

AE 429 - Aircraft Performance and Flight Mechanics

Power Required and Available

Power

- Power is energy per unit time

$$POWER = P \equiv \frac{FORCE * DISTANCE}{TIME} = FV_{\infty} \text{ if } V_{\infty} \text{ is constant}$$

- For an airplane in level, unaccelerated flight, power $P_R = T_R V_{\infty}$ required is

$$P_R = T_R V_{\infty} = \frac{W}{C_L/C_D} V_{\infty} \text{ but } V_{\infty} = \sqrt{\frac{2W}{\rho_{\infty} S C_L}}$$

- So

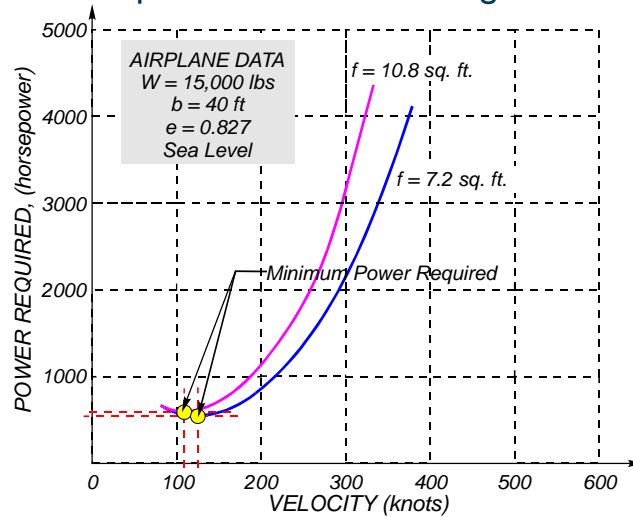
$$P_R = \frac{W}{C_L/C_D} \sqrt{\frac{2W}{\rho_{\infty} S C_L}} = \left(\sqrt{\frac{2W^3}{\rho_{\infty} S}} \right) \left(\sqrt{\frac{C_D^2}{C_L^3}} \right)$$

- That is,

$$P_R \propto \frac{1}{C_L^{3/2}/C_D}$$

Power

- How power required varies with drag



Power

- Let's analyze minimum power required
 - As we observed before for a parabolic drag polar

$$P_R = T_R V_\infty = D V_\infty = q_\infty S \left(C_{D0} + \frac{C_L^2}{\pi e A R} \right) V_\infty$$

$$P_R = \underbrace{q_\infty S C_{D0} V_\infty}_{\text{Parasite power required}} + \underbrace{q_\infty S V_\infty \frac{C_L^2}{\pi e A R}}_{\text{Induced power required}}$$

- In level, unaccelerated flight

$$C_L = \frac{2L}{\rho_\infty V_\infty^2 S} = \frac{2W}{\rho_\infty V_\infty^2 S};$$

$$P_R = \frac{1}{2} \rho_\infty V_\infty^3 S C_{D0} + \frac{1}{2} \rho_\infty V_\infty^3 S \frac{C_L^2}{\pi e A R} = \frac{1}{2} \rho_\infty V_\infty^3 S C_{D0} + \frac{1}{2} \rho_\infty V_\infty^3 S \frac{4W^2}{\rho_\infty^2 V_\infty^4 S^2 \pi e A R}$$

$$P_R = \frac{1}{2} \rho_\infty V_\infty^3 S C_{D0} + \frac{2W^2}{\rho_\infty V_\infty S \pi e A R}$$

Power

- Still looking for the minimum P_R point
 - Differentiating this last expression for P_R and setting this result to zero

