REPORT No. 234

THREE METHODS OF CALCULATING RANGE AND ENDURANCE OF AIRPLANES

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SUMMARY

This report, which was prepared for the National Advisory Committee for Aeronautics, develops new equations which give the range and endurance of airplanes with an accuracy equal to that obtained from a step-by-step integration of the flight. A method of obtaining equally satisfactory results from Breguet's equations is also given in detail. A third method of calculating range and endurance, derived by the writer for use in routine estimating in the Bureau of Aeronautics, is also given in full.

The report contains tables and curves arranged for convenient use and illustrates the three methods by comparative estimates.

INTRODUCTION

It has been customary to calculate the range and endurance of airplanes either from a step-by-step integration or by means of the common equations for range and endurance, usually known as Breguet's equations. Although the work involved in a calculation with the equations is only a small fraction of that required in a step-by-step integration, the latter method has been almost universally used for accurate work, owing to the inability of many engineers to secure accurate or consistent values with the equations.

This study was started for the purpose of developing new equations which would give definite results equal in accuracy to a step-by-step integration. After the new equations were developed it was found that while they fulfilled the requirements of simplicity and accuracy, the same results could be obtained from Breguet's equations by the use of new data on specific fuel consumption. This paper therefore not only supplies the aeronautical engineer with new equations for calculating range and endurance with the accuracy of a step-by-step integration, but it also gives a method whereby the same results can be obtained from Breguet's equations. This is a matter of considerable importance, since it is no longer necessary to carry through a step-by-step integration in order to secure accurate cruising ranges and endurances.

An additional method of calculating cruising range and endurance is given in full in this paper. This method, which was developed by the writer for routine estimating in the Bureau of Aeronautics, may be called the "factor method," since the range and endurance at cruising speed are found by multiplying the range and endurance at high speed by easily determined factors. While the factor method is slightly less accurate than the equations, it gives very satisfactory results for routine estimations.

In this report cruising speed will be the air speed at which the fuel consumption in pounds per mile is a minimum. This designation should be noted carefully, since any normal speed in throttled flight is frequently called cruising speed. If the term "cruising speed" is to apply to any speed of throttled flight, the particular cruising speed used in this report is the "most economical cruising speed" giving the maximum range (but not the maximum endurance).

The basis of the present study is found in two average curves of specific fuel consumption which were originally prepared by Mr. R. M. Parsons and Commander E. E. Wilson, United States Navy, for the Bureau of Aeronautics. The specific fuel consumption of an individual engine may vary slightly from the average, but a great number of comparisons indicate that these curves fairly represent any standard engine or carburetor now in service.

A word of caution is perhaps necessary in regard to the use of the data for special mixture control. This requires that the mixture control be constantly adjusted during the flight so as to give the minimum fuel consumption at all times. Owing to the special training and skill required for this purpose, the cases where the special mixture control data can be used are very few.

BREGUET'S EQUATIONS FOR RANGE AND ENDURANCE

Breguet's equations for range and endurance of airplanes are—

$$\text{Range miles} = 863.5 \left(\frac{L}{D}\right)\left(\frac{\eta}{c}\right)\log_{10}\left(\frac{W_0}{W_1}\right) \tag{1}$$

$$\text{Endurance hours} = 750\ \frac{V_c}{\sqrt{W}}\left(\frac{L}{D}\right)\left(\frac{\eta}{c}\right)\left[\frac{1}{\sqrt{W_1}}-\frac{1}{\sqrt{W_0}}\right] \tag{2}$$

where (L/D) is the ratio of lift to the drag of the entire airplane at the cruising angle of attack, η the cruising propeller efficiency, c the cruising specific fuel consumption in pounds per B.HP. per hour, W_0 the initial gross weight in pounds, and W_1 the final gross weight in pounds. $(W_0 - W_1)$ is therefore the fuel consumed in the flight. For cruising at a constant angle of attack the ratio of $(V_c/\sqrt{W})$ is constant.

In view of the simple and straightforward derivation of these equations (see reference 1) it has been rather disappointing to many engineers to find that the range and endurance calculated by their use is often very optimistic. This condition is entirely due to the fact that the specific fuel consumption varies between wide limits, which are difficult to determine from the usual engine data. However, the equations are fundamentally correct and can be made to give excellent results, even in the hand of an inexperienced calculator, when the variation of specific fuel consumption with weight and speed range ratio is known.

Data for determining the average specific fuel consumption and explicit instructions for using equations (1) and (2) will be given later.

THE VARIATION OF SPECIFIC FUEL CONSUMPTION WITH AIRPLANE WEIGHT

Consider an airplane in horizontal flight at cruising speed and assume that—

1. The flight is to be made at a constant angle of attack, i. e., C_L and L/D constant.
2. The brake horsepower B.HP. varies as the cube of the R.P.M. for throttled conditions, i. e.

$$\frac{\text{B.HP.}}{\text{B.HP.}_0} = \left(\frac{N}{N_0}\right)^3$$

It then follows that the air speed V varies as the square root of the weight W

$$V = K_1\sqrt{W} \tag{3}$$

and the thrust horsepower required is

$$\text{T.HP.} = \frac{WV}{K_2\left(\frac{L}{D}\right)} = \frac{K_1\ W^{1.5}}{K_2\left(\frac{L}{D}\right)} \tag{4}$$

Since T.HP. $= \eta$ B.HP. $= \eta\ K\ N^3$, it may be shown that (V/N), and consequently the propeller efficiency η, is constant. Therefore,

$$\frac{\text{B.HP.}}{\text{B.HP.}_0} = \left(\frac{V}{V_0}\right)^3 = \left(\frac{W}{W_0}\right)^{1.5} \tag{5}$$

Values of the specific fuel consumption ratio c/c_0 are given for throttled conditions in Table I or Figure 1. Plotting these data logarithmically as in Figure 2, it is found that for general service operation at throttled powers less than 70 per cent of the full power, B.HP.$_0$

$$c=c_1\left(\frac{\text{B.HP.}_0}{\text{B.HP.}}\right)^{.30} \tag{6}$$

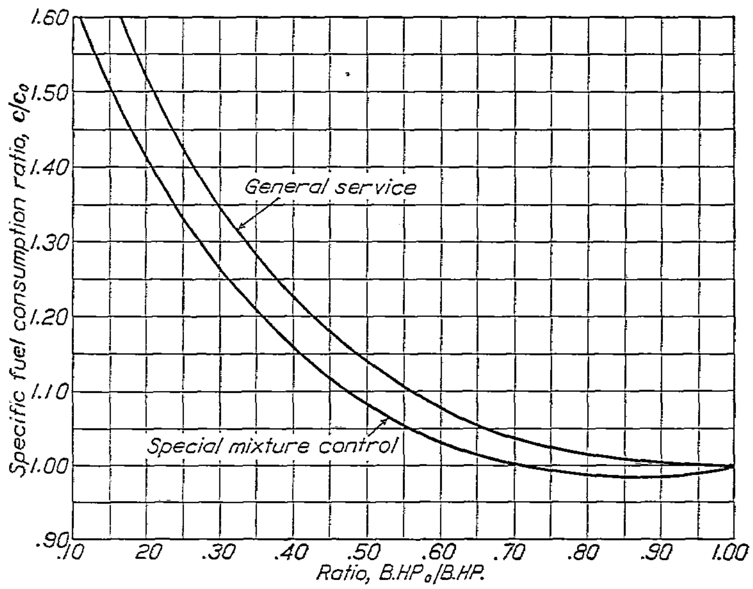

FIG. 1.—Relation between specific fuel consumption and brake horsepower

where c is the instantaneous specific fuel consumption and c_1 the initial specific fuel consumption. In a similar manner it is found that with special mixture control for the leanest mixture practicable

