
REPORT No. 192

**CHARTS FOR GRAPHICAL ESTIMATION OF
AIRPLANE PERFORMANCE**

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SUMMARY.

This report, which was prepared at the request of the National Advisory Committee for Aeronautics, contains a series of charts which were developed in the Bureau of Aeronautics of the Navy Department in order to simplify the estimation of airplane performance. Charts are given for estimating propeller diameter and efficiency, maximum speed, initial rate of climb, absolute ceiling, service ceiling, climb in 10 minutes, time to climb to any altitude, maximum speed at any altitude, and endurance. A majority of these charts are based on the equations given in N. A. C. A. Technical Report No. 173. Plots of pressure and density against altitude in standard air are also given for convenience. It must, of course, be understood that the charts giving propeller diameter, maximum speed, initial rate of climb, absolute ceiling, and speeds at altitudes are approximations subject to considerable error under certain conditions. These particular charts should not be used as a substitute for detailed calculations when accuracy is required, as, for example, in military or naval proposals.

INTRODUCTION.

There is considerable need for a set of charts from which the aeronautical engineer can readily estimate the performance of an airplane, without the necessity of detailed calculation. It is thought that the charts which are published for the first time in this report will fill this need.

While it is not intended that these charts supplant careful detailed calculation when great accuracy is required, it will be found that many of them are based on exact relations. In others the constants may be estimated with considerable accuracy or determined from free flight test data when the effect of a change in weight or power is to be studied.

For the derivation of the equations on which these charts are based the reader is referred to N. A. C. A. Technical Report No. 173. The equations will be given with brief explanations in this report.

PROPELLER DIAMETER.

The first step in estimating the performance of an airplane is to estimate the maximum propeller efficiency. For this purpose the diameter will be required. It has been shown in N. A. C. A. Technical Report No. 173 that the diameter of the average two-bladed propeller is

$$D_2 = \sqrt[4]{\left(\frac{90000}{N}\right)^2 \frac{HP}{V}} \quad (1)$$

where HP is the B. HP. at high speed V in M. P. H. and R. P. M., N . Under similar conditions the diameter of a four-bladed propeller is 86 per cent of the diameter of a two-bladed propeller

$$D_4 = 0.86 D_2. \quad (2)$$

Equation (1) may easily be solved with a slide rule or by the use of a chart. The usual method of plotting this equation is in the form of a double nomogram. In this case, however the simple form, using $\frac{HP}{V}$ as ordinates and D as abscissæ, with curves of constant N is used in Figure 1. The values of D determined by this chart are usually correct within 0.1 foot. This accuracy is sufficient for performance calculations.

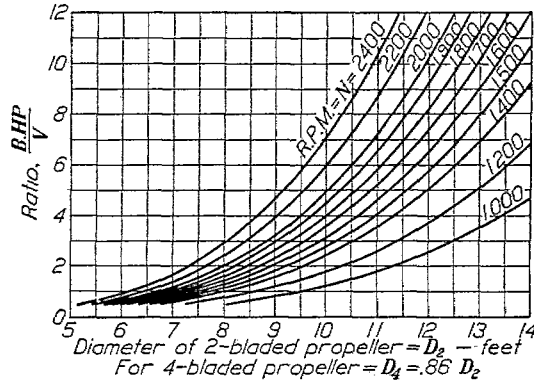


FIG. 1.—Propeller diameter based on the equation

$$D = \sqrt[4]{\left(\frac{90000}{R.P.M.}\right)^3 \left(\frac{HP}{V}\right)}$$

PROPELLER EFFICIENCY.

It is shown in N. A. C. A. Technical Report No. 168 that the curve drawn through the highest maximum efficiency at each $\left(\frac{V}{ND}\right)$ may be considered as the curve of maximum efficiency against $\left(\frac{V}{ND}\right)$. For two-bladed propellers this curve is defined by the empirical equation

$$\eta_{2m} = 0.94 - \frac{0.11}{\left(\frac{V}{ND}\right)_{2m}} \quad (3)$$

where $\left(\frac{V}{ND}\right)_{2m}$ is the value of $\left(\frac{V}{ND}\right)$ at which the maximum efficiency η_{2m} occurs. Also the efficiency of a four-bladed propeller is, on the average, 95 per cent of the efficiency of a two-bladed propeller, according to test data. That is

$$\eta_{4m} = 0.95 \eta_{2m}. \quad (4)$$

Equations (3) and (4) are plotted on Figure 2. Having given the value of $\left(\frac{V}{ND}\right)$ at which it is desired that the efficiency be a maximum, Figure 2 gives the value of this maximum efficiency. The value of the efficiency at any other $\left(\frac{V}{ND}\right)$ may be obtained from Figure 3 by substituting the η_m just found. The derivation of this curve is given in N. A. C. A. Technical Report No. 168.