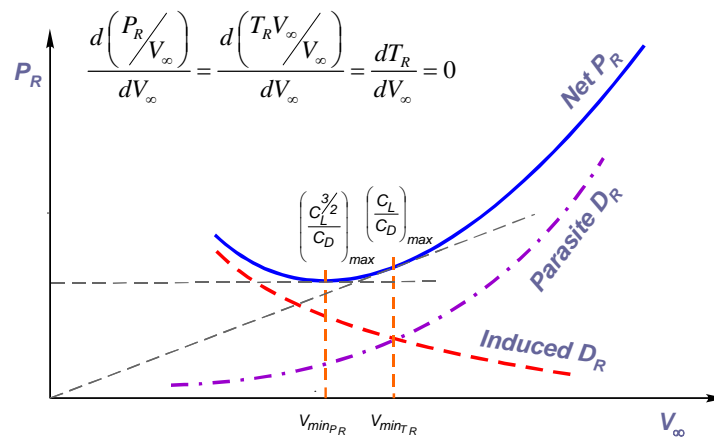
$$\begin{aligned} \frac{dP_R}{dV_\infty} &= \frac{3}{2} \rho_\infty V_\infty^2 S C_{D_0} - \frac{2W^2}{\rho_\infty V_\infty^3 S \pi e A R} \\ &= \frac{3}{2} \rho_\infty V_\infty^2 S \left(C_{D_0} - \frac{4W^2}{3\rho_\infty^2 V_\infty^4 S^2 \pi e A R} \right) \\ &= \frac{3}{2} \rho_\infty V_\infty^2 S \left(C_{D_0} - \frac{C_L^2}{3\pi e A R} \right) \\ &= \frac{3}{2} \rho_\infty V_\infty^2 S \left(C_{D_0} - \frac{C_{D_i}}{3} \right) \end{aligned}$$

- Thus, the relationship between C_{D_0} and C_{D_i} at minimum power required is

$$C_{D_0} = \frac{C_{D_i}}{3}$$

Power

- In level flight, $D = T_R$



Altitude effects on P_R

- Power required and velocity for level unaccelerated flight

- At sea level

$$V_0 = \sqrt{\frac{2W}{\rho_0 S C_L}} \Rightarrow P_{R,0} = \sqrt{\frac{2W^3 C_D^2}{\rho_0 S C_L^3}}$$

- At altitude

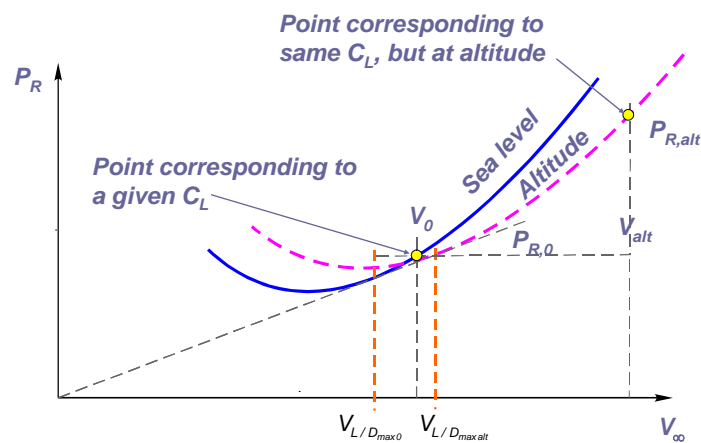
$$V_{alt} = \sqrt{\frac{2W}{\rho S C_L}} \Rightarrow P_{R,alt} = \sqrt{\frac{2W^3 C_D^2}{\rho S C_L^3}}$$

- Dividing each altitude equation by the s.l. one

$$V_{alt} = V_0 \sqrt{\frac{\rho_0}{\rho}} = V_0 \sqrt{\frac{1}{\sigma}} \Rightarrow P_{R,alt} = P_{R,0} \sqrt{\frac{\rho_0}{\rho}} = P_{R,0} \sqrt{\frac{1}{\sigma}}$$

- On a power required curve, any point associated with a given C_L at sea level, moves to the right and up as altitude is increased

Altitude effects on P_R



Weight effects on P_R

$$P_R = \frac{W}{C_L/C_D} \sqrt{\frac{2W}{\rho_\infty S C_L}}$$

$$= W^{3/2} \sqrt{\frac{2}{\rho_\infty S}} \left(\sqrt{\frac{1}{C_L^{3/2}/C_D}} \right)$$

