REPORT No. 548

EFFECT OF TIP SHAPE AND DIHEDRAL ON LATERAL-STABILITY CHARACTERISTICS

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SUMMARY

Tests were conducted in the N. A. C. A. 7- by 10-foot wind tunnel to determine the effect of wing-tip shape and dihedral on some of the aerodynamic characteristics of Clark Y wings that affect the performance and lateral stability of airplanes. Force tests at several angles of yaw and rotation tests at zero yaw were made. From these tests the rates of change of rolling-moment, yawing-moment, and cross-wind force coefficients with angle of yaw and the rate of change of rolling-moment coefficient with rolling were determined.

The tests showed that the plan form of a wing tip as well as the elevation shape had considerable effect on the rate of change of rolling- and yawing-moment coefficients with angle of yaw. The tests also showed that with dihedral of only the outer one-fourth of each semispan, the dihedral effect was maintained to a much higher angle of attack than when the complete semispan had dihedral. At normal angles of attack, the increments of rate of change of rolling moment with angle of yaw due to dihedral may be calculated with satisfactory accuracy.

INTRODUCTION

As part of a general program on the improvement of safety in flight the N. A. C. A. has instituted a research of various means for improving the lateral stability of airplanes. That part of the investigation reported herein was made to provide fundamental data on the effect of wing-tip shape and dihedral on some of the lateral-stability factors. The part of the program dealing with tip shape was requested by the Army Air Corps.

The particular wing-tip shapes tested were chosen after a study of existing pertinent data. The pressure-distribution tests of reference 1 revealed that the high local loading near the extreme tips of rectangular wings could be reduced by rounding the tips and also that the centers of pressure could be located nearly on spanwise straight lines by having the extreme tip at least as far forward as 35 percent of the basic chord from the leading edge. Force tests of references 2 to 5 showed that the maximum L/D ratio of a wing with a square tip could be improved by substituting practically any other shape of tip, the greatest improvement being found with tips having a ratio of the tip length to the basic chord of between 0.75 and 1.50. (See fig. 6.) The maximum lift coefficient was increased with tip-length/basic-chord ratios greater than 0.75 but was decreased with shorter tips. For the present tests, two tips were designed having tip-length/basic-chord ratios of 0.75 and 1.00 and similar plan forms with the extreme tip 35 percent of the basic chord aft of the leading edge. In addition, several modifications so small as to be classed as "tip fairings" rather than as new shapes were tested to determine the effect of such fairings on the aerodynamic characteristics of a rectangular wing since in actual practice small fairings are used to improve the otherwise blunt-end appearance of rectangular wings.

The effect of elevation shape of a particular tip on lift and drag was tested by the Navy (unpublished data). It was found that, although the effect was not great, there was a definite improvement in general performance characteristics in having the lower surface of the tip curve upward. In order to measure this improvement and to determine the effective dihedral angle of such shapes, three elevation shapes of a particular tip were included in the present tests.

The effect of dihedral on the aerodynamic characteristics of airplanes has been experimentally determined in several instances but with small wing models at low air speeds (references 6 to 8). The tests reported herein included the determination of the effect of dihedral angle of the outer 25, 50, and 93 percent of the semispan of a rectangular wing equipped with the rounded tip determined to be the optimum in the first part of the tests. Tests of a rectangular wing with dihedral were included for comparison.

The effect of the various wing shapes on the rates of change of rolling-moment, yawing-moment, and cross-wind force coefficients with angle of yaw and the rate of change of rolling-moment coefficient with rolling was determined in addition to the general aerodynamic characteristics. The results are presented in such a form that they may be directly applied in lateral-stability calculations.

APPARATUS

The N. A. C. A. 7- by 10-foot wind tunnel, which was used in this investigation, has a closed return passage and an open test section (reference 9). The

