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INDUCED DRAG OF MULTIPLANES.

By L. Prandtl

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Summary.

The most important part of the resistance or drag of a wing system, the induced drag, can be calculated theoretically, when the distribution of lift on the individual wings is known. The calculation is based upon the assumption that the lift on the wings is distributed along the wing in proportion to the ordinates of a semi-ellipse (Fig. 1). Formulas and numerical tables are given for calculating the drag. In this connection, the most favorable arrangements of biplanes and triplanes are discussed and the results are further elucidated by means of numerical examples.

Notation.

$V$  = velocity of flight in meters per second;  
 $v$  = horizontal velocity of disturbance  
 $w$  = vertical velocity of disturbance } produced by the wings;  
 $L_1$  = lift of wing 1;  
 $D_{11}$  = self-induced drag of wing 1;  
 $D_{12}$  = additional drag induced by wing 1 on wing 2;  
 $\sigma$  = coefficient of  $D_{12}$ ;

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\* Technische Berichte, Volume III, No.7, pp. 309-315.

$k$  = coefficient of total drag  $D$ ;

$r$  =  $b_2/b_1$  = ratio of spans.

### 1. Introduction.

During experiments with extra large wing models in the Göttingen wind tunnel, it was found that, with an increase in the characteristic number,\* the wing-section drag\*\* became an ever smaller part of the total drag. In the case of very large characteristic numbers, it consisted only of frictional drag, provided a good wing section was experimented with and that the angle of attack came within the range to which the particular wing section was adapted.

From the results thus established, a theoretical calculation of the induced drag of multiplanes attains a high practical value, for, since the frictional drag can be approximately estimated,\*\*\* it is also possible to calculate the total drag by computing the induced drag. It follows that the total drag, conformably with the properties of induced drag, depends only upon the outer contour dimensions of the wing system and upon the distribution of the lift on its various parts. It also follows that the wing-section is of importance only in so far as it must be suitably selected for the purpose in view.

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\* Characteristic number = product of wing chord and velocity (Compare Technische Berichte, Vol. I, No.4, p.85).

\*\* Wing-section drag in monoplanes = difference between measured drag and "induced drag".  $\mathbb{W} = \frac{A_2}{\pi a b^2}$  (Compare Technische Berichte, Vol.I, No.5, p.145). The "induced drag" is there called "Randwiderstand" (wing-edge drag).

\*\*\* Systematic experiments on frictional drag are in course of

A method for calculating the induced drag of biplanes and multiplanes has already been proposed. It is contained in two somewhat recondite articles by A. Betz (Technische Berichte, Vol. I, No.4, pp.98 and 103). The method consists in first calculating the drag of the monoplane, from which the multiplane is made up, from  $A \tan \phi$  (Technische Berichte, Vol. I, No.4, p.101).

To this is then added the additional drag due to the mutual influence of the wings. This is given by the equation

$W_{12} = A_0 \alpha'_0 = A_U \alpha'_U$  (Technische Berichte, Vol. I, No.4, pp.105-107). The total induced drag of a system of  $n$  wings is, therefore, the sum of  $n^2$  such individual drags.

This method, however, shows an inconsistency, of the nature that the monoplane drag is calculated on the assumption that the lift is distributed along the wing span in proportion to the ordinates of a semi-ellipse (Fig. 1), while, as regards the interference, the same lift is assumed to be uniformly distributed over the whole span. Since the actual distribution of the lift comes nearer the elliptical than the rectangular distribution, it might be expected that the assumption of elliptical distribution in both cases would furnish the basis for a theory which would not only be more consistent, but would also accord more closely with the actual conditions.

Theoretical investigation is more complicated throughout, with the assumption of elliptical distribution of the lift, than with the method of Betz. The problems which Betz worked out for

the case of biplanes, have not all been solved for elliptical distribution. As yet, only the problem of ascertaining the total induced drag of the wing system has been solved, but this has been accomplished in a very satisfactory manner.

The computations are only summarized here, in so far as they conduce to a full understanding of the results. The principal results are reproduced, moreover, in practical form. In addition to tables and curves, approximate formulas are given which will be found useful in computations.\*

### 3. Drag Formulas.

We will first state two general laws discovered by Munk, which will be of great assistance in what follows.

1. Any system, as regards its total induced drag, is equivalent to a simpler system having the same front view, in which the centers of pressure of all the constituent wing surfaces, while maintaining the same lift distribution, are shifted into one and the same plane, at right angles to the direction of flight (unstaggered wing system).

2. In an unstaggered wing system, the drag  $D_{12}$ , induced by wing 1 on wing 2, equals the drag  $D_{21}$  induced by wing 2 on wing 1.