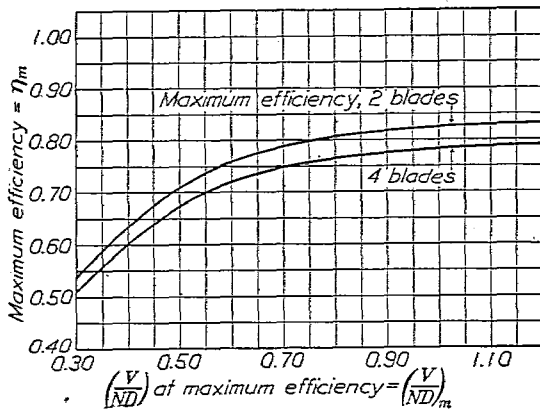


FIG. 2.—Maximum efficiency of air propellers based on Durand's tests.

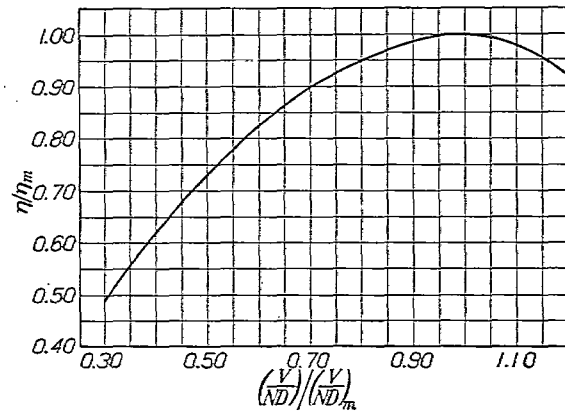


FIG. 3.—General efficiency curve for air propellers based on Durand's tests.
NOTE.—The maximum efficiency η_m occurs when $(\frac{V}{ND}) = (\frac{V}{ND})_m$.

SPEED RANGE AND MAXIMUM SPEED.

It is shown in N. A. C. A. Technical Report No. 173 that the speed range ratio of an airplane is given by

$$\frac{V_M}{V_s} = \frac{K \cdot \eta_m^{1/3}}{\left(V_s \cdot \frac{W}{HP} \right)^{1/3}} \tag{5}$$

where V_M is the maximum speed in M. P. H., V_s the *stalling speed* (without power), η_m the maximum propeller efficiency, W the weight in lb. and HP the maximum B. HP developed at high speed V_M .

The value of K is substantially constant for any given design, and normally varies but little from one design to another. Also it may easily be shown that K is substantially proportional to the cube root of $(\frac{L}{D})_{max}$, where $(\frac{L}{D})_{max}$ is the overall value for the airplane. A study of test data indicates that the best average value is given by

$$K = 10.2 \left(\frac{L}{D} \right)_m^{1/3} \tag{5a}$$

At sea level the stalling speed V_s is given by

$$\begin{aligned} V_s &= 19.8 \sqrt{\frac{W}{C_{L_{max}} \cdot S}} \\ &= \sqrt{\frac{W}{K_{y_{max}} \cdot S}} \end{aligned} \tag{6}$$

Equation (5) is plotted in Figure 4 so that the value of V_M corresponding to any normal values of V_s , η_m and $\frac{W}{HP}$ may be read directly. The following table gives the variation of K with $\left(\frac{L}{D}\right)_{max}$:

$\left(\frac{L}{D}\right)_m$	6	7	7.5	8	8.5	9	10
K	18.5	19.5	20.0	20.4	20.8	21.2	21.9