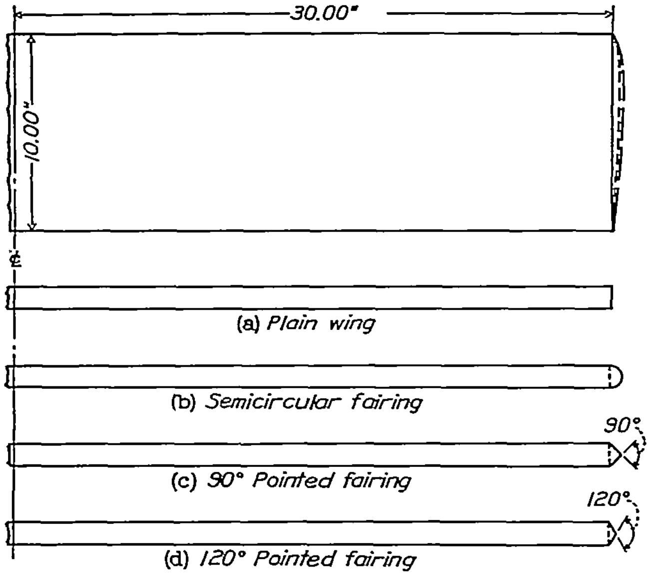

FIGURE 1.—A rectangular wing with small tip fairings.

model under test is attached to a small mounting plate resting on the balance tripod so that the center of moments on the model is at the midspan quarter-chord point. The six-component balance indicates the forces and moments with respect to the wind, or tunnel, axes. The mounting plate is fastened to a

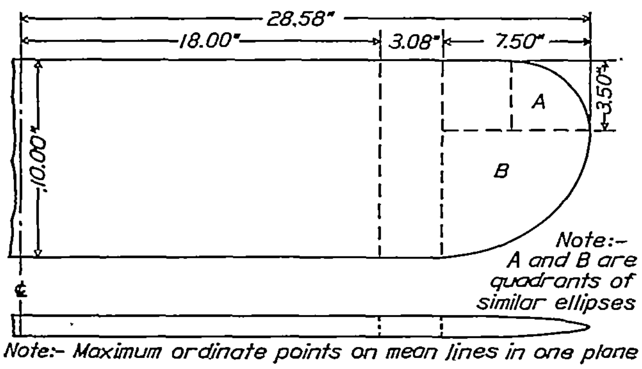

FIGURE 2.—A wing with three-quarters-chord length modified elliptical tip.

spindle that may be rotated in yaw about a vertical axis through a system of gears actuated from without the tunnel. The angle of attack may be changed while the tunnel is in operation. In order to apply a rolling velocity to the model so that the rolling moment accompanying the rolling about the wind axis may be measured, a special rotation mounting replaces the force-test tripod.

Eleven wing models were used in the present investigation. All but the ones with small tip fairings shown in figure 1 are listed in the first column of table I. Laminated mahogany was used in the construction of the models, which were accurate to within ±0.005 inch of the specified Clark Y ordinates. The small tip fairings were screwed directly to the ends of the rectangular wing of aspect ratio 6. For the other models, a common center segment of 10-inch chord and 36-inch span was used, the tips under test being sufficiently long to make the aspect ratio of the wing equal to 6 when they were attached to the ends fo the common center segment.

For the wings with rounded tips Clark Y sections were maintained without washout to the tips. Two of the tips had the same elevation shape (maximum ordinate points on mean lines in one plane) but different plan forms. One of these had a length equal to three-quarters of the basic wing chord (fig. 2) and the other a length equal to the basic wing chord (fig. 3 (b)). The tips were similar in that they were composed of two quadrants of similar ellipses and a rectangle with the extreme tip 35 percent of the basic chord back of the leading edge of the center section. The other two tips had the same plan form as the above-described larger tip but had different elevation

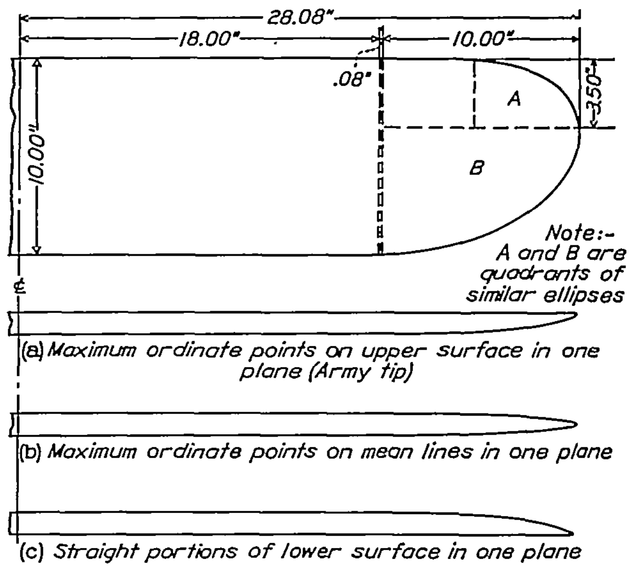

FIGURE 3.—A wing with one-chord length modified elliptical tip having three elevation shapes.

shapes. For one the maximum ordinate points on the upper surface were in one plane (fig. 3 (a)), adopted as the standard Army tip, and for the other the straight portions of the lower surface were in one plane (fig. 3 (c)). In order to find the effect of aspect ratio, the Army tip was attached to the 60-inch span rectangular wing, making the aspect ratio 8.39 instead of 6.