$$c=c_1\left(\frac{\text{B.HP.}_0}{\text{B.HP.}}\right)^{.28} \tag{7}$$

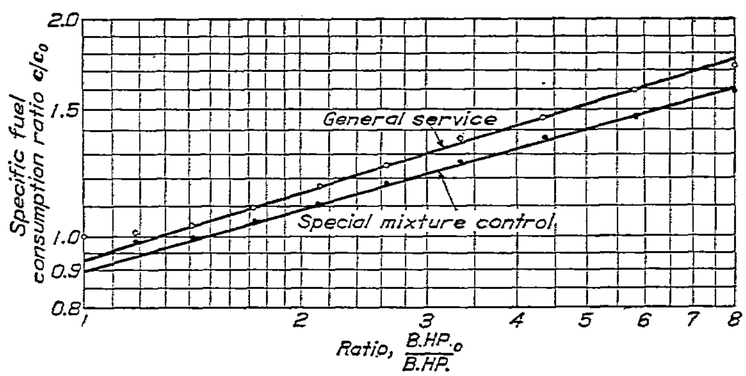

FIG 2.—Relation between specific fuel consumption and brake horsepower

General service, $c/c_0=.93\left(\frac{\text{B. HP}_0}{\text{B. HP}}\right)^{.30}$

Special mixture control, $c/c_0=.90\left(\frac{\text{B. HP}_0}{\text{B. HP}}\right)^{.28}$

Substituting equations (6) and (7) into (5), one obtains for general service operating conditions

$$c=c_1\left(\frac{W_0}{W}\right)^{.45} \tag{8}$$

and for special mixture control

$$c=c_1\left(\frac{W_0}{W}\right)^{.42} \tag{9}$$

The initial specific fuel consumption at cruising speed c_1 is really determined from the speed range ratio with full load. The calculations may be found in Table IX and a plot of c_1 against V_m/V_s is given in Figure 9.

c_1 must not be confused with the specific fuel consumption at full throttle c_0. The latter is a function of the compression ratio and has the average values given in Table II. These values may be represented by the empirical equation

$$c_0 = 0.75 - 0.04\ C.R. \tag{10}$$

where $C.R.$ is the compression ratio and c_0 the specific fuel consumption at full throttle, in lb./B.HP./hr.

DERIVATION OF NEW EQUATIONS FOR RANGE

The rate of change of weight is

$$\frac{dw}{dt} = -\text{B.HP.}\ c = \frac{-\text{T.HP.}}{\eta}\ c \tag{11}$$

from which

$$dt = \frac{-\eta\ dw}{\text{T.HP.}\ c}$$

Substituting the value of T.HP. from (4) gives

$$dt = \frac{-K_2\eta\left(\frac{L}{D}\right)dw}{K_1\ W^{1.5}\ c}$$

Consider the general case of equations (8) and (9), where

$$c = c_1\left(\frac{W_0}{W}\right)^n$$

Substituting this gives

$$dt = \frac{-K_2\eta\left(\frac{L}{D}\right)dw}{c_1 K_1\ W_0^n\ W^{(1.5-n)}} \tag{12}$$

The range is

$$R = \int V dt = \int K_1\sqrt{W}\,dt$$

$$R = \int_{W_0}^{W_1} \frac{K_2\eta\left(\frac{L}{D}\right)dw}{c_1\ W_0^n\ W^{1-n}}$$

$$= \frac{K_2\eta\left(\frac{L}{D}\right)}{c_1\ W_0^n n}\left[W_0^n - W_1^n\right] \tag{13}$$

For the usual units, W in lb., V in M.P.H., c_0 in lb./B.HP./hr., and R in miles, K_2 has the value 375. The general equation for range is therefore

$$R = \frac{375\eta\left(\frac{L}{D}\right)}{c_1 n}\left[1-\left(\frac{W_1}{W_0}\right)^n\right] \tag{13a}$$

Upon substitution of the proper values for n, this equation is applicable to the two cases under consideration:

CASE I. GENERAL SERVICE—NO MIXTURE CONTROL

$$n = 0.45$$

$$R = 833.3\left(\frac{\eta}{c_1}\right)\left(\frac{L}{D}\right)\left[1-\left(\frac{W_1}{W_0}\right)^{.45}\right] \tag{14}$$

CASE II. SPECIAL MIXTURE CONTROL FOR LEANEST MIXTURE PRACTICABLE

$$n=0.42$$

$$R=892.9\left(\frac{\eta}{c_1}\right)\left(\frac{L}{D}\right)\left[1-\left(\frac{W_1}{W_0}\right)^{0.42}\right] \tag{15}$$

DERIVATION OF NEW EQUATIONS FOR ENDURANCE

The endurance may be obtained by integrating equation (12):

$$dt=\frac{K_2\,\eta\left(\frac{L}{D}\right)dw}{c_1 K_1\,(W_0)^n\,W^{(1.5-n)}} \tag{12}$$

$$t=\frac{K_2\,\eta\left(\frac{L}{D}\right)}{c_1 K_1\,(W_0)}\int_{W_0}^{W_1}\frac{dw}{W^{1.5-n}}$$

$$=\frac{K_2\,\eta\left(\frac{L}{D}\right)}{c_1 K_1\,(W_0)^n\,(n-0.5)}\left[W_0^{(n-0.5)}-W_1^{(n-0.5)}\right]$$

$$=\frac{K_2\,\eta\left(\frac{L}{D}\right)}{c_1 K_1\,W_0^{0.5}\,(n-0.5)}\left[1-\left(\frac{W_1}{W_0}\right)^{n-0.5}\right]$$

Since $K_1\,W_0^{0.5}=V_0$ and $K_2=375$

$$t=\frac{375\,\eta\left(\frac{L}{D}\right)}{c_1 V_0\,(n-0.5)}\left[1-\left(\frac{W_1}{W_0}\right)^{n-0.5}\right] \tag{16}$$

Upon substitution of the proper values for n this general equation is applicable to the two cases under consideration.

CASE I.—GENERAL SERVICE—NO MIXTURE CONTROL

$$n=0.45$$

$$T=7500\left(\frac{\eta}{c_1}\right)\left(\frac{L}{D}\right)\frac{1}{V_{c_0}}\left[\left(\frac{W_0}{W_1}\right)^{.05}-1\right] \tag{17}$$

CASE II. SPECIAL MIXTURE CONTROL FOR LEANEST MIXTURE PRACTICABLE

$$n=0.42$$

$$T=4687.5\left(\frac{\eta}{c_1}\right)\left(\frac{L}{D}\right)\frac{1}{V_{c_0}}\left[\left(\frac{W_0}{W_1}\right)^{.08}-1\right] \tag{18}$$

In equations (17) and (18), V_{c_0} is the initial cruising speed in M.P.H. Unless great exactness is required V_{c_0} may be taken as 1.4 times the stalling speed as determined by the initial weight, the maximum lift coefficient, and the wing area. This approximation gives a speed which rarely differs more than two M.P.H. (or about 3%) from the true cruising speed.

