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ANALYTICAL METHOD FOR PLOT DRAG POLAR OF LIGHT AIRCRAFT OPERATING UNDER SUBSONIC FLIGHT CONDITIONS - APPLICATIONS FOR SAE-AERODESIGN

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Summary
This article has as objective to present an analytical model that allows the determination of the equation that defines the drag polar of a light aircraft operating under subsonic flight conditions. Basically the whole relationship between the lift force and drag force and important details about the performance of an aircraft can be obtained from the direct reading of the drag polar curve, fundamental questions such as what is a drag polar and, what its importance for the performance of the aircraft are discussed in detail on the development and submission of this article.

1.Introduction

The precise plot of the curve that defines the drag polar of an aircraft is essential to a great project. During the early stages of the project of a new aircraft, often there is the need for a series of refinements and iterations until they reach an equation that defines the ideal drag polar for the purpose of the project in question. The concepts discussed in this article for the determination of the drag polar are used in the aviation industry and serve as reference for the application in the development of aircraft to participate in the SAE-AeroDesign competition.

2. Definition of drag polar and how it can be obtained

The drag polar is a curve that shows the relationship between the drag coefficient and lift coefficient for a full aircraft. This relationship is expressed by an equation that can be represented by a graph called drag polar, Raymer (1992).

For whole body with aerodynamic shape moving through to the air, there is a relation between the lift coefficient (C_L) and the drag coefficient (C_D) that can be expressed by an equation or represented by a graph. Both the equation as the graph who representing the relationship between (C_L) and (C_D) are called drag polar.

The drag polar shows, all the information necessary for an aerodynamic and performance analysis of the aircraft. The equation that defines the drag polar of an aircraft can be obtained from the total drag generated in it. The total drag is obtained from the sum of parasite drag with the wave drag and the induced drag due to lift generation of the aircraft, thus the equation that defines the total drag of an aircraft as aerodynamic coefficients can be written to follows.

$$C_D = C_{D0} + C_{Dw} + C_{Di} \qquad (1)$$

In this equation, the term referring to the wave drag C_{DW} can be neglected during the calculations of the project of an aircraft to participate in the SAE-AeroDesign, since this portion of drag is only present in transonic or supersonic speeds, the is not done on the aircraft involved SAE-AeroDesign that normally perform flights with a range of speeds between 10 m/s and 30 m/s. Thus, the equation (1) may be rewritten as follows.

$$C_D = C_{D0} + \frac{C_L^{\ 2}}{\pi \cdot e_0 \cdot AR} \qquad (2)$$

The first term on the right side of the equation (2) represents the parasite drag of the aircraft and the second represents the drag due the production of lift force (induced drag). In order to simplify this equation, the induced drag may be written in the form of a coefficient of proportionality as the equation (3).

$$C_D = C_{D0} + K \cdot C_L^{\ 2} \qquad (3)$$

The coefficient of proportionality K is calculated by.

$$K = \frac{1}{\pi \cdot e_0 \cdot AR} \qquad (4)$$

As e_0 called Oswald efficiency factor. According to Anderson (1999), the Oswald coefficient represents about 75% of the wingspan efficiency factor, and can be obtained as follows.

$$e_0 = 0{,}75 \cdot e \qquad (5)$$

Usually for a complete aircraft, e_0 is a number that is between 0.6 and 0.8, this is due to the effects of interference between the wing and fuselage, and due to the effects of the tail and other parts of the aircraft.

The equation (3) represents the polar drag of an aircraft and, in this equation, C_D represents the total drag coefficient of the aircraft, C_{D0} represents the parasite drag coefficient and KC_L^2 represents the induced drag due the lift production of the aircraft. A graph of generic drag polar of an aircraft is shown in Figure 1.

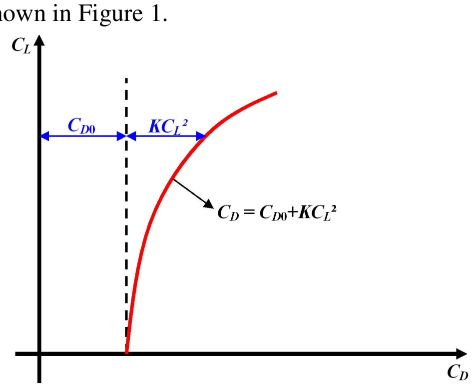


Figure 1 – Generic curve of a drag polar of an aircraft.

The curve shown in Figure 1 assumes that a generic way for any aircraft on a subsonic flight.

The origin of this form can be easily viewed from the aerodynamic forces that act on an aircraft in flight as shown in Figure 2.