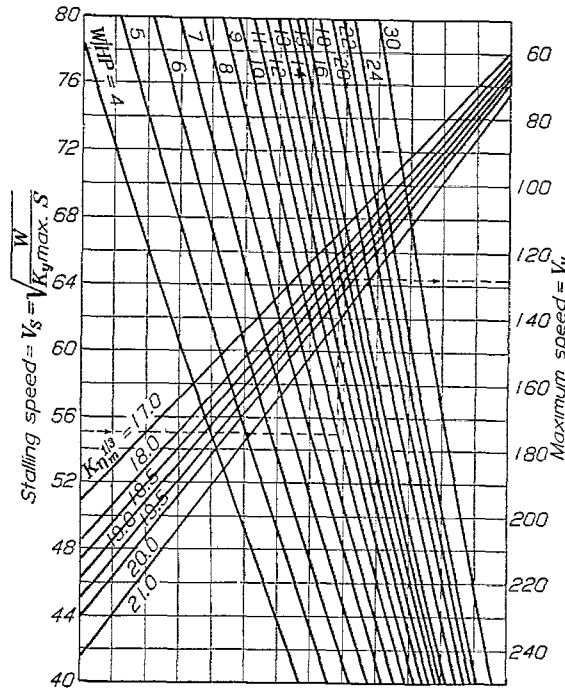


FIG. 4.—Maximum speed of airplanes based on the formula

$$V_M = \frac{K \eta_m^{1/3} V_s}{\sqrt{V_s W / HP}}$$

Directions: Pass horizontally from V_s to W/HP , then vertically to proper value of $K \eta_m^{1/3}$, then horizontally to maximum speed V_M , as shown by broken line.

NOTE.—This chart to be used when $V_M > 1.7 V_s$ average value of $K \eta_m^{1/3} = 19.0$.

The average value of $K \eta^{1/3}$ is 19.0.

INITIAL RATE OF CLIMB.

It was demonstrated in N. A. C. A. Technical Report No. 173 that the initial rate of climb of an airplane is given very closely by

$$C = 33000 \left(\frac{\eta_m \cdot \left(\frac{V_M}{V_s}\right)^{-0.27}}{\frac{W}{HP} \cdot 1125 \left(\frac{L}{D}\right)} \right) \tag{7}$$

where $\left(\frac{L}{D}\right)$ is the overall $\left(\frac{L}{D}\right)$ for the airplane and, substantially, its maximum value for the best climb.

For all ordinary purposes this value will be 8.0. The following tabulation indicates variations to be expected:

Designs with very high resistance.....	L/D=6.5-7.5
Average designs.....	L/D=7.5-8.5
Very clean designs.....	L/D=8.5-9.5

Equation (7) is plotted in Figures 5 and 6. Figure 5 gives the climb corresponding to the power-available term in (7). Figure 6 gives the correction corresponding to the power-required term in (7). Very little practice is required to become familiar with these charts.

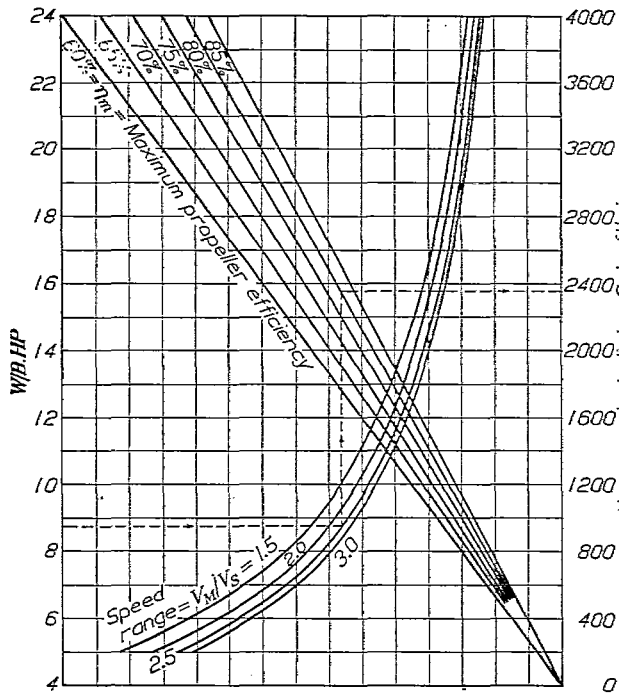


FIG. 5.—Initial rate of climb for conventional airplanes. Chart I uncorrected climb. See Chart II for climb corrections. Directions: Pass horizontally from W/B.H.P. to V_M/V_S , then vertically to η_m , then horizontally to uncorrected climb C_1 . Obtain C_2 from Chart II and subtract from C_1 to find actual climb C . NOTE.—Use actual maximum B.H.P. at high speed. These charts are based on the formula $\frac{dz}{dt}$ = Equation (7)