COMPARISON OF THE EQUATIONS FOR RANGE AND ENDURANCE

Equation (1) may be written in the form

$$R=B_R\left(\frac{L}{D}\right)\left(\frac{\eta}{c}\right) \tag{1a}$$

where $B_R = 863.5 \log_{10}\left(\frac{W_0}{W_1}\right)$. B_R is therefore a constant for any given value of (W_0/W_1), or for any given value of (W_f/W_0) where W_f is the fuel load. In a similar manner equation (2) may be written

$$T = B_E\left(\frac{L}{D}\right)\left(\frac{\eta}{c}\right)\frac{1}{V_{c_0}} \tag{2a}$$

where V_{c_0} is the initial crusing speed.

The new equations may also be written in the same form as follows:

GENERAL SERVICE

$$R = K_R\left(\frac{L}{D}\right)\left(\frac{\eta}{c_1}\right) \tag{14a}$$

$$T = K_E\left(\frac{L}{D}\right)\left(\frac{\eta}{c_1}\right)\frac{1}{V_{c_0}} \tag{17a}$$

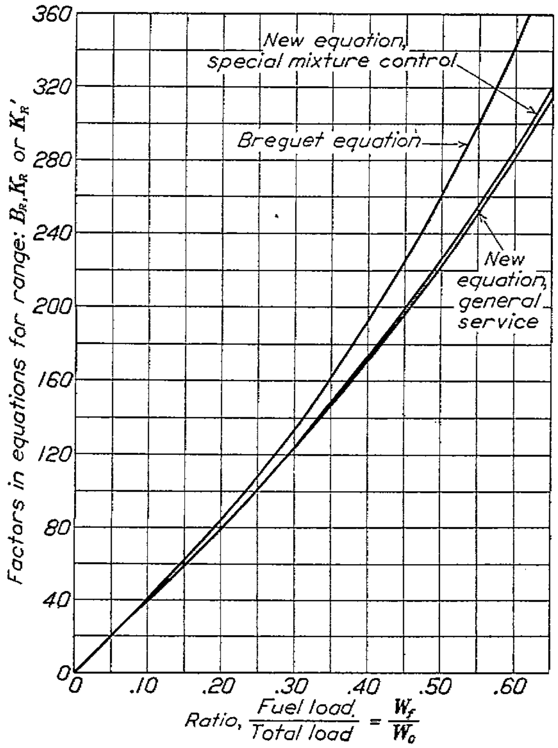

FIG. 3.—Factors in the equations for range

Range = Factor $\left(\frac{L}{D}\right)\left(\frac{\eta}{c}\right)$ miles

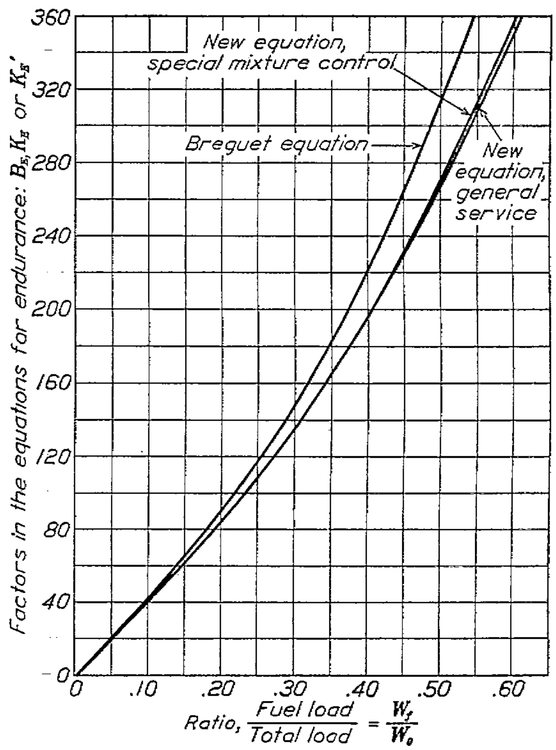

FIG. 4.—Factor in the equations for endurance

Endurance, hrs. = $\frac{\text{Factors}}{V_{c_0}}\left(\frac{L}{D}\right)\left(\frac{\eta}{c}\right)$ hrs.

SPECIAL MIXTURE CONTROL

$$R = K_R'\left(\frac{L}{D}\right)\left(\frac{\eta}{c_1}\right) \tag{15a}$$

$$T = K_E'\left(\frac{L}{D}\right)\left(\frac{\eta}{c_1}\right)\frac{1}{V_{c_0}} \tag{18a}$$

A comparison is now readily made, since the equations are in the same form. Tables III, IV, V, and VI contain the calculations for the factors B_R, B_E, K_R, K_E, K_R', and K_E'. The factors for range are plotted in Figure 3, and the factors for endurance in Figure 4. It will be seen that the factors for range and endurance in the new equations are but slightly affected by the mixture control, the chief effect of which is therefore to reduce the initial specific fuel consumption. The factors in the Breguet equations are considerably higher than those in the new equations, particularly at high values of the ratio W_f/W_0. This difference is exactly compensated by the higher value of the specific fuel consumption used in the former. It has been pointed out that in equations (1) and (2) or (1a) and (2a), c is the *average value* of the specific fuel consumption for the flight, while in equations (14), (15), (17) and (18), c_1 is the *initial value* of the specific fuel consumption. When c_1 and c are accurately determined by a method to be given later, the two equations give identical results on either range or endurance.

In order that the factors, B_R, B_E, K_B, K_E, K_R', and K_E' may be determined accurately, Table VII has been prepared to give directly the desired factor for any value of W_f/W_0 likely to be used.

AVERAGE CRUISING SPEED

Equations (14a) and (17a) or (15a) and (18a) supply a simple method for determining the ratio of the average cruising speed to the initial cruising speed, (V_c average)/V_{c_0}. This ratio is obviously the ratio K_B/K_E or K_R'/K_E'. Since it is frequently required, it has been calculated in Tables V and VI and plotted in Figure 5.

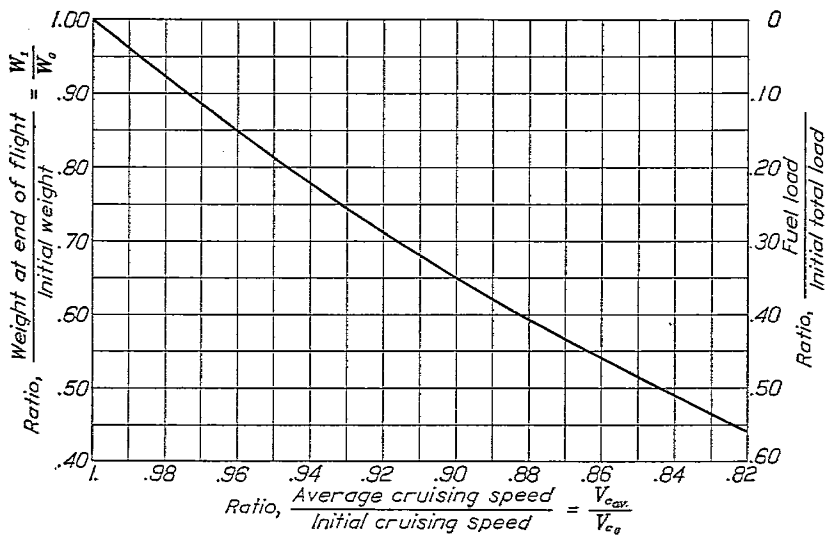

FIG. 5.—Average cruising speed variation with fuel load

THE FACTOR METHOD OF CALCULATING RANGE AND ENDURANCE

The "factor method" of calculating range and endurance was developed by the writer for routine estimating work in the Bureau of Aeronautics. Briefly stated, the method consists of a determination of the factors F_R and F_E where

$$F_R = \text{Ratio}\ \frac{\text{Range at cruising speed}}{\text{Range at high speed}}$$

and

$$F_E = \text{Ratio}\ \frac{\text{Endurance at cruising speed}}{\text{Endurance at high speed}}$$

While this method has no outstanding general advantages over the simplified equations given in this report, it is particularly adapted to certain kinds of performance estimating on account of its simplicity and directness. Since the method is entirely new its derivation will be given in full.