The drag  $D_{12}$  is due to the fact that wing 1 produces a

\* Mr. Pohlhausen gave me valuable assistance in the calculations and diagrams and Messrs. Munk and Grammel contributed some important details.

downward air current toward wing 2, the effect being that the resultant air pressure on wing 2 is inclined rearward at an angle  $\epsilon$ , thus producing a new component,  $L_2 \sin \epsilon$ , of the drag. Here,  $\tan \epsilon = w_{12}/V$ , in which  $V$  is the velocity of flight and  $w_{12}$  is the downward velocity produced by wing 1. Since only small angles are considered here,  $\sin \epsilon$ , and  $\tan \epsilon$  may be interchanged. The velocity  $w_{12}$  is not actually uniform throughout the span and we must, therefore, write

$$D_{12} = \int \frac{w_{12}^2}{V} dL_2 \quad (1)$$

In order to evaluate  $w_{12}$ , we make use of the condition proposed by Munk, that the flow below and above a monoplane with flow around a plate moved at right angles to its plane.\* From this flow we evaluate (Fig. 2) the vertical velocities for a series of suitable distances from the plate. The result, for which I have to thank Mr. K. Pohlhausen, is shown in Fig. 3. The span of the wings is here taken as 2 units and the velocity  $w$ , at the wing itself, as 1 unit. The actual velocities are, therefore,

$$w_{12} = \frac{2L_1}{\pi V b_1^2} z \quad (2)$$

where  $z$  is taken from Fig. 3 for the ratio  $G/b_1$  coming under consideration ( $G$  = the vertical distance between the two unstaggered wings,  $b_1$  = the span of the first wing).

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\* Compare Lamb's Textbook of Hydrodynamics, pp. 100 and 101 of the German translation ("Lehrbuch der Hydrodynamik von Lamb") by Friedel.

The integral must now be formed according to equation (1), on the assumption that the lift  $L_2$  is distributed elliptically along the span  $b_2$ . This integral has been evaluated by planimetry of the curves obtained for  $r = b_2/b_1 = 1.0, 0.8$  and  $0.6$ , for different values of  $G/b_1$ . The results are shown in Figs. 4 and 5, wherein is plotted the unnamed quantity  $\sigma$  which is expressed by the equation\*

$$D_{12} = D_{21} = \frac{\sigma}{\pi q} \frac{L_1}{b_1} \frac{L_2}{b_2} \quad (3)$$

In Fig. 4, the ratio between the gap  $G$  of the biplane and the arithmetical mean of the spans,  $b_1$  and  $b_2$ , is taken as the abscissa, but in Fig. 5, the ratio  $\frac{b_1 - b_2}{b_1 + b_2} = \frac{1 - r}{1 + r}$  is employed. In Fig. 4, the curves are plotted for  $r = \text{constant}$  and in Fig. 5 for  $G/\frac{b_1 + b_2}{2} = \text{constant}$ . Since  $D_{12} = D_{21}$ , we obtain the same values of  $\sigma$  for  $r = \frac{b_2}{b_1} > 1$ , as for  $r' = \frac{1}{r}$ . Table 1 contains the values of  $\sigma$  taken from a diagram similar to Fig. 4.

Table I. Values of  $\sigma$

$G/\frac{b_1 + b_2}{2}$	0	0.05	0.1	0.15	0.2
$r = 1.0$	1.000	0.780	0.655	0.561	0.485
0.8	0.800	0.690	0.600	0.523	0.459
0.6	0.600	0.540	0.485	0.437	0.394

\* This formula is constructed in a similar manner to the formula for self-induced drag  $D_{11} = \frac{L_1^2}{\pi q b_1^2}$  into which it passes when  $L_1 = L_2$ ,  $b_1 = b_2$  and  $G = 0$ , whereby  $\sigma$  equals 1.

Table I. Values of  $\sigma$  (Cont.)

$G/\frac{b_1+b_2}{2}$	0.25	0.3	0.35	0.4	0.45	0.5
$r = 1.0$	0.420	0.370	0.327	0.290	0.258	0.230
0.8	0.401	0.355	0.315	0.282	0.252	0.225
0.6	0.351	0.315	0.285	0.255	0.231	0.210

The values of  $\sigma$  in the most important case, where  $r = 1$  and, therefore,  $b_1 = b_2 = b$ , are represented (with good approximation between  $G/b = 1/15$  and  $G/b = 1/4$ ) by

$$\sigma_1 \approx \frac{1}{1 + 5.3 G/b} \quad (4)$$

More exact is the approximation formula

$$\sigma_1 \approx \frac{1 - 0.66 G/b}{1.05 + 3.7 G/b} \quad (5)$$

which obtains between  $G/b = 1/15$  and  $G/b = 1/2$ . The approximate formula for  $r \geq 1$  is less simply constructed. We first calculate the value of  $\sigma_1$ , corresponding to  $b_m = \frac{b_1 + b_2}{2}$  and, further, the auxiliary quantities  $0.8 \times \sigma_1 (1 - \sigma_1) - 0.1 = s$  and  $\frac{0.56}{\sigma_1 + s - 0.22} = t$  and, if (for the sake of brevity) we assume  $\frac{b_1 - b_2}{b_1 + b_2} = \frac{1 - r}{1 + r} = \tau$ , we then have

$$\sigma = \sigma_1 + s - \sqrt{s^2 + (\tau/t)^2} \quad (6)$$

Numerical Example.- Let a biplane have an upper-wing span  $b_1 = 12$  m (39.37 ft.) and a lower-wing span  $b_2 = 10$  m (32.8 ft.) and let the gap  $G = 2$  m (6.56 ft.) to calculate the coefficient of mutual influence  $\sigma$  for the drag  $D_{12}$ .



We first calculate  $\sigma_1$ , for the mean span  
 $b_m = (b_1 + b_2)/2 = 11.0 \text{ m (36.09 ft.)}$ . Then  $G/b_m = 2/11 = 0.1818$ .  
According to equation (4), we now obtain

$$\sigma_1 = \frac{1}{1 + 5.3 \times 0.1818} = 0.509$$

If, for the sake of comparison, we make the calculation from  
equation (5), we get

$$\frac{1 - 0.66 \times 0.1818}{1.055 + 3.7 \times 0.1818} = 0.509$$

Lastly, by interpolation from Fig. 3, we get  $\sigma_1 = 0.511$ .  
The agreement is, therefore, quite satisfactory.