For the dihedral tests, the wing was cut at the proper sections and held at the desired dihedral angles by metal straps set in the surface. With the Army tip in use, these cuts were made 25, 50, and 93 percent of the wing semispan from the tip. (See fig. 4.) With the square tips, dihedral straps were used only at the 93-percent semispan cut. (See fig. 5.)

TESTS

The tests were made at a dynamic pressure of 16.37 pounds per square foot, which corresponds to 80 miles per hour under standard conditions, making the aver-

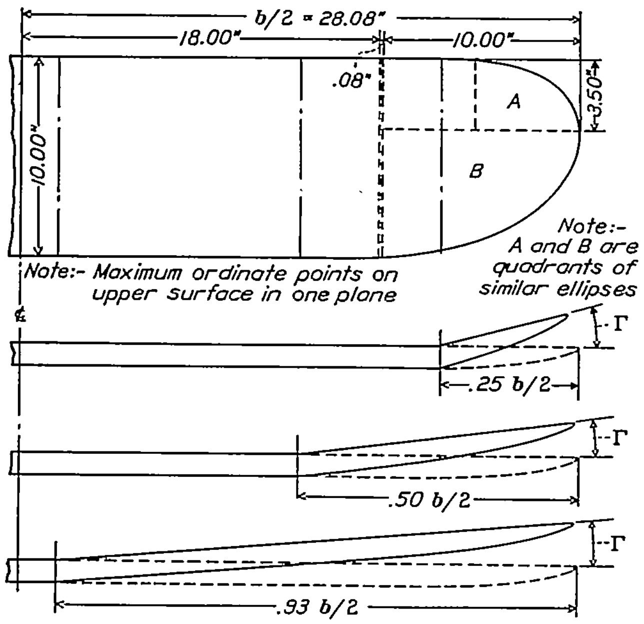

FIGURE 4.—A wing with Army tip showing various dihedral arrangements.

age Reynolds Number 609,000 computed on the basic chord of the wing.

With the rectangular wing, force tests were made at a large number of angles of yaw (ψ') to determine the

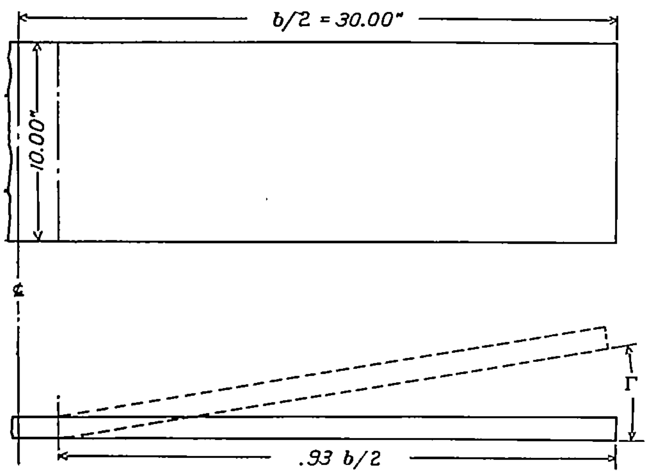

FIGURE 5.—A rectangular wing with dihedral.

minimum number of angles necessary to define the slope of the curves of rolling- and yawing-moment and cross-wind force coefficients against angle of yaw. Values of $\psi'= -5°$, 0°, and 5° having been found sufficient, the tests of the other models were made at these settings and also at $\psi'=10°$ and $\psi'=20°$ to cover the range likely to be encountered in flight.

Rolling tests were made with the rectangular wing driven in rotation at values of $\frac{p'b}{2V}$ of 0.02 and 0.05 in both directions, where p' is the angular velocity in roll about the wind axis. These tests showed that the slope of the curves of rolling-moment coefficient against rate of rotation at zero rotation could be determined from the tests at $\frac{p'b}{2V}=0.05$ in both directions for angles of attack encountered in normal flying below the stall. Consequently, the remainder of the rotation tests were made only at this rate.