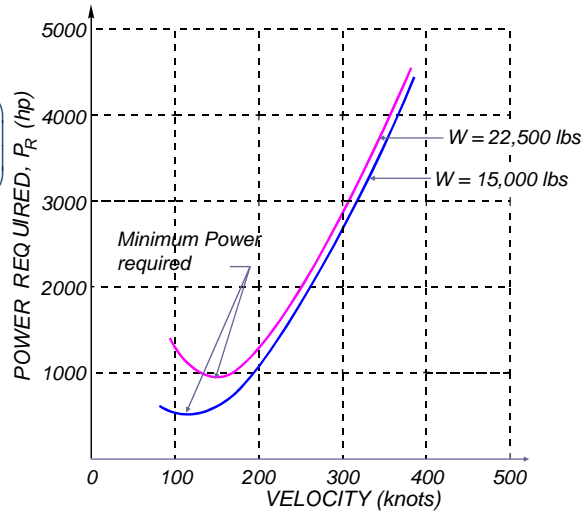
$$V = \sqrt{\frac{2W}{\rho_\infty S C_L}}$$

- Define

$$\omega \equiv \frac{W}{W_{std}}$$

$$V = V_{std} \sqrt{\omega}$$

$$P_R = P_{Rstd} \sqrt{\omega^3}$$



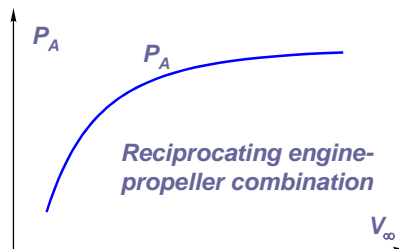
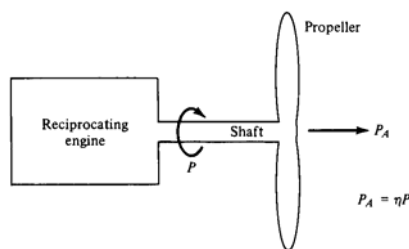
Power available P_A

- Propeller efficiency is important for any powerplant driving a propeller

$$P_A = \eta P$$

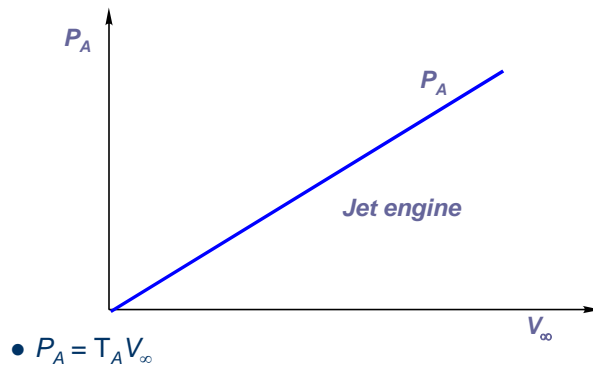
- η is propeller efficiency
- P is brake horsepower
- P_A is available horsepower $< P$

$$1 \text{ hp} = 550 \text{ ft} - \text{lb} / \text{sec} = 746 \text{ watts}$$



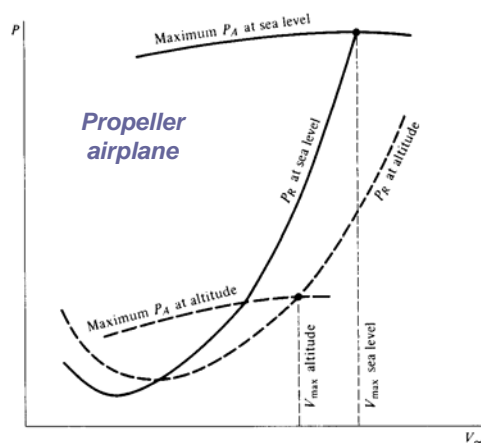
Power available P_A

- Power available from a jet engine has quite a different shape



Power available P_A

- Altitude effects on P_A are kin to P_R
 - Propeller aircraft



Power available P_A

- Similarly for a jet-powered airplane