It may be shown by theory or flight test data that for an *average* airplane the variation of maximum speed with gross load is very small. This means that the maximum speed will remain substantially constant during a flight in which the fuel load is consumed. Consequently the endurance (T_m) at maximum speed is equal to the fuel load (W_f) divided by the hourly fuel consumption (W_f/T_m) of the engine or engines, at full power and full throttle. The hourly fuel consumption at full throttle is obviously the product of the specific fuel consumption c_0 by the maximum brake horsepower B.HP.$_m$. The range at high speed is equal to the product of the maximum speed by the endurance at maximum speed.

An analysis of test data, both model and free flight, shows the cruising speed V_c to bear a substantially constant relation to the stalling speed V_s with an average value of $V_c=1.40\ V_s$. This means that regardless of the high speed, or the speed range ratio,[1] except as they affect the propeller efficiency and fuel consumption, the cruising speed depends only on the stalling speed. Since at high speeds the power required for horizontal flight varies substantially as the cube of the speed it should be possible to express the thrust power at any speed greater than cruising speed in terms of the thrust power at cruising speed, and this ratio should show little variation from one airplane to another. Table VII contains the ratios of thrust power at speed V to thrust power at speed V_{CR} for 10 representative airplanes as obtained from wind tunnel test data. The averages of these values form a composite or "average thrust power" curve against speed range ratio as plotted in Figure 6. This thrust power curve gives directly the ratio of the thrust power at high speed to the thrust power at cruising speed.

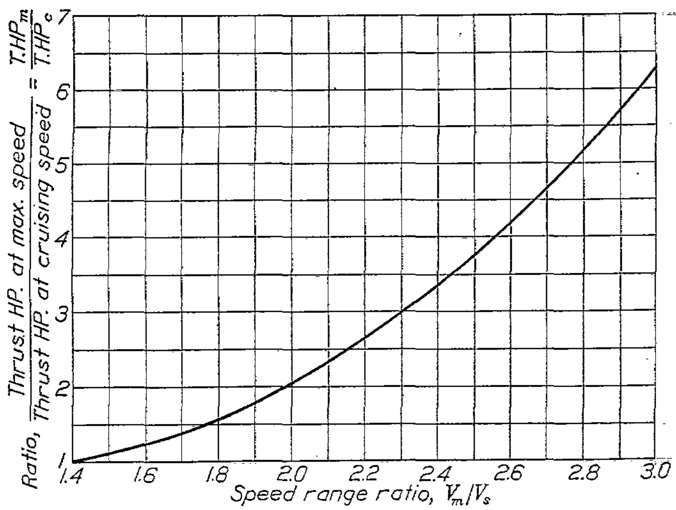

FIG. 6.—Thrust horsepower composite curve

In order to convert the thrust power ratios to brake power ratios the propeller efficiency ratios must be determined. This may readily be done by means of Durand's power coefficient $C_2=\frac{P}{\rho V^3 D^2}$. Multiplying by the propeller efficiency gives $\eta C_2=C_2'=\frac{\eta P}{\rho V^3 D^2}$, where ηP is the thrust power in ft. lb. per sec. That is, $C_2'=550$ T.HP.$/\rho V^3 D^2$. Since T.HP. varies substantially as N^3 over that part of the power-required curve under consideration:

$$C_2' \propto \frac{N^3}{V^3} \propto \left(\frac{V}{ND}\right)^{-3} \tag{19}$$

The ratio of C_2' at maximum and cruising speeds is

$$\frac{(C_2')_m}{(C_2')_c}=\frac{550\ (\text{T.HP.})_m}{\rho V^3_m D^2}\times\frac{\rho V_c^3 D^2}{550\ (\text{T.HP.})_c}=\frac{\text{T.HP.}_m}{\text{T.HP.}_c}\times\left(\frac{V_c}{V_m}\right)^3 \tag{20}$$

From equation (19) the ratio of $\left(\frac{V}{ND}\right)$ at maximum and cruising speeds is

$$\frac{\left(\frac{V}{ND}\right)_m}{\left(\frac{V}{ND}\right)_c}=\left[\frac{(C_2')_c}{(C_2')_m}\right]^{1/3} \tag{21}$$

Given any value of $\left(\frac{V}{ND}\right)_c\Big/\left(\frac{V}{ND}\right)_m$ the corresponding ratio of the propeller efficiencies η_c/η_m, is readily found from the general efficiency curve of N. A. C. A. Technical Report No. 168 (Reference 2). The ratio of brake horsepower at cruising speed to the maximum brake horsepower is

$$\frac{\text{B.HP.}_c}{\text{B.HP.}_m}=\frac{\text{T.HP.}_c}{\text{T.HP.}_m}\frac{\eta_m}{\eta_c} \tag{22}$$

[1] Provided that the initial value of the speed range ratio is greater than 1.4.

Table IX contains calculations for B.HP.$_c$/B.HP.$_m$.

The endurance factor, F_E, is found from

$$F_E = \frac{\text{B.HP.}_m}{\text{B.HP.}_c}\left(\frac{c_0}{c}\right) \tag{23}$$

and the range factor, F_R, from

$$F_R = F_E\left(\frac{V_c}{V_m}\right) \tag{24}$$

Table X contains the calculations for F_E and F_R which are plotted in Figures 7 and 8

APPLICATION OF RANGE AND ENDURANCE FACTORS

In order to use the factor method of calculating range and endurance at cruising speed, the following data must be available:

(1) Maximum speed, V_m, M.P.H.

(2) Stalling speed, V_s M.P.H. for initial and final gross loads.