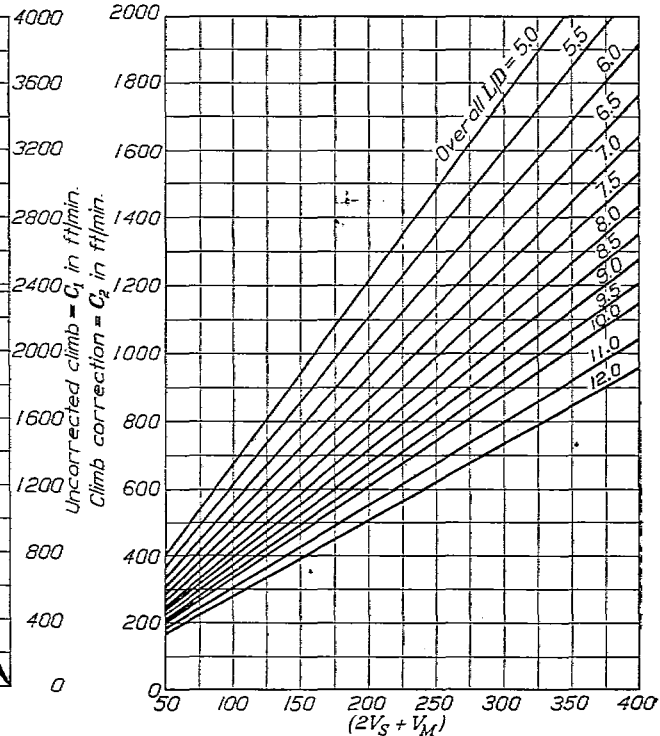


FIG. 6.—Initial rate of climb for conventional airplanes Chart II, climb correction. See Chart I for uncorrected climb. Directions: Pass vertically from $(\frac{1}{2}V_S + V_M)$ to overall L/D and horizontally to climb correction C_2 . Subtract C_2 from C_1 (Chart I) to obtain actual climb C . NOTE.—When overall L/D is known use 8 for average airplanes. Actual climb = $C = C_1 - C_2$ ft./min.

ABSOLUTE CEILING.

In N. A. C. A. Technical Report No. 173, it was shown that the absolute ceiling of an airplane was determined indirectly by the equation

$$\frac{HP_{ao}}{HP_{ro}} = \frac{K_4 \left(\frac{L}{D}\right)}{\left(\frac{1}{\eta_m} \cdot V_s \cdot \frac{W}{HP}\right)^{.90}} \tag{8}$$

where HP_{ao} and HP_{ro} are the power available and power required at the ground. The value of $K_4 = 61.7$ originally used was based on the assumption of no decrease in engine R. P. M. with altitude. Subsequent investigations show that for the average engine now in service, it is advisable to assume the normal decrease in R. P. M. with altitude. The value of K_4 will then

be 61.7 and the curve of HP_{ao}/HP_{ro} versus Z_a as in Figure 7 which is taken from N. A. C. A. Technical Report No. 171. Combining Figure 7 and equation (8) we can plot Figure 8 giving the absolute ceiling directly in terms of the factors $\left(\frac{1}{\eta_m} V_s \frac{W}{HP}\right)$ and $\left(\frac{L}{D}\right)$.