RESULTS

The results, uncorrected for tunnel effects, are presented in the form of standard absolute coefficients with respect to wind axes that intersect on the wing model at the midspan quarter-chord point of the basic chord. The wing area, span, and average chord used in computing the coefficients were those of the wing with no dihedral.

A copy of the extensive tabulated results of the tests herein analyzed is available upon request from the National Advisory Committee for Aeronautics.

The force-test data were plotted against angle of yaw; the rates of change of rolling-moment coefficient $\left(\frac{dC_l'}{d\psi'}\right)_0$, yawing-moment coefficient $\left(\frac{dC_n'}{d\psi'}\right)_0$, and cross-wind force coefficient $\left(\frac{dC_c}{d\psi'}\right)_0$ with angle of yaw were determined at 0° yaw. In addition, increments of rates of change due to dihedral (designated by Δ) were determined for angles of attack of 0°, 10°, and 15° and are plotted against angle of dihedral. From the rotation tests the rates of change of rolling-moment coefficient with rate of rotation were determined by summing the values for positive and negative rotation at a rate of $\frac{p'b}{2V}=0.05$ and dividing by 0.10. These rates of change are designated $\left(\frac{dC_l'}{d\frac{p'b}{2V}}\right)_0$.

The above-mentioned slopes at zero yaw are sufficient for the normal range of angles of attack and yaw encountered in flight and for computations based on the theory of small oscillations. At the angles of attack above the stall and for displacements in yaw greater than 10°, however, the factors cannot be computed directly from the slopes at zero yaw. Unsymmetrical stalling of the wing and the generally unsteady conditions encountered at high angles of attack cause the results to vary widely.

The addition of small tip fairings had such a minor effect on the characteristics of the rectangular wing that the results have not been included. The interesting result of the tests is that small tip fairings may be added to a rectangular wing of aspect ratio 6 without appreciably altering the aerodynamic characteristics.

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ACCURACY OF RESULTS

The dynamic pressure was maintained constant within ±0.25 percent. The angles of attack, yaw, and dihedral were accurate within ±0.10°. The coefficients for a particular model are believed to be accurate within the following limits:

$C_{L_{max}}$ ±0.005

$C_{D_{min}}$ ±0.0005

C_l' ±0.001 except at angles of attack above 25° where angles of yaw of opposite sign may give values differing by 0.020.

C_n' ±0.001 except at high angles of attack where it may be ±0.010.

C_c ±0.002

Because of slight inaccuracies in the models resulting from alterations made during the tests, it is believed that for purposes of comparison between models the accuracy should be considered to be ±0.001 for $C_{D_{min}}$ and ±0.010 for $C_{L_{max}}$.

PERFORMANCE CRITERIONS

The criterions used to measure the effect of the wing shapes on airplane performance are: The maximum lift coefficient $C_{L_{max}}$, which gives an indication of the landing speed; the minimum drag coefficient $C_{D_{min}}$, which is a high-speed criterion; the ratio of maximum lift coefficient to minimum drag coefficient $C_{L_{max}}/C_{D_{min}}$, a speed-range criterion; and the lift-drag ratio L/D at a lift coefficient of 0.70, which is a criterion of the rate of climb. The values of these criterions for the wings tested have been tabulated in table I for a direct comparison.

TABLE I.—PERFORMANCE CRITERIONS OF RECTANGULAR WINGS HAVING VARIOUS TIP SHAPES AND DIHEDRAL ANGLES

[All wings not otherwise designated have an aspect ratio of 6]