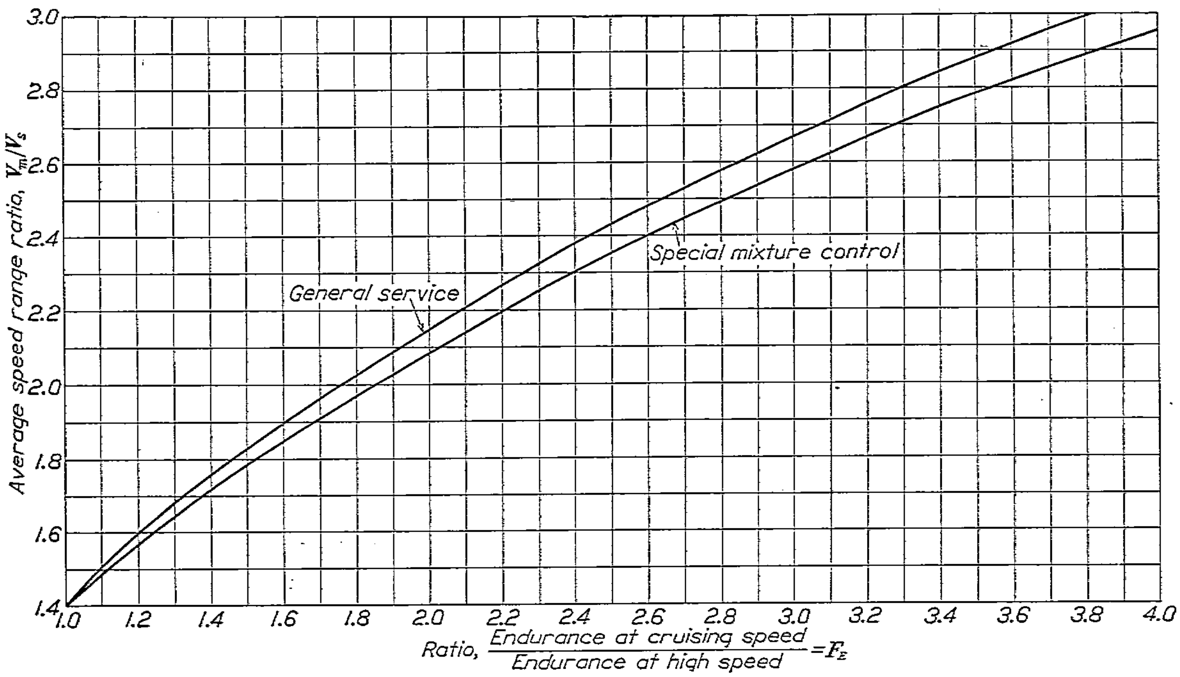

FIG. 7.—Variation with speed range ratio. Endurance factor, F_E

(3) B.HP. at full throttle = B.HP.$_m$.

(4) Specific fuel consumption at full throttle, c_0 lb./B.HP./hr.

(5) Fuel load, W_f lb.

The steps in applying the method are as follows:

I. Find the hourly fuel consumption at full throttle $\left(\frac{W_f}{T_m}\right) = \left(\text{B.HP.}_m\right) c_0$ lb./hr.

II. Find endurance at maximum speed T_m = fuel load divided by hourly fuel consumption $= W_f \div \left(\frac{W_f}{T_m}\right)$.

III. Find range at maximum speed R_m = endurance at maximum speed × maximum speed $= T_m V_m$.

IV. Find the average speed range ratio, $(V_m/V_s)_{av}$ and read corresponding endurance and range factors F_E and F_R from Figures 7 and 8.

V. Endurance at cruising speed, T_c = endurance factor × endurance at maximum speed $= F_E T_m$.

VI. Range at cruising speed, R_c = range factor × range at maximum speed = $F_R R_m$.

The average speed range ratio is the average of the speed range ratios for initial gross load and for final gross load. The final gross load W_1 is the initial gross load W_0 less the fuel load W_f. The maximum speed is assumed constant and the stalling speed is determined either from wind tunnel test data or from direct calculation. It is usually satisfactory to take the value of V_s determined by the gross load, wing area, and maximum lift coefficient of the wing section.

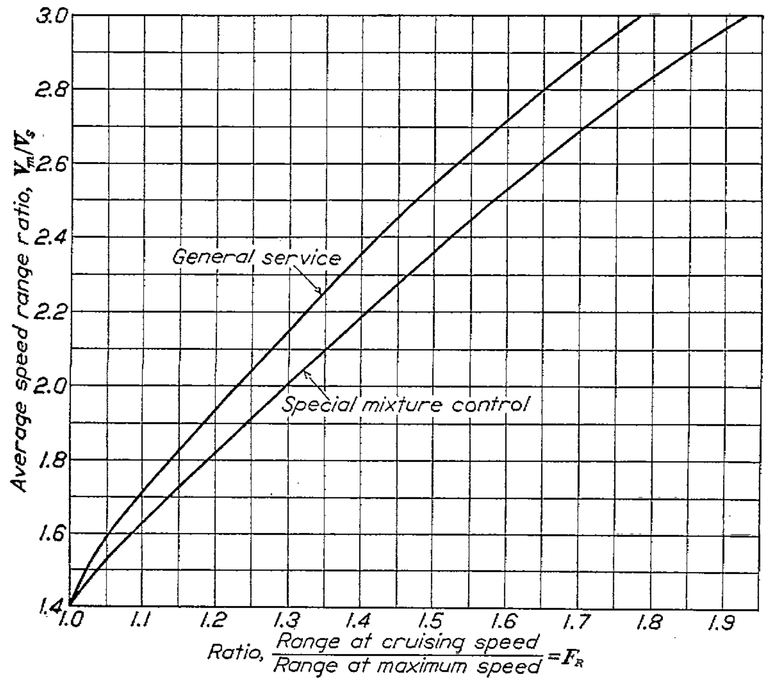

FIG. 8.—Variation with speed range ratio. Range factor, F_R

Should an airplane be so heavily loaded that its initial speed range ratio is less than 1.4, the high speed will probably increase considerably with decrease in load. In this case an accurate value of the average speed range ratio can be obtained by plotting the speed range ratio against gross weight and integrating the area under the curve.

SPECIFIC FUEL CONSUMPTION AT CRUISING SPEED

The calculations of Table IX supply data for determining the specific fuel consumption at cruising speed, required in using the range and endurance formulas. Figure 9 contains plots of the ratio

$$\frac{\text{specific fuel consumption at cruising speed}}{\text{specific fuel consumption at maximum speed}}$$

against the speed range ratio, V_m/V_s, for the two conditions designated as "General service" and "Special mixture control."

In using these curves with Breguet's equations (1a) and (2a) in which the specific fuel consumption c is an average value. The average value of the speed range ratio V_m/V_s must be determined from the initial and final gross loads. The value of (c/c_0) or (c'/c_0') corresponding to the average value of (V_m/V_s) is then read from Figure 9. This value of (c/c_0) is then multiplied by the specific fuel consumption at full throttle, c_0, to obtain the average specific fuel consumption at cruising speed.

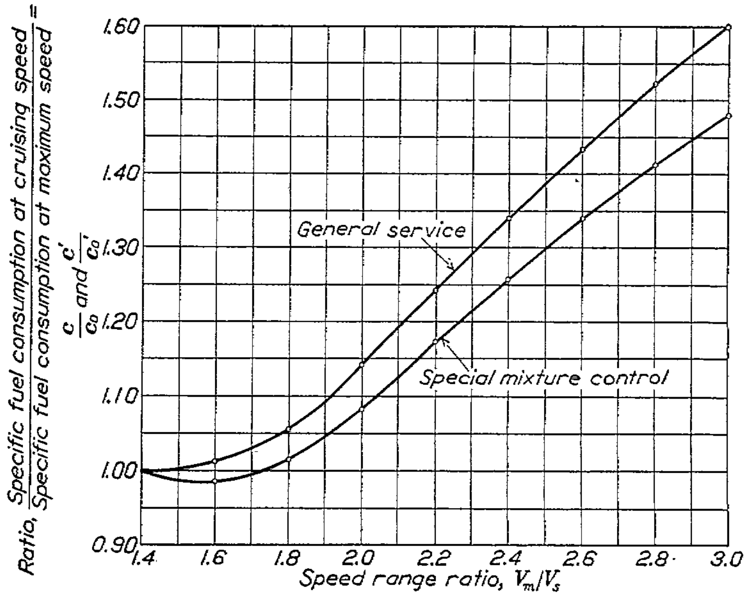

FIG. 9.—Specific fuel consumption at cruising speed

In using the curves with the new equations (14a), (15a), (17a), and (18a) in which the specific fuel consumption c_1 is the initial value, the speed range ratio is determined for the initial condition only. The value of c/c_0 [or (c'/c'_0)] corresponding to the initial value of V_m/V_s is then used in the same manner as the average value was used in Breguet's equations.