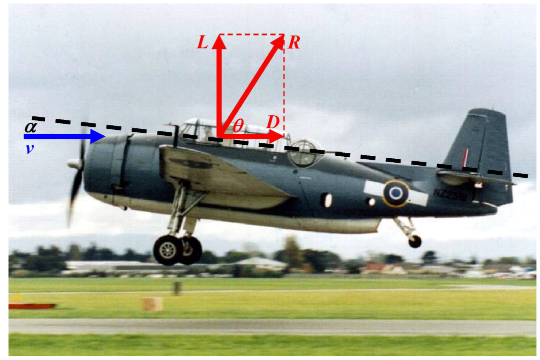


Figure 2 – Aerodynamic forces acting during the flight.

From the analysis of Figure 2, it is possible to see that for a determinate angle of attack α, the aerodynamic force R makes a θ angle with relative wind. Thus, if R and θ will be plot in a appropriate scale, it is possible to trace the drag polar of an aircraft as a whole, it is true that for every angle of attack evaluated, a new value of R and a new value of θ to be obtained. Figure 3 shows the design of the drag polar for various values of R and θ.

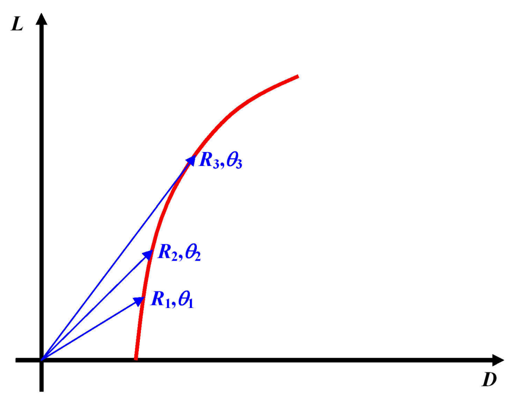


Figure 3 – Aerodynamic force on drag polar.

Hence, the drag polar is nothing more that the representation of aerodynamic force designed in polar coordinates. It is important to note that each point of the drag polar corresponds to a different angle of attack, it is also important to note that the graph shown in Figure 3 has its values data on aerodynamic forces to lift and drag, but usually the drag polar curve for an aircraft is presented in terms of aerodynamic coefficients C_D and C_L. In both situations, the curve obtained will be exactly the same.

For a greater aerodynamic efficiency, it is possible to see that who greater is the θ angle, higher is the value of relationship achieved between the lift force and drag force and consequently lower is the parasite drag, making this way with the drag polar curve is very close to vertical axis. The ideal situation for the project must be a θ angle equal to 90°, because that way, the whole drag would be removed from the aircraft, but this is a impossible situation to achieve in practice, and therefore a very effective way to improve the drag polar of an aircraft is to try to reduce the parasite an induced drag as possible.

For all drag polar there is a point where the relationship between C_L and C_D takes its peak value, this point is called the aerodynamic point and represented in the nomenclature of $(L/D)_{Max}$ or maximum efficiency $E_{máx}$.

It is important to say that this point is the point who the pilot cam fly the aircraft getting the maximum lift with the lowest penalty of drag resulting in important performance characteristics of the aircraft.

As a way to determine the design point of an aircraft from its drag polar, Figure 4 shows the location of that point and the Equations of (6a) to (6j) provide an allowance for determining the mathematical lift coefficient called C_L* with which it is possible to obtain the maximum aerodynamic efficiency of aircraft.

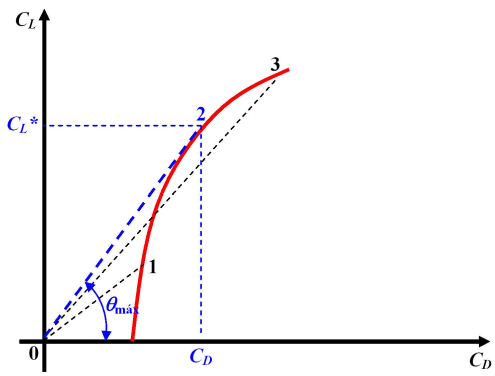


Figure 4 – Determination of $(L/D)_{máx}$.

It is possible to see can see in Figure 4 who the maximum value of θ and consequently the greatest relation C_L/C_D occurs from a tangent line to the drag polar curve from the origin of the system of coordinates (line 0.2). For any other position than that of the graph, the aerodynamic efficiency of the aircraft will be less.

