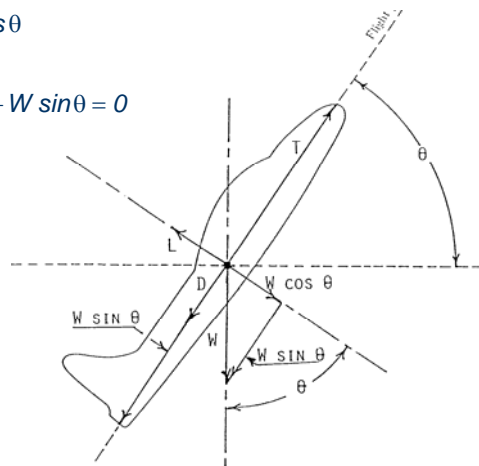
- Notice the difference in velocity for θ_{\max} and the velocity for $(R/C)_{\max}$

Rate of Climb R/C: High performance climb

- Lift forces Eq. $L = W \cos \theta$
- Drag forces Eq. $T - D - W \sin \theta = 0$
- Solving the two equations for θ
 - solve each for qS
 - set $qS = qS$
 - solve the resulting quadratic for θ

$$\sin \theta = \frac{C_L^2 T}{W(C_L^2 + C_D^2)}$$

$$-\sqrt{\left(\frac{C_L^2 T}{W(C_L^2 + C_D^2)}\right)^2 - \frac{C_L^2 T^2 - C_D^2 W^2}{W(C_L^2 + C_D^2)}}$$



Gliding flight

- Forces in a power-off glide

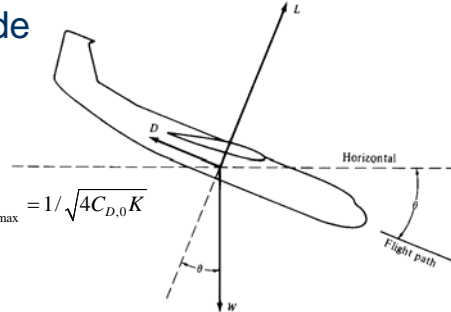
$$D = W \sin \theta \quad L = W \cos \theta$$

- Dividing drag by lift

$$\tan \theta = \frac{1}{L/D} \quad \tan \theta_{\min} = \frac{1}{(L/D)_{\max}} \quad (L/D)_{\max} = 1/\sqrt{4C_{D,0}K}$$

- Once again, an important performance parameter is set by L/D

- the smallest θ gives maximum gliding range
- this maximum range occurs when L/D is maximum for V_∞ constant
- airplanes with good aerodynamic efficiency (high L/D) can glide 20-50 times as far as their altitude
- Does not depend on wing loading or altitude.



$$V_\infty = \sqrt{\frac{2 \cos \theta W}{\rho_\infty C_L S}}$$

Equilibrium glide velocity

$$L = \frac{1}{2} \rho_\infty V_\infty^2 S C_L = W \cos \theta$$

$$V_\infty = \sqrt{\frac{2 \cos \theta W}{\rho_\infty C_L S}} \quad \text{Equilibrium glide velocity}$$

$$(L/D)_{\max} = 1/\sqrt{4C_{D,0}K}$$

$$\cos \theta \approx 1$$

$$\left(\frac{L}{D}\right)_{\max} = \left(\frac{C_L}{C_D}\right)_{\max} = \sqrt{\frac{1}{4C_{D,0}K}}$$

$$L = W = \frac{1}{2} \rho_\infty V_\infty^2 S C_L$$

$$V_{(L/D)\max} = \left(\frac{2}{\rho_\infty} \sqrt{\frac{K W}{C_{D,0} S}}\right)^{1/2}$$

$$V_V = V_\infty \sin \theta$$

$$V_H = V_\infty \cos \theta$$

$$D = W \sin \theta$$

$$D V_\infty = W \sin \theta V_\infty = W V_V$$

$$V_V = \frac{D V_\infty}{W} = \frac{P_R}{W}$$

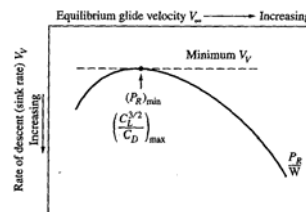


Figure 5.42 Rate of descent versus equilibrium glide velocity.