COMPARATIVE ESTIMATES

Consider a flying boat having the following characteristics:

Initial gross weight,	$W_0 = 16{,}500$ lbs.
Final gross weight,	$W_1 = 10{,}500$ lbs.
Fuel load,	$W_f = 6{,}000$ lbs.
Maximum B.HP.	$= 1{,}020$
Compression ratio	$= 5.7$
Maximum propeller efficiency,	$\eta_m = .78$
Maximum (L/D)	$= 8.48$
Maximum speed (V_m)	$= 116$ M.P.H.
Stalling speed, initial,	$V_s = 61.6$
Stalling speed, final,	$V_s = 49.2$
Stalling speed, average,	$V_s = 55.4$

From which—

Initial specific fuel consumption $c_0 = .522$ lb./B.HP./hr. $W_f/W_0 = .364$
Speed range ratio, initial, $(V_m/V_s) = 1.885$
Speed range ratio, final, $(V_m/V_s) = 2.361$
Speed range ratio, average, $(V_m/V_s) = 2.123$

Estimates for cruising range and endurance by the various methods are as follows:

I. BREGUET'S FORMULAS

For $W_f/W_0 = .364$ $B_R = 169.1$ and $B_E = 190.2$.
For the average $V_m/V_s = 2.123$ $c/c_0 = 1.205$ (fig. 9).
$\therefore$ $c = 1.205 \times .522 = .630$ lb./B.HP./hr.
Initial cruising speed $= 1.4 \times 61.6 = 86.2$.

$$\therefore \text{ Range} = 169.1 \times 8.48 \times \frac{.78}{.63} = 1{,}775 \text{ miles.}$$

and

$$\text{Endurance} = 190.2 \times 8.48 \times \frac{.78}{.63} \times \frac{1}{86.2} = 23.2 \text{ hours.}$$

II. NEW FORMULAS

For $W_f/W_0 = .364$ $K_R = 153.3$ and $K_E = 171.5$.
For initial $V_m/V_s = 1.885$ $c/c_0 = 1.086$ (fig. 9).
$\therefore$ Initial $c_1 = 1.086 \times .522 = .569$ lb./B.HP./hr.

$$\therefore \text{ Range} = 153.3 \times 8.48 \times \frac{.78}{.569} = 1{,}780 \text{ miles.}$$

and

$$\text{Endurance} = 171.5 \times 8.48 \times \frac{.78}{.569} \times \frac{1}{86.2} = 23.1 \text{ hours.}$$

III. FACTOR METHOD

Fuel per hr. $= 1{,}020 \times .522 = 533$ lb.

$$\text{Endurance at full throttle} = \frac{6000}{533} = 11.25.$$

Range at maximum speed $= 11.25 \times 116 = 1{,}306$.
For average $(V_m/V_s) = 2.123$, $F_R = 1.29$, $F_E = 1.965$.
Range $= 1.29 \times 1306 = 1{,}690$ miles.
Endurance $= 1.965 \times 11.25 = 22.1$ hours.
A step-by-step integration of the flight gives
Cruising range $= 1{,}739$ miles.
Cruising endurance $= 22.0$ hours.

The comparative values are as follows:

	Breguet's formulas	New formulas	Factor method	Step-by-step integration
Cruising range	1,775	1,780	1,690	1,739
Cruising endurance	23.2	23.1	22.1	22.0

From this comparison it is to be concluded that the three methods give results which are well within the limits of error of the known data.

TABLE I

BUREAU OF AERONAUTICS STANDARD FUEL CONSUMPTION DATA FOR THROTTLED CONDITIONS

R.P.M.	Specific fuel consumption ratio		$\frac{\text{B.HP.}}{\text{B.HP.}_0}$	$\frac{\text{B.HP.}_0}{\text{B.HP.}}$
	General service	Special mixture control	$\left(\frac{\text{R.P.M.}}{1,800}\right)^3$	
1,800	1.000	1.000	1.000	1.000
1,700	1.008	.983	.8423	1.1872
1,600	1.037	1.000	.7024	1.4237
1,500	1.090	1.040	.5787	1.728
1,400	1.163	1.100	.4705	2.1254
1,300	1.250	1.180	.3767	2.6546
1,200	1.352	1.266	.2963	3.375
1,100	1.463	1.363	.2282	4.382
1,000	1.586	1.463	.1715	5.831
900	1.712	1.565	.1250	8.000

TABLE II

VARIATION OF SPECIFIC FUEL CONSUMPTION WITH COMPRESSION RATIO

Compression ratio	Specific fuel consumption lb./B.HP./hr.
C. R.	c_0
4.6	0.566
4.8	.558
5.0	.550
5.2	.542
5.4	.534
5.6	.526
5.8	.518
6.0	.510
6.2	.502
6.4	.494
6.6	.486
6.8	.478
7.0	.470

$c_0=0.75-0.04\ C.\ R.$

TABLE III

CALCULATION OF CONSTANT B_R IN BREGUET'S FORMULA FOR RANGE

$$\text{Range}=863.47\ \frac{\eta}{c}\left(\frac{L}{D}\right)\log_{10}\left(\frac{W_0}{W_1}\right)=B_R\ \frac{\eta}{c}\left(\frac{L}{D}\right)\ \text{Miles.}$$

$\frac{W_1}{W_0}$	$\frac{W_0}{W_1}$	$\log_{10}\left(\frac{W_0}{W_1}\right)$	B_R $863.47\log_{10}\left(\frac{W_0}{W_1}\right)$
1.000	1.000	0	0
.90	1.1111	.0457574	39.510
.85	1.17647	.0705811	60.945
.80	1.2500	.0969100	83.679
.75	1.3333	.1249388	107.881
.70	1.42857	.1549019	133.752
.65	1.53846	.1870868	161.544
.60	1.66667	.2218488	191.560
.55	1.818182	.2596373	224.189
.50	2.000000	.3010300	259.930
.45	2.222220	.3467875	299.441
.40	2.500000	.3979400	343.609

TABLE IV

CALCULATION OF CONSTANT B_E IN BREGUET'S FORMULA FOR ENDURANCE

$$\text{Endurance}=\frac{750}{Vc_0}\ \frac{\eta}{c}\left(\frac{L}{D}\right)\left(\frac{1}{\sqrt{K}}-1\right)=\frac{B_E}{Vc_0}\ \frac{\eta}{c}\left(\frac{L}{D}\right)\text{hours.}$$

$\frac{W_1}{W_0}$ K	$\sqrt{K}$	$\frac{1}{\sqrt{K}}$	$\frac{1}{\sqrt{K}}-1$	B_E $750\left(\frac{1}{\sqrt{K}}-1\right)$
1.00	1.000	1.000	0	0
.90	.94868	1.05409	.05409	40.570
.85	.92195	1.08465	.08465	63.490
.80	.89443	1.11803	.11803	88.523
.75	.86603	1.15470	.15470	116.025
.70	.83666	1.19523	.19523	146.423
.65	.80623	1.24035	.24035	180.263
.60	.77460	1.29099	.29099	218.243
.55	.74162	1.34840	.34840	261.300
.50	.70711	1.41421	.41421	310.658
.45	.67082	1.49071	.49071	368.033
.40	.63246	1.58114	.58114	435.855