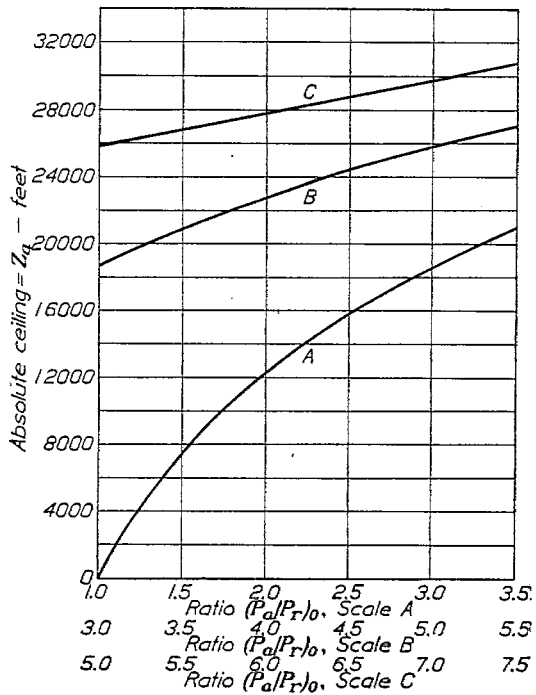


FIG. 7.—Absolute ceiling as determined by the ratio at sea level of $\frac{\text{Power available}}{\text{Power required}} = \left(\frac{P_a}{P_r}\right)_0$.

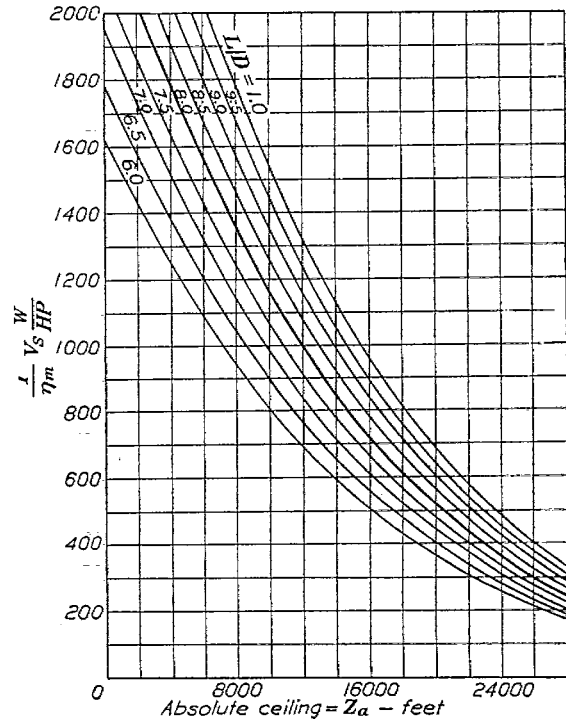


FIG. 8.—Absolute ceiling for conventional airplanes. NOTE.—These curves are based on the equation

$$\frac{HP_{ao}}{HP_{ro}} = \frac{61.7 (L/D)}{\left(\frac{1}{\eta_m} V_s \frac{W}{HP}\right)^{0.80}}$$

(See Repts. 171 and 173.)

$$V_s = \text{Stalling speed in M. P. H. } \frac{W}{HP} = \frac{1b}{B.H.P.}$$

SERVICE CEILING.

The relation between service ceiling Z_s , absolute ceiling Z_a and initial rate of climb C_o is:

$$\frac{Z_s}{Z_a} = \frac{C_o - 100}{C_o} \tag{9}$$

This relation is so simple that a plot is hardly required, although Figure 9 is included for the sake of completeness.

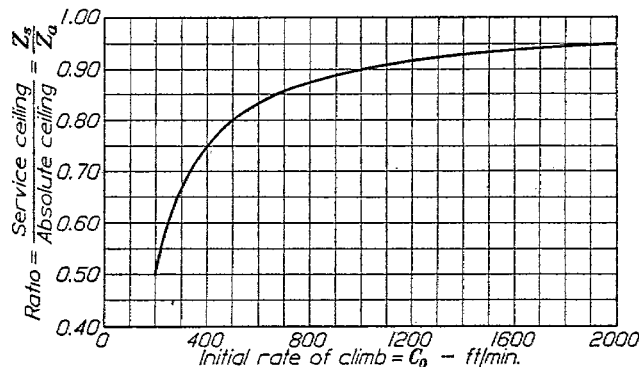


FIG. 9.—Service ceiling as determined by initial rate of climb and absolute ceiling.

CLIMB IN 10 MINUTES.

The climb in 10 minutes Z_{10} depends upon the initial rate of climb C_0 , and the absolute ceiling Z_a , according to the well-known equation

$$\frac{Z_a - Z_{10}}{Z_a} = e^{-10 \frac{C_0}{Z_a}} \tag{10}$$

There are several ways of plotting this equation according to the variables used. Figure 10 is a "folded" plot of $\frac{Z_{10}}{Z_a}$ versus $\frac{C_0}{Z_a}$ and Figure 11 is a plot of $\frac{Z_{10}}{C_0}$ versus $\frac{C_0}{Z_a}$. These two plots are very useful although they are indirect. Figures 12 and 13 are direct reading.