Taking  $\sigma_1 = 0.511$ , we obtain the auxiliary values:

$$s = 0.8 \times 0.511 \times 0.489 - 0.1 = 0.100$$

$$t = \frac{0.56}{0.511 + 0.100 - 0.22} = 1.432$$

$$r = b_2/b_1 = 10/12 = 0.833$$

$$\tau = \frac{1 - r}{1 + r} = 0.0909$$

$$\text{Therefore } \tau/t = \frac{0.0909}{1.432} = 0.0635.$$

$$\begin{aligned} \text{Whence } \sigma &= 0.511 + 0.1 - \sqrt{0.1^2 + 0.0635^2} = \\ &= 0.611 - 0.1185 = 0.4925 \end{aligned}$$

Interpolation in Figure 5 gives  $\sigma = 0.490$ .

### 3. The Biplane.

The induced drag of the upper wing, for the unstaggered biplane, is

$$D_1 = D_{11} + D_{12} = \frac{1}{\pi q} \left( \frac{L_1^2}{b_1^2} + \sigma \frac{L_1 L_2}{b_1 b_2} \right)$$

and that of the lower wing is

$$D_2 = D_{21} + D_{22} = \frac{1}{\pi q} \left( \sigma \frac{L_1 L_2}{b_1 b_2} + \frac{L_2^2}{b_2^2} \right)$$

Where there is a positive stagger, as is generally the case, the drag of the upper wing is diminished by the upward air currents produced by the lower wing; but, on the other hand, the drag of the lower wing is increased, to exactly the same extent, by the downward air current produced by the upper wing, so that the total drag is the same as in the case of an unstaggered biplane and

$$D = D_1 + D_2 = \frac{1}{\pi q} \left( \frac{L_1^2}{b_1^2} + 2 \sigma \frac{L_1 L_2}{b_1 b_2} + \frac{L_2^2}{b_2^2} \right) \quad (7)$$

With the given values of the total lift,  $L$ , and of  $b_1$ ,  $b_2$ , and  $G$ , the question naturally arises as to how the lift must be distributed on the two wings so that the total drag will be the same as that of an unstaggered biplane.

For this purpose, let  $L_2 = Lx$ , or  $L_1 = L(1 - x)$  and

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\* The approximate formula (given in Technische Berichte, Volume II, No. 2, p.275) for the induced drag, based on rather uncertain analogies, does not satisfactorily stand the test by the more exact formula (7). Its agreement with the measurements of Munk seems to point to inaccuracies in these measurements, which were made in the old wind tunnel.

let us seek a value of  $x$  for which the expression in brackets in equation (7) is a minimum. Taking  $b_2/b_1 = r$ , a very simple calculation gives

$$x = \frac{r - \sigma}{r + \frac{1}{r} - 2\sigma} \quad (8)$$

If this value of  $x$  is put in equation (7), we have, for the minimum value of the induced drag,

$$D_{\min} = \frac{L^2}{\pi q b_1^2} \frac{1 - \sigma^2}{r \left( r + \frac{1}{r} - 2\sigma \right)} \quad (9)$$

In the special case when  $b_1 = b_2$  and, therefore,  $r = 1$ , the formulas become simpler. As is easily seen,  $x = \frac{1}{2}$ , or, in other words, the lift is equally divided between the two wings\*.

We also have

$$D_{\min} = \frac{L^2}{\pi q b^2} \frac{1 + \sigma}{2} \quad (9a)$$

$\frac{L^2}{\pi q b_1^2}$  is the induced drag,  $D_I$ , of a monoplane with a span  $b_1$  which gives the same lift as the biplane under consideration. The factor following this expression in equations (9) and (9a) thus gives the ratio  $D/D_I = \kappa$ \*\*. In Figure 6 the course of  $\kappa$  is plotted against  $G/b_1$  and  $r = b_2/b_1$ .

\* These relations are not quite exact, since the influence of the component of the disturbed flow,  $v$ , parallel to  $V$ , has been neglected for simplicity. With more precise computation, it appears that it is not the lift, but the circulations of the two wings which must be equal, in order to obtain the minimum drag. The lifts are then in the ratio  $V + v$  to  $V - v$ . The effect of this correction on the magnitude of the drag, however, vanishes for all practical purposes.

\*\*The quantity  $\kappa$ , introduced by Munk (Technische Berichte, Volume II, No. 2, p.187) is equivalent to  $1/\sqrt{\kappa}$

It is seen that all biplanes have less drag than the equivalent monoplane and that their minimum drag is obtained when  $r = 1$ , that is, when the upper and lower wings have the same span. It is also seen that, with the same span, the drag decreases as the gap increases.

The result must not, however, be misunderstood. It does not mean that the biplane is once and for all superior to the monoplane. The analysis merely states (apart from the fact that it enables the drag to be calculated in each particular case) that, among monoplanes and biplanes having the same maximum span and the same total load, the biplane, both of whose wings have the given maximum span, is superior to other arrangements. It is only necessary to compare a monoplane with the same load as a given biplane and with a span  $1/\sqrt{k}$  times greater than that of the biplane, in order to be convinced that both have the same total drag. In the same way, it is seen that a biplane with wing spans of 12 m (39.37 ft.) and 10 m (32.8 ft.) is a little superior to one with two wings of 11 m (36.09 ft.) span. Figure 6 and the corresponding Table II, give information on all these relations with a very little calculation.

