TECHNICAL NOTES
NATIONAL ADVISORY COMMITTEE FOR AERONAUTICS

MEASURED MOMENTS OF INERTIA OF 32 AIRPLANE

By William Gracey
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By William Gracey

SUMMARY

A compilation of the experimentally determined moments of inertia of 32 airplanes is presented. The measurements were obtained at the laboratories of the NACA by means of a pendulum method. The airplanes tested are representative of several types of aircraft of gross weight less than 10,000 pounds.

The results are presented in coefficient as well as in dimensional form. An elementary analysis of the data disclosed the possibility of grouping the results according to wing type of the airplane, as low-wing monoplanes, parasol and high-wing monoplanes, and biplanes. The data are shown to provide a convenient means of rapidly estimating the moments of inertia of other airplanes. A three-view drawing of each of the 32 airplanes is included.

This note supersedes NACA Technical Note No. 375.

INTRODUCTION

The determination of the moments of inertia of airplanes by means of a pendulum method has been described in detail in reference 1. The precision of the results obtained by this method is within \( \pm 2.5 \) percent, \( \pm 1.3 \) percent, and \( \pm 0.8 \) percent, for the X, Y, and Z axes, respectively; whereas, the precision of estimates computed on the basis of a weight schedule has been shown to be about 10 percent (reference 2). The pendulum method has been in use at the laboratories of the NACA for the past 12 years, during which time measurements have been obtained on several types of airplane of gross weight less than 10,000 pounds. Because these measurements represent the most accurate data available, it appeared desirable to compile and publish all the data accumulated up to the present time.
The results of the tests of a few of the airplanes have already been published in an appendix of reference 1 but are included herein for completeness. These same data had also been published earlier in reference 3 but were slightly in error because the methods of correcting for the additional-mass effect had not been developed at that time. As the corrected values of reference 1 are presented herein, the information contained in the present paper should be considered as superseding that of reference 3. Since the publication of reference 3 the practice of determining the angle between the principal and the reference axes has been abandoned, this angle having been found to be less than $3^\circ$ for conventional airplanes.

METHOD AND APPARATUS

The measurements reported herein were obtained by the method described in reference 1. According to this procedure, the moments of inertia are determined about three reference axes, the origin of which is the center of gravity of the airplane. These axes are: the $X$ axis, parallel to the thrust line in the plane of symmetry; the $Y$ axis, perpendicular to the plane of symmetry; and the $Z$ axis, perpendicular to the thrust line in the plane of symmetry.

The moments of inertia about the $X$ and $Y$ axes are found by swinging the airplane as a compound pendulum, whereas the moment of inertia about the $Z$ axis is determined by oscillating the airplane as a bifilar-torsional pendulum. In each case the true moments of inertia are determined by correcting the measured moments of inertia for (1) the buoyancy of the structure, (2) the air entrapped within the structure, and (3) the additional-mass effect. The apparent additional moment of inertia about each axis is evaluated on a basis of (1) the size and the shape of the airplane normal to the direction of motion and (2) the results of tests of the additional-mass effect of flat plates (reference 1).

The airplanes tested are listed in Table I. Most of the airplanes are representative of several types of military aircraft, both Army and Navy. A few commercial and experimental types are also included. With the exception of the twin-engine OA-4A, all of the airplanes tested were of the single-engine type and, except for the Hummon Y-1,
the airplanes were all of the tractor type. All of the airplanes except the amphibian OA-4A were landplanes.

In general, the airplanes were tested for the normal full-load condition. In all cases the gas and the oil tanks were filled. As a rule, the pilot and each passenger of the airplane was represented by 175 to 200 pounds of ballast. In some cases, however, only the pilot was so represented and, in other cases, no ballast at all was added. For this reason both the weight of the airplane as tested and the weight of the airplane minus the ballast for the pilot and the passengers will be noted.

The airplanes with fixed landing gear were usually tested with the landing gear in flying position, that is, with the oleo extended. For an airplane with a retractable landing gear, tests were conducted with the landing gear either retracted or extended (with the oleo extended). In some few instances, the wheels were fixed in the taxying condition, that is, with the oleo compressed.