TABLE V

CALCULATION OF CONSTANTS IN NEW FORMULAS FOR RANGE AND ENDURANCE

GENERAL SERVICE

$\frac{W_1}{W_0}$	Range			Endurance			$\frac{K_R}{K_E}$
	$\left(\frac{W_1}{W_0}\right)^{.45}$	$1-\left(\frac{W_1}{W_0}\right)^{.45}$	K_R	$\left(\frac{W_0}{W_1}\right)^{.05}$	$\left(\frac{W_0}{W_1}\right)^{.05}-1$	K_E	
1.00	1.00	0	0	1	0	0	1.00
.90	.95369	.04631	38.590	1.005282	.005282	39.615	.9741
.85	.92948	.07052	58.767	1.008159	.008159	61.193	.9604
.80	.90446	.09554	79.617	1.011220	.011220	84.150	.9461
.75	.87857	.12143	101.192	1.014488	.014488	108.660	.9313
.70	.85171	.14829	123.575	1.017994	.017994	134.955	.9157
.65	.82378	.17622	146.850	1.021773	.021773	163.297	.8993
.60	.79464	.20536	171.133	1.025870	.025870	194.025	.8820
.55	.76412	.23588	196.567	1.030343	.030343	227.573	.8638
.50	.73204	.26796	223.300	1.035265	.035265	264.488	.8443
.45	.69814	.30186	251.550	1.040733	.040733	305.498	.8234
.40	.66211	.33789	281.575	1.046880	.046880	351.600	.8008

$$\text{Range} = K_R \frac{\eta}{c_1} \left(\frac{L}{D}\right) \text{ miles.}$$

$$\text{Endurance} = \frac{K_E}{Vc_0} \frac{\eta}{c_1} \left(\frac{L}{D}\right) \text{ hours.}$$

$$\text{Average cruising speed} = \frac{K_R}{K_E} \times \text{initial cruising speed.}$$

TABLE VI

CALCULATION OF CONSTANTS IN NEW FORMULAS FOR RANGE AND ENDURANCE

SPECIAL MIXTURE CONTROL

$\frac{W_1}{W_0}$	Range			Endurance			$\frac{K'_R}{K'_E}$
	$\left(\frac{W_1}{W_0}\right)^{.42}$	$1-\left(\frac{W_1}{W_0}\right)^{.45}$	K'_R	$\left(\frac{W_0}{W_1}\right)^{.08}$	$\left(\frac{W_0}{W_1}\right)^{.08}-1$	K'_E	
1.00	1.000	0	0	1.00	0	0	1.00
.90	.95671	.04329	38.652	1.008464	.008464	39.675	.9742
.85	.93402	.06598	58.911	1.013086	.013086	61.341	.9604
.80	.91054	.08946	79.875	1.018012	.018012	84.431	.9460
.75	.88619	.11381	101.616	1.023281	.023281	109.130	.9311
.70	.86088	.13912	124.214	1.028945	.028945	135.680	.9155
.65	.83449	.16551	147.777	1.035063	.035063	164.358	.8991
.60	.80691	.19309	172.402	1.041712	.041712	195.525	.8817
.55	.77795	.22205	198.259	1.048989	.048989	229.636	.8634
.50	.74742	.25258	225.518	1.057018	.057018	267.272	.8437
.45	.71507	.28493	254.402	1.065965	.065965	309.211	.8227
.40	.68056	.31944	285.214	1.076057	.076057	356.517	.8000

$$\text{Range} = K'_R \frac{\eta}{c_1} \left(\frac{L}{D}\right) \text{ miles.}$$

$$\text{Endurance} = \frac{K'_E}{Vc_0} \frac{\eta}{c_1} \left(\frac{L}{D}\right)$$

$$\text{Average cruising speed} = \frac{K'_R}{K'_E} \times \text{initial cruising speed.}$$

TABLE VII

CONSTANTS IN THE EQUATIONS FOR RANGE AND ENDURANCE

$\frac{W_f}{W_0}$	Breguet's equations		New equations			
			General service		Special-mixture control	
	Range	Endurance	Range	Endurance	Range	Endurance
	B_R	B_E	K_R	K_E	K'_R	K'_E
0.01	3.8	3.8	3.8	3.8	3.8	3.8
.02	7.6	7.6	7.5	7.6	7.5	7.6
.03	11.4	11.5	11.3	11.4	11.3	11.4
.04	15.3	15.4	15.2	15.3	15.2	15.3
.05	19.2	19.4	19.0	19.3	19.0	19.3
.06	23.2	23.5	22.9	23.2	22.9	23.3
.07	27.2	27.7	26.8	27.3	26.8	27.3
.08	31.3	31.9	30.7	31.3	30.7	31.4
.09	35.4	36.2	34.6	35.5	34.7	35.5
.10	39.5	40.6	38.6	39.6	38.7	39.7
.11	43.7	45.0	42.6	43.8	42.7	43.9
.12	47.9	49.5	46.6	48.1	46.7	48.2
.13	52.2	54.1	50.6	52.4	50.7	52.5
.14	56.6	58.8	54.7	56.8	54.8	56.9
.15	61.0	63.5	58.8	61.2	58.9	61.3
.16	65.4	68.3	62.9	65.7	63.1	65.8
.17	69.9	73.2	67.0	70.2	67.2	70.4
.18	74.4	78.2	71.2	74.8	71.4	75.0
.19	79.0	83.3	75.4	79.4	75.6	79.7
.20	83.7	88.5	79.6	84.2	79.9	84.4
.21	88.4	93.8	83.9	88.9	84.2	89.2
.22	93.2	99.2	88.2	93.8	88.5	94.1
.23	98.0	104.7	92.5	98.7	92.8	99.1
.24	102.9	110.3	96.8	103.6	97.2	104.1
.25	107.9	116.0	101.2	108.7	101.6	109.1
.26	112.9	121.9	105.6	113.8	106.1	114.3
.27	118.0	127.8	110.0	119.0	110.6	119.5
.28	123.2	133.9	114.5	124.2	115.1	124.8
.29	128.4	140.1	119.0	129.6	119.6	130.2
.30	133.8	146.4	123.6	135.0	124.2	135.7
.31	139.1	152.9	128.2	140.5	128.9	141.2
.32	144.6	159.5	132.8	146.0	133.5	146.9
.33	150.2	166.3	137.4	151.7	138.2	152.6
.34	155.8	173.2	142.1	157.5	143.0	158.4
.35	161.5	180.3	146.9	163.3	147.8	164.4
.36	167.4	187.5	151.6	169.3	152.6	170.4
.37	173.3	194.9	156.4	175.3	157.5	176.5
.38	179.3	202.5	161.3	181.4	162.4	182.7
.39	185.4	210.3	166.2	187.7	167.4	189.1
.40	191.6	218.2	171.1	194.0	172.4	195.5
.41	197.9	226.4	176.1	200.5	177.5	202.1
.42	204.3	234.8	181.2	207.1	182.6	208.8
.43	210.8	243.5	186.3	213.8	187.8	215.6
.44	217.4	252.3	191.4	220.6	193.0	222.6
.45	224.2	261.3	196.6	227.6	198.3	229.6
.46	231.1	270.6	201.8	234.7	203.6	236.9
.47	238.1	280.2	207.1	241.9	209.0	244.2
.48	245.2	290.1	212.4	249.3	214.4	251.8
.49	252.5	300.2	217.8	256.8	220.0	259.4
.50	259.9	310.7	223.3	264.5	225.5	267.3
.51	267.5	321.4	228.8	272.3	231.2	275.3
.52	275.2	332.5	234.4	280.4	236.9	283.5
.53	283.1	344.0	240.0	288.6	242.6	291.9
.54	291.2	355.8	245.8	297.0	248.5	300.4
.55	299.4	368.0	251.6	305.5	254.4	309.2
.56	307.9	380.7	257.4	314.3	260.4	318.2
.57	316.5	393.8	263.3	323.3	266.5	327.4
.58	325.3	407.3	269.3	332.5	272.6	336.9
.59	334.4	421.4	275.4	341.9	278.9	346.6
.60	343.6	435.9	281.6	351.6	285.2	356.5