If the span of the lower wing is taken as smaller than that of the upper wing, then the portion of the lift that must be assigned to the lower wing, in order to produce the minimum drag, is smaller than in proportion to the spans. If we adopt equal loading on both wings (which would seem to be most desirable),

then the lower wing will have a smaller chord than the upper wing. The ratio  $x$ , which gives the fraction of the total lift assigned to the smaller wing, is shown in Figure 6 by the dotted lines and may also be taken from Table II.

Numerical Example.- A biplane, of the dimensions given in the previous numerical example, is expected to fly with a load of 1500 kg (3307 lb.) and a velocity  $V = 40$  m/sec. (131.23 ft./sec.) at an altitude of 6 km (19685 ft.). Let the density be  $\rho = \frac{\gamma}{g} = 0.065$  and the head pressure  $q = \frac{\rho V^2}{2} = 52$  kg/m<sup>2</sup> (10.65 lb./ft<sup>2</sup>). What are the fractions of the total lift on the upper and lower wings for the best distribution of lift between them and what is the induced drag for this particular distribution of lift?

From the former calculation,  $r = 0.833$  and  $\sigma = 0.490$ . Hence, from equation (8),

$$x = \frac{0.833 - 0.490}{0.833 + 1.200 - 0.980} = 0.326$$

The lift of the lower wing ( $Lx$ ) becomes  $1500 \times 0.326 = 490$  kg (1080.3 lb.) and that on the upper wing,  $1500 - 490 = 1010$  kg (2237 lb.). If we then assume a load of  $37.5$  kg/m<sup>2</sup> (7.68 lb./ft.<sup>2</sup>), the total area of the wings will be  $40$  m<sup>2</sup> (430.6 ft.<sup>2</sup>), of which  $27$  m<sup>2</sup> (290.6 ft.<sup>2</sup>) falls to the upper wing and  $13$  m<sup>2</sup> (140 ft.<sup>2</sup>) to the lower wing. Hence, for the given spans, the wing chords are  $2.35$  m (7.38 ft.) and  $1.30$  m (4.27 ft.),

respectively.\*

\* If we consider the horizontal component  $v$  of the disturbed flow in the same way as Betz (Technische Berichte, Vol. I, No.4, p.107), then the chord of the upper wing is somewhat less and that of the lower wing correspondingly greater. This correction is, however, reduced again by the influence mentioned in the footnote on p.10.

The coefficient of equation (9) now becomes

$$\kappa = \frac{1 - 0.490^2}{0.833 (0.833 + 1.20 - 0.980)} = 0.865$$

and hence

$$D = \kappa \frac{L^2}{\pi q b_1^2} = \frac{0.865 \times 1500^2}{3.14 \times 52 \times 144} = 82.7 \text{ kg (182.3 lb.)}$$

Table II.

Values of  $\kappa = D/D_I$  for the Biplane.

G/b <sub>1</sub>	0	0.05	0.1	0.15	0.2
r = 0.6	1.000	0.990	0.974	0.954	0.932
0.7	1.000	0.982	0.956	0.926	0.897
0.8	1.000	0.974	0.932	0.892	0.855
0.9	1.000	0.950	0.893	0.847	0.807
1.0	1.000	0.890	0.827	0.779	0.742

G/b <sub>1</sub>	0.25	0.3	0.35	0.4	0.45	0.5
r = 0.6	0.911	0.892	0.875	0.861	0.848	0.839
0.7	0.871	0.849	0.830	0.812	0.797	0.793
0.8	0.825	0.800	0.778	0.758	0.740	0.728
0.9	0.773	0.744	0.719	0.699	0.683	0.671
1.0	0.710	0.684	0.662	0.645	0.629	0.615

Values of  $\kappa = \frac{L_2}{L_1 + L_2}$  for the Biplane

G/b <sub>1</sub>	0	0.05	0.1	0.15	0.2
r = 0.6	0	0.060	0.104	0.134	0.157
0.7	0	0.105	0.164	0.202	0.228
0.8	0	0.172	0.246	0.285	0.310
0.9	0	0.303	0.359	0.387	0.402
1.0	0/0	0.500	0.500	0.500	0.500

Table II (Cont.)

Values of  $x = \frac{L}{L_1 + L_2}$  for the Biplane.

G/b <sub>1</sub>	0.25	0.3	0.35	0.4	0.45	0.5
r = 0.6	0.176	0.191	0.202	0.211	0.218	0.224
0.7	0.248	0.262	0.272	0.281	0.288	0.294
0.8	0.327	0.338	0.347	0.355	0.361	0.364
0.9	0.412	0.419	0.425	0.429	0.431	0.433
1.0	0.500	0.500	0.500	0.500	0.500	0.500

In like manner, in the case of the biplane with two wings of 11 m (36.09 ft.) span ( $\sigma = 0.511$ ),  $\kappa = 0.755$  and

$$D = \frac{0.755 \times 1500^2}{3.14 \times 52 \times 121} = 86.0 \text{ kg (189.6 lb.)}$$

Note: The frictional drag may be taken as 0.008 qS. In the above example, for  $S = 40 \text{ m}^2$  (430.6 ft.<sup>2</sup>) this gives 16.6 kg (36.6 lb.) which, together with the induced drag, represents the total drag on the wing.