RESULTS

The results of the tests on the various airplanes are summarized in table I. The data presented include the true moments of inertia of the airplane and the additional moments of inertia about the reference axes. The true moments of inertia are based on the weight of the airplane as tested.

The data are also presented as radii of gyration and in coefficient form. The radii of gyration are computed from the true moments of inertia from the expressions:

\[ k_X = \sqrt{\frac{A}{W/s}} \]
\[ k_Y = \sqrt{\frac{B}{W/s}} \]
\[ k_Z = \sqrt{\frac{C}{W/s}} \]

where

\( k_X, k_Y, k_Z \) radii of gyration about X, Y, and Z axes, respectively
A, B, C the true moments of inertia about the
X, Y, and Z axes, respectively

W weight of the airplane as tested

Nondimensional coefficients, useful for comparing the mo-
ments of inertia of airplanes whose size and weight vary
considerably, are expressed in terms of the wing span,
b, and are calculated from the expressions:

\[ C_X = \frac{k_X}{b} \]

\[ C_Y = \frac{k_Y}{b} \]

\[ C_Z = \frac{k_Z}{b} \]

where \( C_X, C_Y, \) and \( C_Z \) are the coefficients for the mo-
ments of inertia about the \( X, Y, \) and \( Z \) axes, respectively. For convenience all the coefficients are
expressed in terms of the wing span even though the over-
all length of the airplane may be the more rational param-
eter for \( C_Y \).

In order to facilitate the comparison of data from
similar airplanes, the results in table I are arranged in
three groups according to wing type, namely, low-wing
monoplanes, parasol and high-wing monoplanes, and biplanes.
This grouping permits a graphical presentation of certain
portions of the data. Thus, for any one of the groups,
the radii of gyration about the \( X \) and the \( Z \) axes may
be plotted as functions of the wing span, and the radii of
gyration about the \( Y \) axis may be plotted against the
over-all length of the airplane. Charts of this type are
given for the low-wing monoplane and the biplane groups
(figs. 1 to 4, inclusive). No charts are shown for the
high-wing monoplane group because the available data were
insufficient to define the curves. It should be noted
that the curves shown in figures 1 to 4 were derived from
data of similar airplanes; namely, military airplanes of
comparatively recent design. The data for commercial and
experimental airplanes obviously do not apply to these
curves. The biplane data given in reference 1 are omitted
from figures 3 and 4 because the airplanes were generally
of older design and because the more recent airplanes were tested with improved apparatus.

In order to give some indication of the mass distribution about the various axes, a three-view diagram of each airplane tested is included. (See figs. 5 to 36.)

DISCUSSION

The information presented provides a convenient method of rapidly approximating the moments of inertia of airplanes similar to those for which measurements are given. The method involves simply the selection from table I of an airplane which is sufficiently like the airplane considered that the radii of gyration or coefficients of the airplane in table I can be used to compute the moments of inertia of the airplane under consideration. The convenience of the method is obvious, because the only numerical data required for its application are the weight and the over-all dimensions of the airplane considered. The method can be applied when the airplanes are similar as regards general type, shape, and structural characteristics but are different in size and weight. That the results from one airplane can be applied to a similar airplane of different size may be seen from the fact that the data of similar airplanes vary uniformly with the over-all dimensions (figs. 1 to 4). In reference to these figures it is interesting to note that, for both low-wing monoplanes and biplane groups, the curves of \( k_x \) and \( k_z \) are parallel and that the curve of \( k_y \) for biplanes is parallel to and above the \( k_y \) curve for the low-wing monoplanes.

It should be appreciated that the indiscriminate application of the method given may lead to very erroneous results. In order to emphasize this point, the data of the P-35 and the NF-1, two very similar airplanes, will be considered. In spite of the close similarity as regards size, shape, and structural design, the moments of inertia of the NF-1 airplane are considerably higher than those of the P-35, particularly about the \( X \) and the \( Z \) axes. These differences are readily accounted for by the fact that the NF-1 was tested with a 100-pound bomb under each wing. Deducting the moments of inertia of the bombs reduces the values of \( A, B, \) and \( C \) to 2653, 4620, and 5795 slug-feet\(^2\) for the case with the landing gear extended.
The values for the two airplanes are thus shown to be in better agreement than the data in table I indicate. The radii of gyration of the NF-1 plotted in figures 1 and 2 were calculated from these corrected values. None of the other airplanes carried bombs or other concentrated loads not included in the normal load condition of the airplane.