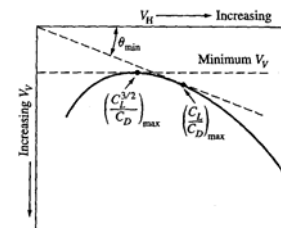


Figure 5.43 Hodograph for unpowered flight.

or

$$V_{\infty} = \sqrt{\frac{2W \cos \theta}{\rho_{\infty} S C_L}} \quad [5.130]$$

Substituting Eq. (5.130) into Eq. (5.128), we have

$$V_V = V_{\infty} \sin \theta = (\sin \theta) \sqrt{\frac{2 \cos \theta W}{\rho_{\infty} C_L S}} \quad [5.131]$$

Dividing Eq. (5.124) by Eq. (5.123), we obtain

$$\sin \theta = \frac{D}{L} \cos \theta = \frac{C_D}{C_L} \cos \theta \quad [5.132]$$

Inserting Eq. (5.132) into Eq. (5.131), we have

$$V_V = \sqrt{\frac{2 \cos^3 \theta W}{\rho_{\infty} (C_L^3 / C_D^2) S}} \quad [5.133]$$

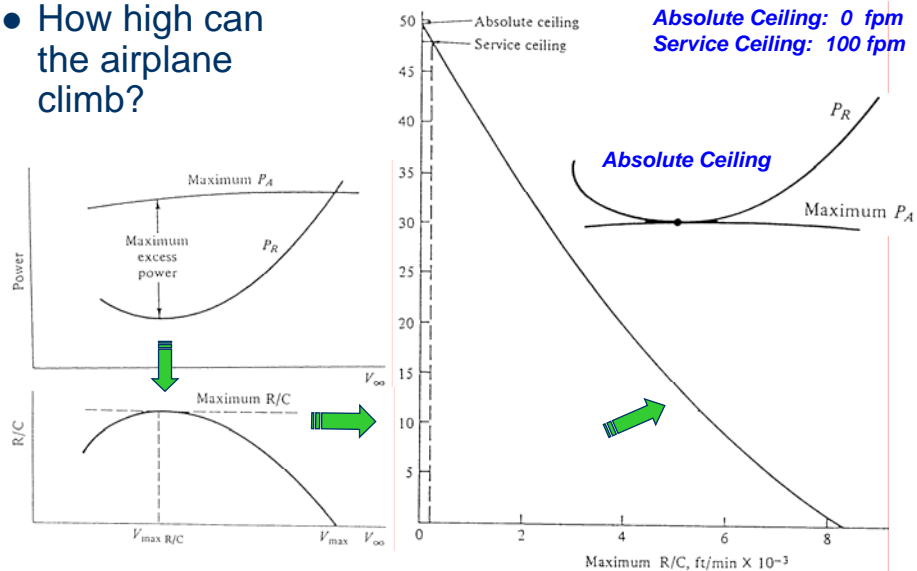
By making the assumption that $\cos \theta = 1$, Eq. (5.133) is written as

$$V_V = \sqrt{\frac{2 W}{\rho_{\infty} (C_L^3 / C_D^2) S}} \quad [5.134]$$

Equation (5.134) explicitly shows that $(V_V)_{\min}$ occurs at $(C_L^{3/2} / C_D)_{\max}$. It also shows that the sink rate decreases with decreasing altitude and increases as the square root of the wing loading.

Ceilings

- How high can the airplane climb?



Ceilings

- The ceiling is the altitude at which R/C has reached some minimum value
 - Absolute ceiling
 - Is defined as the altitude at which the $R/C = 0$
 - Is dictated when P_A is just tangent to the P_R curve
 - Service ceiling
 - is defined as that altitude where $R/C_{max} = 100 \text{ ft/min}$
 - is the practical upper limit for steady, level flight
 - Procedure
 - calculate values of R/C_{max} for different altitudes
 - plot R/C_{max} versus altitude
 - extrapolate this latter curve to 100 fpm and 0 fpm to get the service and absolute ceilings

Time to Climb

- Time to climb

- Needs to be short
- Calculating R/C

$$R/C = V_{\infty} \sin \theta = \frac{dh}{dt}$$

$$dt = \frac{dh}{R/C}$$

- Integrating

$$\int_{t_1}^{t_2} dt = \int_{h_1}^{h_2} \frac{dh}{R/C}$$

- Calculating time-to-climb graphically

- plot $(R/C)^{-1}$ versus h
- Approximate the area under the curve
- Subtract time to climb to the starting altitude