#### 4. The Triplane.

The triplane may be treated in the same way as the biplane. In order to avoid complicating the problem unnecessarily, let it be assumed that all three wings have the same span and that the gap between the upper and middle wings is the same as between the middle and lower wings. Under these conditions, from the results obtained with the biplane it may be assumed at once that the upper and lower wings have the same lift for the minimum drag (See footnote on p.10). The lift of the middle wing, however, is different.

Putting  $L_2 = L x$ , then  $L_1 = L_3 = L (1 - x)/2$ , since the sum of the lifts must equal  $L$ .

For the coefficient of mutual influence,  $\sigma$  we need to distinguish between the adjacent wings, which are  $\frac{G}{2}$  apart, and the top and bottom wings whose distance apart is  $G$ . Let the corresponding coefficients be denoted by  $\sigma_1$  and  $\sigma_2$ . The induced drags of the individual wings are accordingly:

$$D_1 = \frac{1}{\pi q b^2} (L_1^2 + \sigma_1 L_1 L_2 + \sigma_2 L_1 L_3)$$

$$D_2 = \frac{1}{\pi q b^2} [(L_2^2 + \sigma_1 (L_1 L_2 + L_2 L_3))]$$

$$D_3 = \frac{1}{\pi q b^2} (L_3^2 + \sigma_1 L_2 L_3 + \sigma_2 L_1 L_3).$$

If, in the above manner,  $L_1$ ,  $L_2$  and  $L_3$  are expressed in terms of  $L$  and  $x$ , we have

$$D = \frac{L^2}{3\pi q b^2} \left\{ 1 + \frac{\sigma_2 - 2x(1 + \sigma_2 - 2\sigma_1) +}{x^2(3 + \sigma_2 - 4\sigma_1)} \right\} \quad (10)$$

This will be a minimum, when

$$x = \frac{1 + \sigma_2 - 2\sigma_1}{3 + \sigma_2 - 4\sigma_1} \quad (11)$$

The values of  $x$  for different ratios  $G/b$  can be seen from the broken line in Fig. 7. The value of  $x$  is always less than  $1/3$  and the middle wing should, therefore, always have a smaller load than either the top or bottom wing, in order to obtain the minimum drag. The error due to the assumption of equal distribution of the lift between the three wings ( $x = 1/3$ ) is,



however, small, as shown by Figure 7 and Table III.

Table III.

Best Subdivision of Lift for a Triplane -

Values of  $x = \frac{L_2}{(L_1 + L_2 + L_3)}$ .

$G/b_1$	0.0	0.05	0.1	0.15	0.2
$x =$	0	0.161	0.177	0.190	0.202
	Values of $\kappa = D/E_I$				
a) Biplane	1.000	0.890	0.827	0.779	0.742
b) Triplane with $x = 1/3$	1.000	0.889	0.824	0.774	0.732
c) Best triplane	1.000	0.885	0.819	0.767	0.724
d) Best wing system	1.000	0.865	0.787	0.728	0.678

$G/b_1$	0.25	0.3	0.35	0.4	0.45	0.5
$x =$	0.212	0.222	0.231	0.238	0.244	0.251
	Values of $\kappa = D/D_I$					
a) Biplane	0.710	0.684	0.662	0.645	0.629	0.615
b) Triplane with $x = 1/3$	0.695	0.663	0.637	0.612	0.591	0.571
c) Best triplane	0.687	0.656	0.630	0.607	0.585	0.565
d) Best wing system	0.637	0.601	0.572	0.545	0.521	0.500

Two additional curves have been plotted in Fig. 7 for the purpose of comparison, one for a biplane, the other for a wing system like Figure 8, that is, for a biplane closed laterally by panels and so arranged that the upper portion is subject to outward pressure and the lower portion to inward pressure. The induced drag of this wing system has been evaluated according to a method which

cannot here be gone into in detail.\* This wing system has the least induced drag of all wing systems of like span and the same total height (sum of the individual gaps). If we proceed from a triplane to a multiplane, while maintaining the over-all dimensions and inserting further supporting surfaces, the induced drag continues to decrease, the closer the wings are placed together, provided the required subdivision of the load is theoretically maintained, in which the extreme top and bottom wings carry more load than the intermediate wings. If we consider the limiting case of an infinite number of wings within the outside dimensions of  $b$  and  $G$ , we obtain, in the case of the multiplane, the same induced drag as for the wing system of Figure 8.