The precision of the moments of inertia approximated by the method just described is difficult to estimate because it depends on the degree of similarity between the two airplanes considered and on the exactness with which any dissimilarities can be accounted for. If the method is used with due regard to its limitations, it is believed that the precision obtained will in many cases approach that obtained by computation methods.

Langley Memorial Aeronautical Laboratory,
National Advisory Committee for Aeronautics,
Langley Field, Va., September 13, 1940.

REFERENCES


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<tr>
<th>Figure</th>
<th>Airplane</th>
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<th>Type</th>
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<th>Structural details</th>
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<th>Overall length of airplane (ft)</th>
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<th>Additional moments of inertia (slug-ft²)</th>
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* The symbols in the column have the following significance: Am, all-metal construction; Af, all-fabric covered; Wn, metal-covered wing; Wf, fabric-covered wing.

** Weight includes two 100-pound bombs under wings.

*** Wing equipped with several flaps; F-22 fuselage.

**** Wing equipped with slots and flaps.

Converted amphibian, tricycle landing gear replacing standard gear.
Figure 1.- Variation of $k_X$ and $k_Z$ with wing span for seven low-wing monoplanes.
Figure 2. Variation of $k_y$ with over-all length for seven low-wing monoplanes.
Figure 3. - Variation of $k_X$ and $k_Z$ with wing span for five biplanes.
Figure 4. - Variation of $k_Y$ with over-all length for five biplanes.
Figure 5: The P-35 airplane
Test weight, 5877 pounds.

Figure 6: The NF-1 airplane
Test weight, 5992 pounds.
Figure 7. - The BT-9A airplane.
Test weight 4472 pounds

Figure 8. - The PB-2 airplane.
Test weight 5247 pounds.
Figure 9. - The YP-29A airplane.
Test weight, 3433 pounds

Figure 10. - The P-26A airplane.
Test weight, 3091 pounds
Figure 11. - The X5B2U-2 airplane.
Test weight, 6305 pounds

Figure 12. - The Hammond Y-1 airplane.
Test weight, 1883 pounds
Figure 13.— The J-2 airplane. Test weight 1535 pounds

Figure 14.— The McDonnell airplane. Test weight, 1708 pounds
Figure 15.— The XR2K-1 airplane.  
Test weight, 1729 pounds

Figure 16.— The F-22 airplane.  
Test weight, 1388 pounds
Figure 17. The Doyle 0-2 airplane. Test weight, 1,388 pounds.

Figure 18. The OA-4A airplane. Test weight, 8,553 pounds.
Figure 19.— The Aeronca C-2N airplane.
Test weight, 584 pounds

Figure 20.— The XF13C-3 airplane.
Test weight, 4662 pounds
Figure 21.- The XSBF-1 airplane.  
Test weight, 4473 pounds

Figure 22.- The XSB3U-1 airplane.  
Test weight, 5330 pounds
Figure 23. - The F3F-2 airplane.
Test weight, 4673 pounds.

Figure 24. - The XF2F-1 airplane.
Test weight, 3550 pounds.
Figure 25. - The XBM-1 airplane.
Test weight, 4763 pounds.

Figure 26. - The O-11 airplane.
Test weight, 4258 pounds.
Figure 27. — The F4B-1 airplane.
Test weight, 2540 pounds.

Figure 28. — The F4B-2 airplane.
Test weight, 2816 pounds.
Figure 29. - The PW-9 airplane.
Test weight, 2685 pounds.

Figure 30. - The PT-1 airplane.
Test weight, 2512 pounds.
Figure 31. - The NY-1 airplane.
Test weight, 2622 pounds.

Figure 32. - The XN2Y-1 airplane.
Test weight 1567 pounds.
Figure 33.— The 02U-3 airplane.
Test weight, 3550 pounds.

Figure 34.— The 03U-1 airplane.
Test weight, 4057 pounds.
Figure 35. – The VE-7 airplane.
Test weight, 2208 pounds.

Figure 36. – The NB-1 airplane.
Test weight, 2544 pounds.