It should be noted that Figures 12 and 13 readily give C_0 and Z_a for the case where only Z_{10} and Z_s are known, as in the ordinary tabulation of performance.

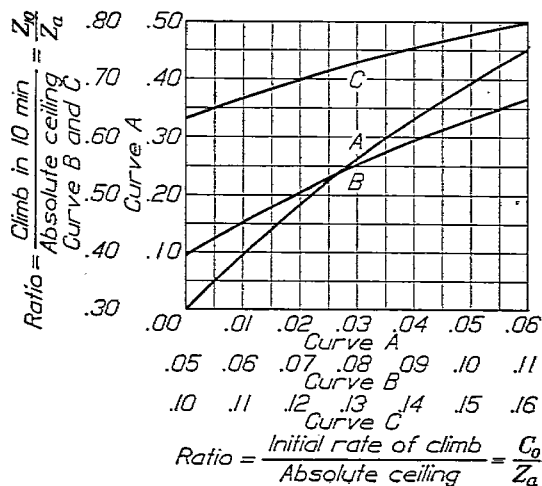


Fig. 10.—Relation between initial rate of climb, climb in 10 minutes, and absolute ceiling.

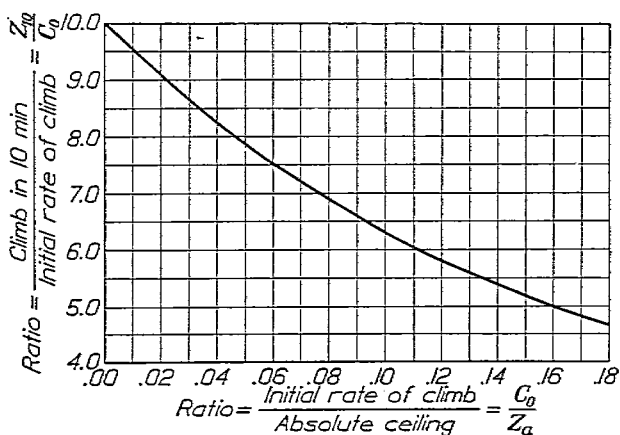


Fig. 11.—Climb in 10 minute as determined by initial rate of climb and absolute ceiling.

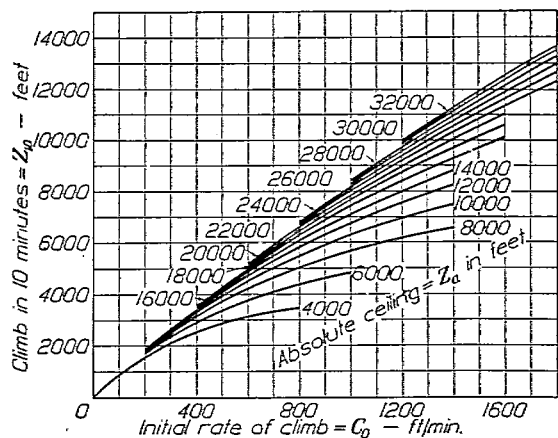


Fig. 12.—Climb in 10 minutes as determined by absolute ceiling and initial rate of climb.

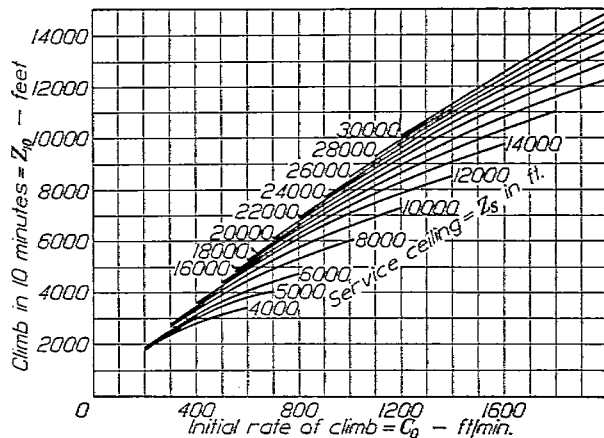


Fig. 13.—Climb in 10 minutes, from initial rate of climb and service ceiling.

TIME TO CLIMB TO ANY ALTITUDE.