For the calculation of this drag, I am indebted to Messrs. Grammel and Pohlhausen. The results may be taken from Figure 7 and Table III. Approximately

$$D \approx \frac{1 + 0.45 \frac{G}{b}}{1.04 + 2.81 \frac{G}{b}} \quad (13)$$

Numerical Example.- It is desired to obtain the coefficient  $\kappa$  of induced drag for a triplane with a span of 10 m (32.81 ft) and a total height of 2.5 m (8.2 ft.) when the middle wing is exactly at the center of the total height. The mutual-influence coefficients, for  $G/b = 0.125$  and for  $G/b = 0.25$ , are found to be  $\sigma_1 = 0.606$  and  $\sigma_2 = 0.421$ . Hence (from equation (11))  $x = 0.21$  and (from equation (10))  $\kappa = \frac{1}{8}$  (the expression in

\* I have briefly indicated the line of argument in my lecture before the Hamburg meeting of the Wissenschaftliche Gesellschaft für Luftfahrt, and it will be included in the printed report of the lecture.

brackets) = 0.687. If, instead, we take  $x = \frac{1}{3}$ , we get  $\kappa = 0.695$ . The biplane which is derived from the triplane by removing the middle wing, while keeping the same distance between the two outer wings, gives  $\kappa = 0.710$ . The three values thus differ slightly. The biplane, whose gap is equal to the sum of the two gaps of the triplane and has  $G = 1.25$  m (4.1 ft.) will, on the contrary, have a much greater drag. In this case,  $\kappa = 0.803$ . The wing system of Figure 8 gives  $\kappa = 0.637$ .

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Figs.1 & 2.

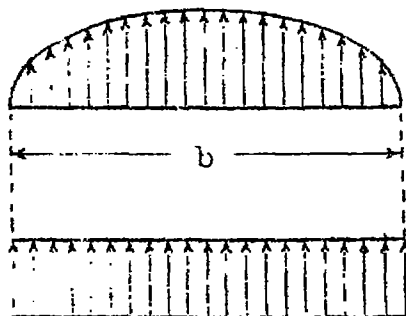


Fig.1. Elliptical and rectangular distribution of lift.

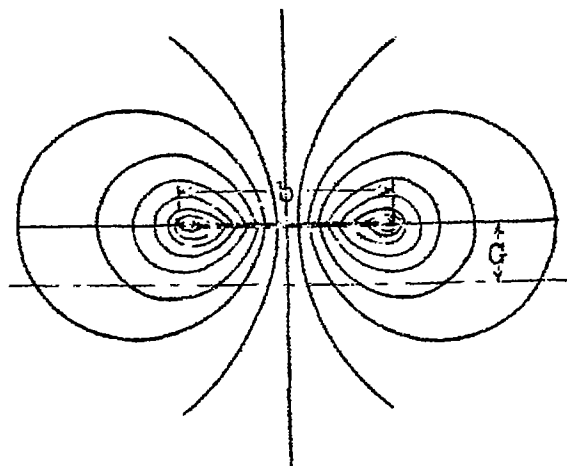


Fig.2. Disturbance of flow around a wing.

Figs.3 & 4.

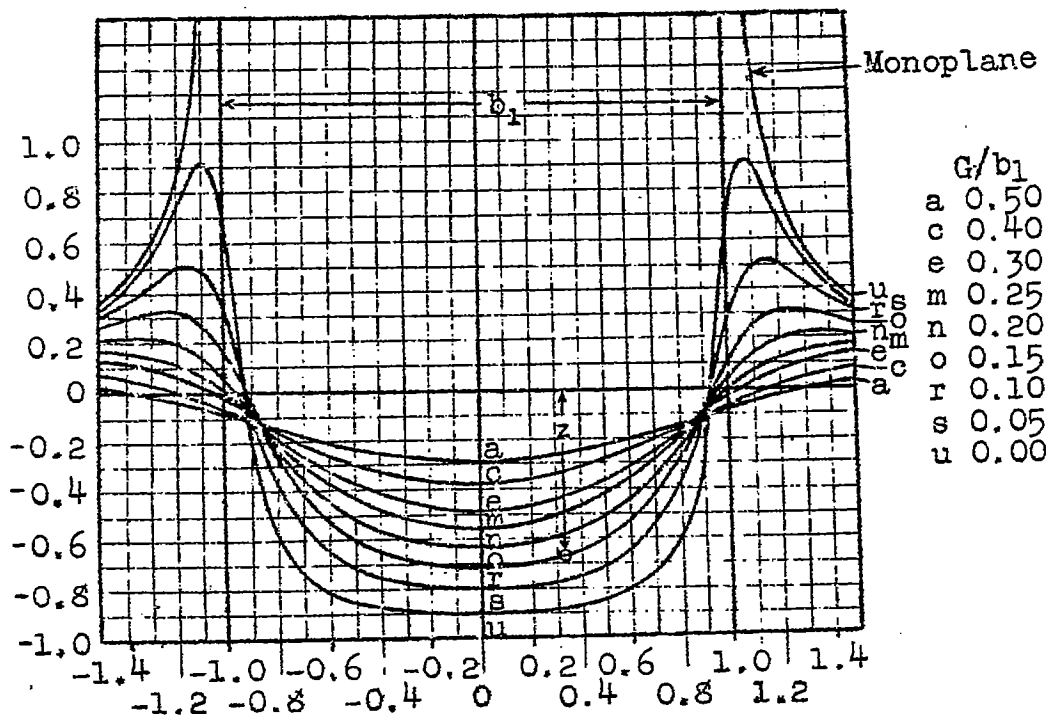


Fig.3. Vertical component of the disturbance velocity plotted against the lateral and the vertical distance from the wing.