From basic definitions of differential and integral calculus, it is possible defines an equation that gives the design lift coefficient, the corresponding drag coefficient and the maximum efficiency of the aircraft. Thus, from the analysis of Figure 4 observe that.

$$tg\theta_{máx} = \frac{C_L^*}{C_D} = E_{máx} \qquad (6a)$$

well, it is possible to write that.

$$\frac{1}{tg\theta_{máx}} = \frac{C_D}{C_L^*} = \frac{1}{E_{máx}} \qquad (6b)$$

or,

$$\frac{1}{tg\theta_{máx}} = \frac{C_{D0} + K \cdot C_L^{*2}}{C_L^*} = \frac{1}{E_{máx}} \qquad (6c)$$

As a way to get the maximum value of efficiency for the aircraft, the fundamental definition of differential and integral calculus says the first derivative of the function must be zero (problems of maximum and minimum), and thus the design lift coefficient C_L* can be obtained as follows.

$$\frac{C_{D0} + K \cdot C_L^{*2}}{C_L^*} \frac{d}{dC_L^*} = 0 \qquad (6d)$$

This equation reduces the system to a single point where the tangent of θ takes its peak value and therefore the aerodynamic efficiency of the aircraft will also be maximum, therefore, Equation (6d) has been write as follows,

$$C_L^{*-1} \cdot (C_{D0} + K \cdot C_L^{*2}) \frac{d}{dC_L^*} = 0 \qquad (6e)$$

$$C_L^{*-1} \cdot C_{D0} + K \cdot C_L^* \frac{d}{dC_L^*} = 0 \qquad (6f)$$

Making the derivative we have who,

$$-C_{D0} \cdot C_L^{*-2} + K = 0 \qquad (6g)$$

$$K = C_{D0} \cdot C_L^{*-2} \qquad (6h)$$

$$K = \frac{C_{D0}}{C_L^{*2}} \qquad (6i)$$

And so, the design lift coefficient that maximizes aerodynamic efficiency of the aircraft can be written as follows,

$$C_L^* = \sqrt{\frac{C_{D0}}{K}} \qquad (6j)$$

with corresponding data for drag coefficient,

$$C_D^* = C_{D0} + K \cdot C_L^{*2} \qquad (7)$$

And maximum aerodynamic efficiency of the aircraft calculated to the design point given by,

$$E_{máx} = \frac{C_L{}^*}{C_D{}^*} \quad (8)$$

During the analysis in this article was found that the parasite drag of the aircraft coincides with the minimum drag, which is the vertex of the parabola coincides with the value of C_{D0} to a condition of C_L = 0. But it is used for aircraft that have wings with symmetrical profile, in case of camber wings when the aircraft is in the angle of attack for zero lift α_L = 0, the parasite drag tends to be greater than the minimum drag of the α_L = 0. Thus, the drag polar characteristic takes a form similar to that shown in Figure 5 and Equation (9) is used to calculate the drag polar of the aircraft.

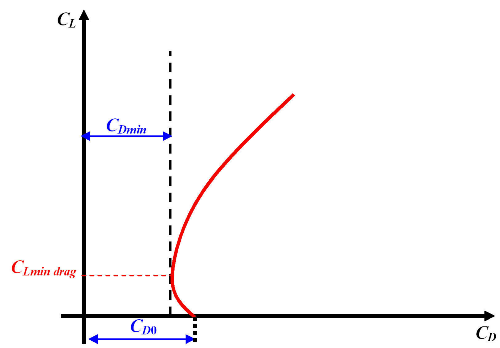


Figure 5 – camber profile drag polar.

$$C_D = C_{D\min} + K(C_L - C_{L\min drag})^2 \quad (9)$$

Usually in practice the difference between the values of C_{D0} and C_{Dmin} is very small and can be neglected during the calculations without cause significant interference in the performance of the aircraft.

3. Example of application of the presented model

Consider a new aircraft in order to participate in the SAE-AeroDesign competition with the following geometric and aerodynamic characteristics:

Trapezoidal wing plantform;
c_r = 0,4 m;
c_t = 0,2 m;
S = 0,75 m²;
b = 2,5 m;
$C_{Lmáx}$ = 2,0;
C_{D0} = 0,045.

Depending on the parameters presented, it is possible plot drag polar curve of the aircraft using the model proposed in this article as follows:

The first point to be determined is the aspect ratio of the wing that can be obtained from the solution of the equation presented below, thus:

$$AR = \frac{b^2}{S}$$

$$AR = \frac{2{,}5^2}{0{,}75}$$

$$AR = 8{,}33$$

The taper ratio which is the ratio between the tip cord and root cord of the wing is obtained as follows:

$$\lambda = \frac{c_t}{c_r}$$

$$\lambda = \frac{0{,}2}{0{,}4}$$

$$\lambda = 0{,}5$$

Known the values of aspect ratio and the taper ratio is possible to determine the induced drag efficiency factor and the wingspan efficiency factor from the chart proposed by Anderson (1999) and represented in the figure below.

The analysis of the graph of Figure 6 shows that δ= 0018.