The time to climb to any altitude Z is given by the general form of equation (10)

$$t = -\frac{Za}{C_0} \cdot \log_e \left(\frac{Z_a - Z}{Z_a} \right) = 2.304 \frac{Za}{C_0} \log_{10} \left(\frac{Z_a}{Z_a - Z} \right) \quad (11)$$

t will be in minutes if the altitudes are in feet and the rate of climb in ft./min. A very convenient plot of this equation is given in Figure 14.

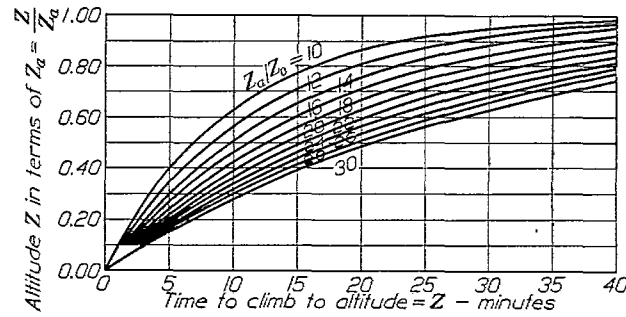


FIG. 14.—Time required to climb to altitude Z . Z_a = Absolute ceiling in feet.
 C_0 = Initial rate of climb in ft./min.

MAXIMUM SPEED AT ALTITUDES.

The maximum speed at any altitude is given with considerable accuracy by the single curve of V_z/V_0 versus Z/Z_a in Figure 15. The derivation of this curve will not be given at this time but it may be easily verified by reference to any accurate performance test.

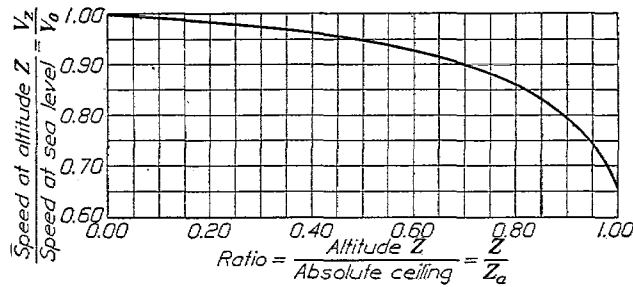


FIG. 15.—Average decrease in maximum speed with increase in altitude.

RANGE.

Breguet's formula for range

$$R = 862 \frac{\eta}{c} \frac{L}{D} \log_{10} \left(\frac{W_1}{W_2} \right) \tag{12}$$

where η = propeller efficiency,
 c = specific fuel consumption,
 and $W_1 - W_2$ = weight of fuel consumed,
 may be plotted in a convenient chart as in Figure 16. The equation for endurance is not easily plotted and must be obtained from the formula

$$T = 750 \frac{\sqrt{W} \eta}{V} \frac{L}{c D} \left(\frac{1}{\sqrt{W_2}} - \frac{1}{\sqrt{W_1}} \right) \text{ hours} \tag{13}$$

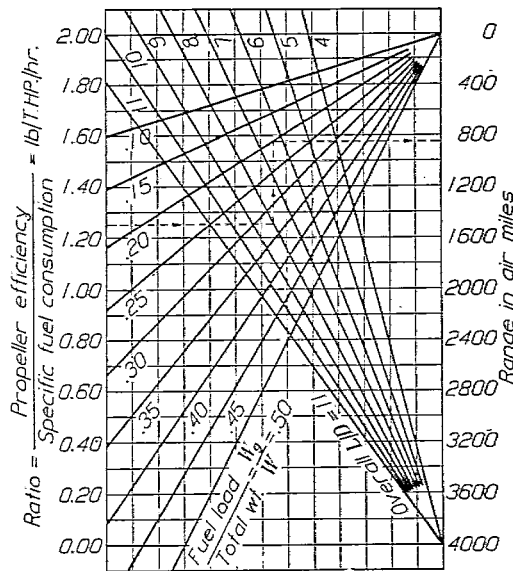


FIG. 16.—Maximum range for airplanes based on Breguet's formula, $R = 862 \frac{\eta}{c} \frac{L}{D} \log_{10} \left(\frac{W}{W - W_f} \right)$.