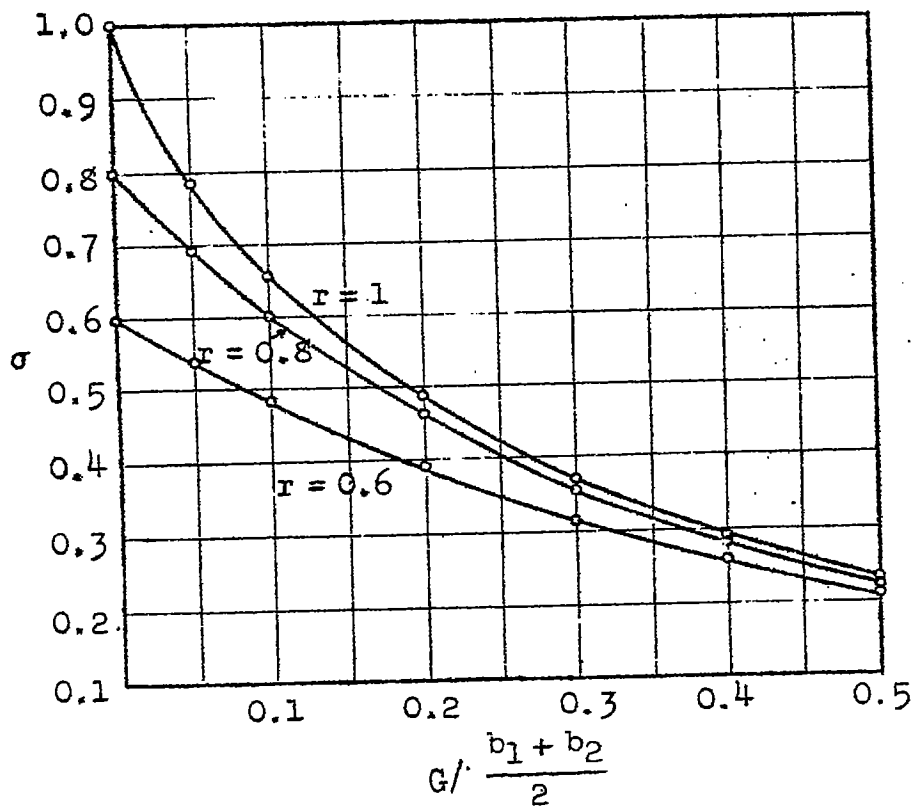


Fig.4. The coefficient of mutual influence  $\sigma$  plotted against  $G / \frac{b_1 + b_2}{2}$ .  $r = b_2 / b_1$

Figs.5 & 6.

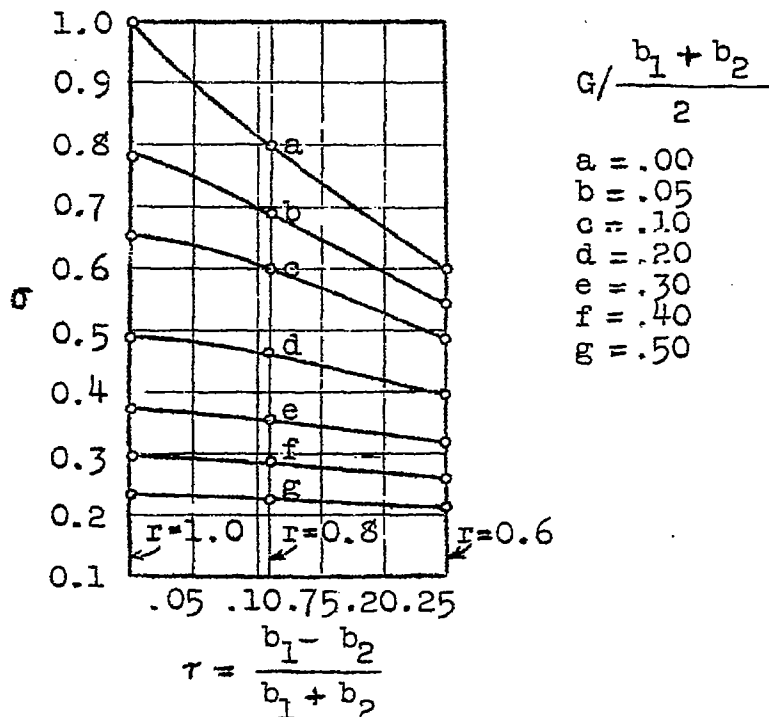


Fig.5. The coefficient of mutual influence,  $\sigma$ , plotted against  $\tau = \frac{b_1 - b_2}{b_1 + b_2}$ .