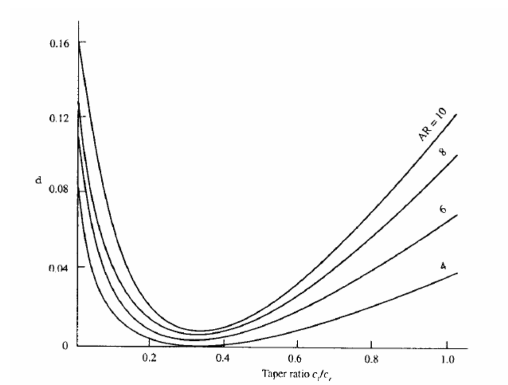


Figure 6 – induced drag factor.

The wingspan efficiency factor is calculated after the second methodology proposed by McCormick (1995).

$$e = \frac{1}{1+\delta}$$

$$e = \frac{1}{1+0{,}018}$$

$$e = 0{,}982$$

For the solution of the Equation (5), comes to the value of the Oswald coefficient to the aircraft.

$$e = 0{,}75 \cdot e$$

$$e = 0{,}75 \cdot 0{,}982$$

$$e = 0{,}736$$

For the solution of the equation (4), comes to the value of K.

$$K = \frac{1}{\pi \cdot e_0 \cdot AR}$$

$$K = \frac{1}{\pi \cdot 0{,}736 \cdot 8{,}33}$$

$$K = 0{,}05194$$

So the equation that defines the drag polar of this aircraft can be written as follows.

$$C_D = 0{,}045 + 0{,}05194 \cdot C_L^{\ 2}$$

For the plot is necessary initially assemble a table of data with the C_L ranging from 0 to $C_{Lmáx}$. In the proposed problem, the table will be mounted considering an increase of 0,2 on the values of C_L, but it is important to mention that the larger the number of points assessed more accurately the curve will be obtained.

Before submitting the table resulting from the analysis will show that the calculation was done to obtain the first two points of the curve.

For $C_L = 0$

$$C_D = 0{,}045 + 0{,}05194 \cdot 0$$

$$C_D = 0{,}045$$

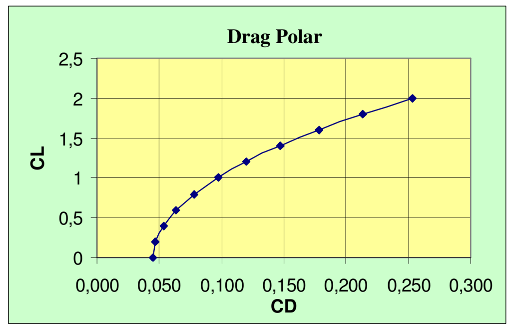
For $C_L = 0{,}2$

$$C_D = 0{,}045 + 0{,}05194 \cdot 0{,}2^2$$

$$C_D = 0{,}047$$

This procedure must be repeated for each point to be evaluated during construction of the graph. The results from the analysis is presented below along with their graph of the drag polar of the aircraft.

C_L	C_D
0	0,045
0,2	0,047
0,4	0,053
0,6	0,064
0,8	0,078
1	0,097
1,2	0,120
1,4	0,147
1,6	0,178
1,8	0,213
2	0,253



The design lift coefficient is obtained by calculating the equation (6j).

$$C_L^{\ *} = \sqrt{\frac{C_{D0}}{K}}$$

$$C_L^{\ *} = \sqrt{\frac{0{,}045}{0{,}05194}}$$

$$C_L^{\ *} = 0{,}93$$

The corresponding drag coefficient is.

$$C_D^{\ *} = C_{D0} + K \cdot C_L^{\ 2}$$

$$C_D^{\ *} = 0{,}045 + 0{,}05194 \cdot 0{,}93^2$$

$$C_D^{\ *} = 0{,}089$$

And finally the maximum efficiency of the aircraft is given by.

$$E_{máx} = \frac{C_L^{\ *}}{C_D^{\ *}}$$

$$E_{máx} = \frac{0{,}93}{0{,}089}$$

$$E_{máx} = 10,44$$

This result indicates that this condition for flight, the aircraft is able to generate 10,44 more lift than drag.

This example has shown clearly and objectively as the drag polar estimate of an aircraft, but other methods can be found in the aeronautic literature. It is important to mention that the model presented is valid only for disposal subsonic flight and that the results are very satisfactory for the proposed SAE-AeroDesign competition.

4.Conclusions

As this article was presented an analytical model for the determination of drag polar of a light aircraft operating under subsonic flight conditions to participate in the SAE-AeroDesign competition. The main conclusions of the model presented can include:

a) The model shown possible to estimate with precision the equation and good curve that represents the drag polar of the aircraft under study.

b) The method presented can be applied to the aerodynamic design and performance of all aircraft operating under conditions of subsonic flight regime.

c) the determination of the drag polar allows the designer establish important performance characteristics of the aircraft from the use of the point of maximum aerodynamic efficiency.

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