Directions: Pass horizontally from effective fuel consumption to $\left(\frac{L}{D} \right)$, then vertically to $\left(\frac{W_f}{W} \right)$, then horizontally to range as shown by arrows on broken lines.

where V is the cruising velocity for the weight W . Note that the term $\frac{\sqrt{W}}{V}$ is constant at any given angle of attack.

STANDARD ATMOSPHERE.

For convenience there are included herewith three charts on the standard atmosphere adopted by the National Advisory Committee for Aeronautics and defined by Technical Report No. 147 and Technical Note No. 99. It will be remembered that the temperature decreases uniformly in this standard atmosphere at the rate of 6.5° C. per 1,000 meters from 15° C. (59° F.) at sea level to the altitude at which it is -55° C. (-67° F.). Above this altitude, approximately 36,000 feet, t is constant. Therefore, below 36,000 feet

$$\begin{aligned} T^{\circ}C &= 288 - 0.0065 Z \text{ meters} \\ T^{\circ}F &= 518.6 - 0.003567 Z \text{ ft.} \end{aligned} \tag{14}$$

Figures 17 and 18 give the variation of p/p_0 and ρ/ρ_0 with Z while Figure 19 gives $\sqrt{\frac{\rho_0}{\rho}}$ against Z .

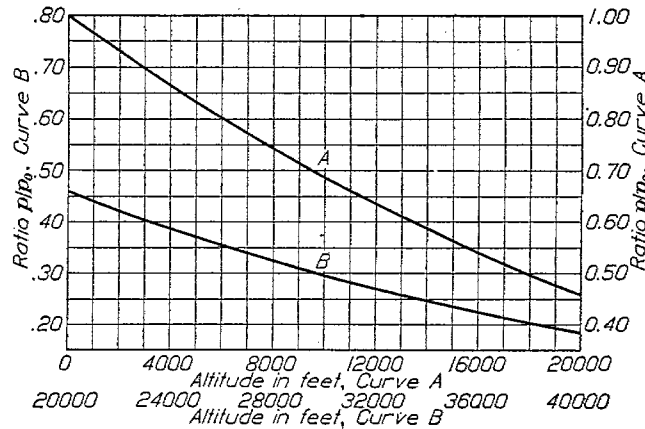


FIG. 17.—Variation of pressure with altitude. Standard atmosphere.
 $P_0=29.921'' \text{ Hg}=2,115.6 \text{ lb/sq. ft.}$

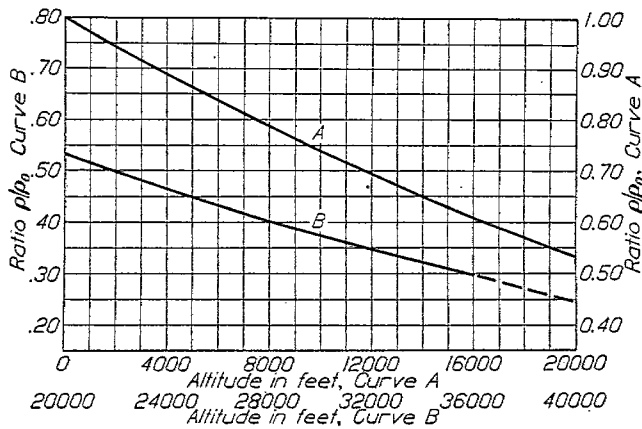


FIG. 18.—Variation of density with altitude. Standard atmosphere.

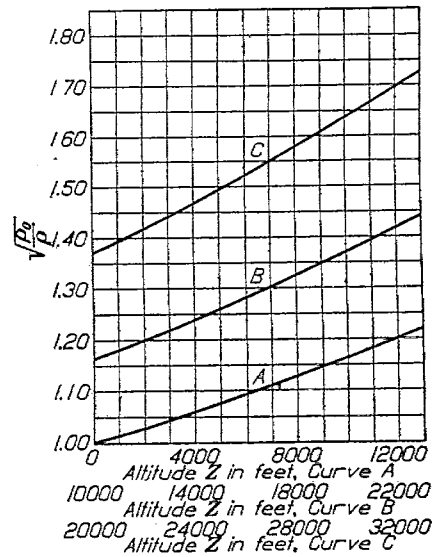


FIG. 19.—Variation of $\sqrt{\frac{\rho_0}{\rho}}$ with altitude. Standard atmosphere.