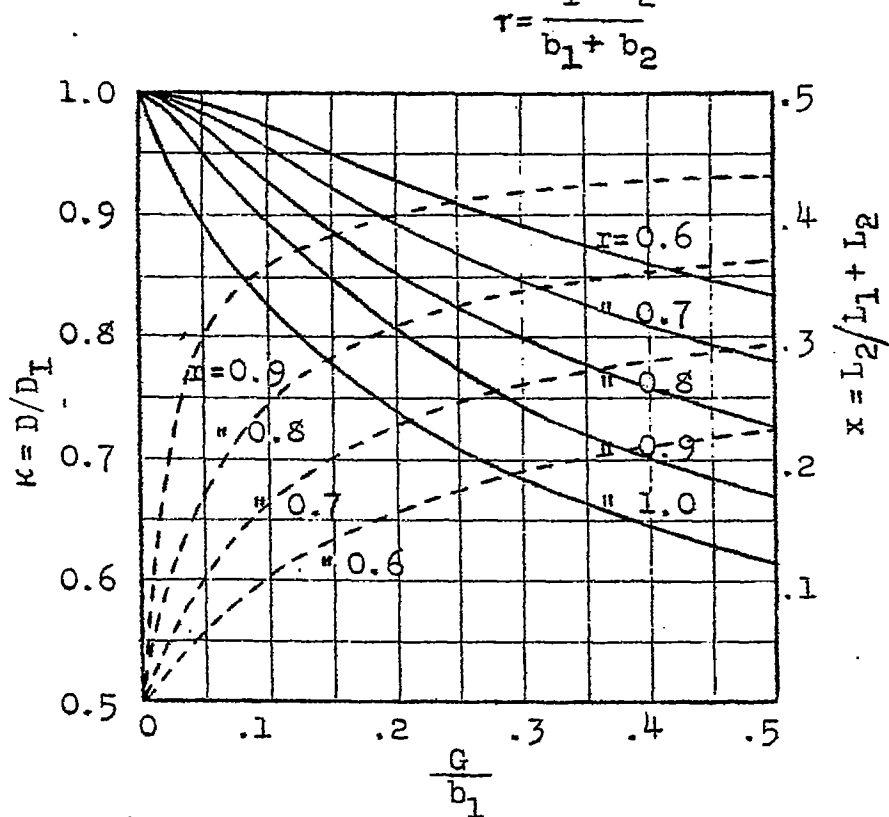


Fig.6. Distribution coefficient  $x$  and efficiency  $K$  of biplane plotted against  $G/b_1$ , with  $r = b_2/b_1$ .

Figs.7&8.

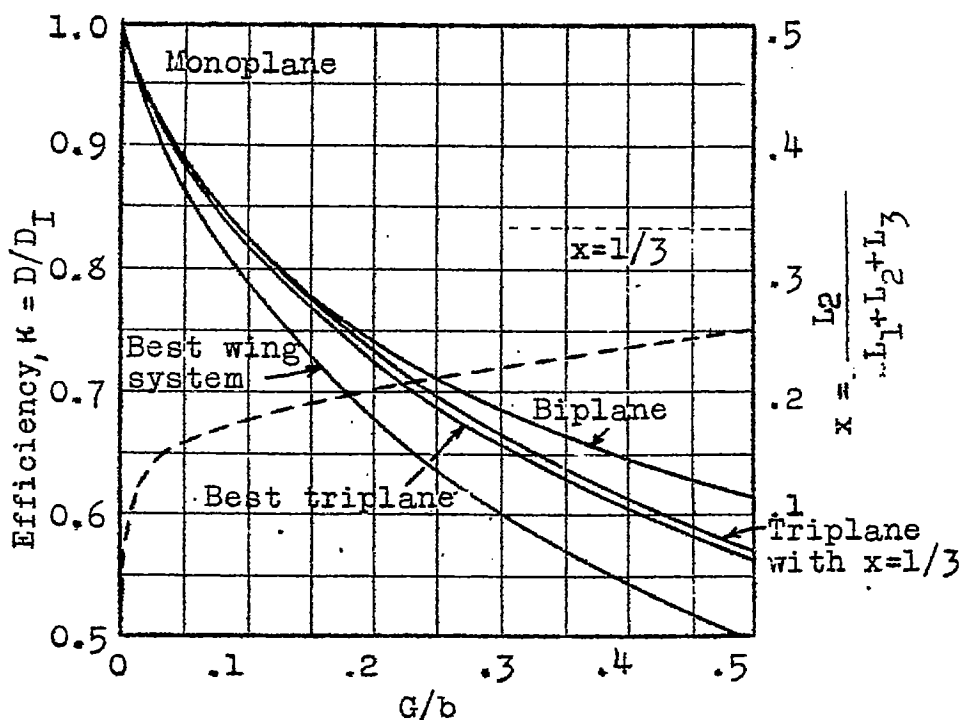


Fig.7. Efficiency  $\kappa$  of various wing systems and coefficient of distribution of wing loads  $x$ , for the triplane.

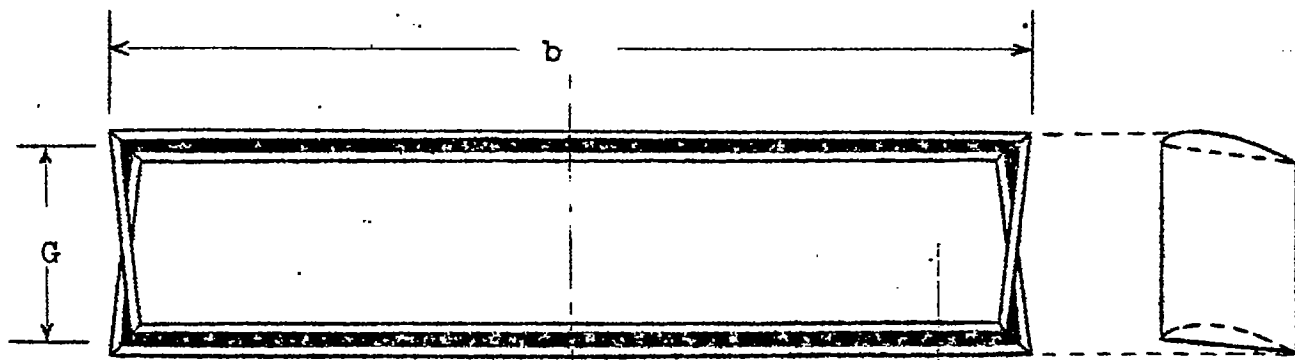


Fig.8. Best wing system.