NOTE:

$$\text{Range miles} = B_R\left(\frac{L}{D}\right)\left(\frac{\eta}{c}\right) \quad \text{Breguet's equation.}$$

$$= K_R\left(\frac{L}{D}\right)\left(\frac{\eta}{c_1}\right) \quad \text{New equation—General service.}$$

$$= K'_R\left(\frac{L}{D}\right)\left(\frac{\eta}{c_1}\right) \quad \text{New equation—Special-mixture control.}$$

$$\text{Endurance, hours} = \frac{B_E}{V_{c_0}}\left(\frac{L}{D}\right)\left(\frac{\eta}{c}\right) \quad \text{Breguet's equation.}$$

$$= \frac{K_E}{V_{c_0}}\left(\frac{L}{D}\right)\left(\frac{\eta}{c_1}\right) \quad \text{New equation—General service.}$$

$$= \frac{K'_E}{V_{c_0}}\left(\frac{L}{D}\right)\left(\frac{\eta}{c_1}\right) \quad \text{New equation—Special-mixture control.}$$

$$\frac{W_f}{W_0} = \text{Ratio } \frac{\text{Weight of fuel}}{\text{Initial gross load}}.$$

TABLE VIII

VARIATION IN T. HP. REQUIRED ASSUMING CRUISING SPEED=1.40 V_s

$\frac{V}{V_s}$	VE-7	PB-1	D-27	D-26	DH-4	CS-1	MO-1	TG	TS	EM	Average
1.4	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
1.6	1.25	1.22	1.24	1.23	1.30	1.22	1.22	1.16	1.24	1.17	1.225
1.8	1.65	1.59	1.56	1.56	1.68	1.56	1.53	1.46	1.58	1.48	1.565
2.0	2.15	2.08	1.99	2.03	2.18	2.01	1.95	1.94	2.06	1.97	2.036
2.2	2.86	2.71	2.55	2.57	2.91	2.62	2.46	2.54	2.64	2.59	2.645
2.4	3.48	3.44	3.26	3.27	3.70	3.28	3.06	3.34	3.29	3.27	3.339
2.6	4.25	4.27	4.10	4.15	4.65	4.02	3.84	4.44	4.12	4.02	4.186
2.8	5.10	5.29	5.21	5.15	5.75	4.90	4.72	5.24	5.06	4.93	5.135
3.0	6.18	6.36	6.33	6.59	7.00	5.90	5.65	6.75	6.14	5.98	6.288

TABLE IX

CRUISING B.HP. IN TERMS OF MAXIMUM B. HP.

$\frac{V_m}{V_s}$	$\frac{V_m}{V_c}$	$\frac{\text{T.HP.}_m}{\text{T.HP.}_c}$	$\frac{(c_2')_c}{(c_2')_m}$	$\frac{\left(\frac{V}{ND_c}\right)}{\left(\frac{V}{ND_m}\right)}$	$\frac{\eta_c}{\eta_m}$	$\frac{\text{B.HP.}_m}{\text{B.HP.}_c}$	$\frac{\text{B.HP.}}{\text{B.HP.}_m}$	Specific fuel consumption ratio: General service	Specific fuel consumption ratio: Special mixture control
	$\frac{V_m}{1.4\,V_s}$	From Figure 6	See note 1	See note 2	See note 3	See note 4		$\frac{c}{c_0}$	$\frac{c'}{c'_0}$
1.4	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
1.6	1.143	1.225	1.218	.936	.997	1.221	.819	1.012	.984
1.8	1.286	1.565	1.360	.903	.988	1.546	.647	1.055	1.014
2.0	1.429	2.036	1.433	.888	.984	2.003	.499	1.141	1.082
2.2	1.571	2.645	1.467	.880	.981	2.595	.385	1.241	1.172
2.4	1.714	3.339	1.508	.872	.978	3.266	.306	1.340	1.258
2.6	1.857	4.186	1.538	.866	.976	4.086	.245	1.434	1.340
2.8	2.000	5.135	1.558	.863	.975	5.007	.200	1.522	1.412
3.0	2.143	6.288	1.570	.860	.974	6.125	.163	1.605	1.480

NOTES:

(1) $\frac{(c_2')_c}{(c_2')_m} = \frac{\text{T.HP.}_c}{\text{T.HP.}_m} \times \left(\frac{V_m}{V_c}\right)^3$

(2) $\frac{\left(\frac{V}{ND}\right)_c}{\left(\frac{V}{ND}\right)_m} = \left[\frac{(c_2')_c}{(c_2')_m}\right]^{-1/3}$

(3) $\frac{\eta_c}{\eta_m}$ is from the general efficiency curve in N. A. C. A. Technical Note No. 168.

(4) $\frac{\text{B.HP.}_m}{\text{B.HP.}_c} = \frac{\text{T.HP.}_m}{\text{T.HP.}_c} \times \frac{\eta_c}{\eta_m}$

42488—27——8

TABLE X

CALCULATION OF RANGE AND ENDURANCE FACTORS

$\frac{V_m}{V_s}$	$\frac{V_m}{V_c}$	$\frac{B.HP._m}{B.HP._c}$	$\frac{c}{c_0}$	General service		Special mixture control		
				Endurance factor F_E	Range factor F_R	$\frac{c'}{c_0'}$	Endurance factor F_E'	Range factor F_R'
1.4	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
1.6	1.143	1.221	1.012	1.206	1.055	.984	1.241	1.086
1.8	1.286	1.546	1.055	1.465	1.139	1.014	1.525	1.186
2.0	1.429	2.003	1.141	1.755	1.228	1.082	1.852	1.295
2.2	1.571	2.595	1.241	2.092	1.333	1.172	2.213	1.409
2.4	1.714	3.266	1.340	2.437	1.421	1.258	2.596	1.514
2.6	1.857	4.086	1.434	2.849	1.534	1.340	3.049	1.642
2.8	2.000	5.007	1.522	3.290	1.645	1.412	3.546	1.773
3.0	2.143	6.125	1.605	3.816	1.780	1.480	4.139	1.931

$$F_E = \text{Ratio } \frac{\text{Endurance at cruising speed}}{\text{Endurance at maximum speed}} = \frac{B.HP._m}{B.HP._c} \frac{c_0}{c}$$

$$F_R = \text{Ratio } \frac{\text{Range at cruising speed}}{\text{Range at maximum speed}} = F_E \frac{V_c}{V_m}$$

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