Wing shape	Dihedral angle Γ	Criterion: $C_{L_{max}}$	$C_{D_{min}}$	$\frac{C_{L_{max}}}{C_{D_{min}}}$	$\frac{L}{D}$ at C_L=0.70
	°				
Rectangular tip, 0.93 b/2 dihedral	0	1.268	0.0158	80.3	15.2
Do	5	1.242	.0162	76.6	14.9
Do	10	1.222	.0168	72.7	14.7
0.75c rounded tip [1]	0	1.265	.0160	79.5	15.9
1.00c rounded tip [1]	0	1.278	.0154	82.9	16.6
Do.[2]	0	1.277	.0155	82.3	16.3
1.00c rounded tip,[3] 0.25 b/2 dihedral	0	1.281	.0156	82.1	16.2
Do	7	1.293	.0159	81.3	16.4
Do	15	1.269	.0163	79.0	15.9
Do	30	1.262	.0167	75.5	15.5
Do	45	1.225	.0168	73.0	15.3
1.00c rounded tip,[3] 0.50 b/2 dihedral	0	1.298	.0162	80.1	----
Do	5	1.295	.0162	80.0	15.4
Do	10	1.288	.0162	79.9	15.4
Do	15	1.262	.0169	74.7	15.3
1.00c rounded tip,[3] 0.93 b/2 dihedral	0	1.288	.0162	79.5	----
Do	2	1.295	.0168	77.0	15.2
Do	5	1.275	.0164	77.7	15.2
Do	10	1.248	.0162	77.0	15.2
Do	15	1.205	.0156	77.2	14.8
1.00c rounded tip [3] (A. R. 8.39)	0	1.321	.0148	89.3	18.5

[1] Maximum ordinate points on mean lines in one plane.
[2] Straight portions of lower surface in one plane.
[3] Maximum ordinate points on upper surface in one plane (Army tip).

Effect of plan form.—The rectangular wing with rounded tips of a length equal to three-quarters the basic chord had practically the same performance characteristics as the plain rectangular wing of the same aspect ratio except for the rate-of-climb criterion, which was increased in the order of 4 percent. The rounded tip equal to the basic chord in length had, however, improved performance characteristics; the speed-range ratio was increased about 3 percent and the climb criterion about 9 percent. Although these percentages are within the experimental error, it is believed that they indicate a definite trend. The effect of these two similarly shaped tips of different lengths is in fair agreement with previous systematic tests of the effect of plan form as shown in figure 6. The improvement in rate-of-climb criterion is explained by the fact that the span load distribution for the wings with rounded tips approaches the ideal, which results in a lower induced drag.

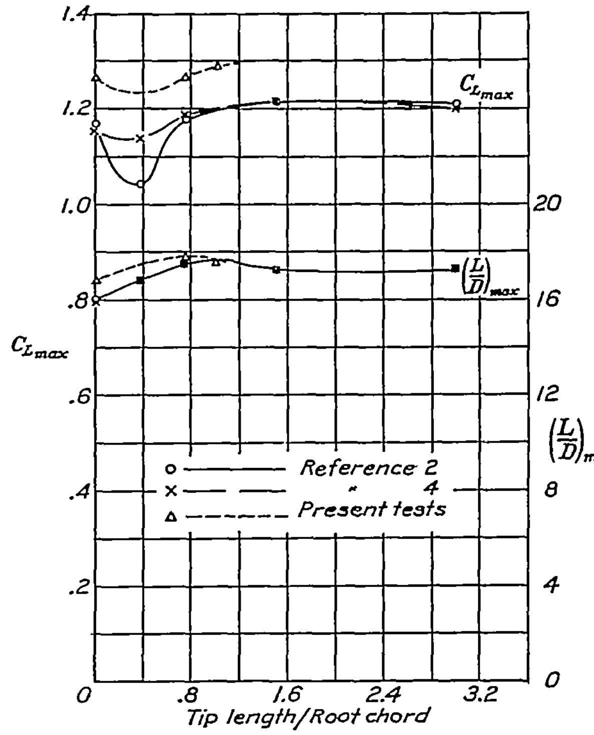

FIGURE 6.—Effect of tip length on $C_{L_{max}}$ and $(L/D)_{max}$.

Effect of elevation shape.—The effect of elevation shape of the one-chord length tip on the performance criterions is shown in table I. The results of these tests are in agreement with previous unpublished Navy tests in that, although the effect of elevation shape on the performance criterions is not great, there is a certain advantage in raising the extreme tip at least as high as the midpoint. The tip adopted by the Army as their standard rounded tip is the longer rounded tip with the maximum ordinate points on the upper surface in one plane.

Effect of aspect ratio.—The wing with aspect ratio 8.39 was made by adding the Army tip to the rectangular wing of aspect ratio 6, which makes the tip portion a smaller percentage of the span than the wing of aspect ratio 6 with Army tip. An appreciable improvement was obtained in all the performance criterions with the wing of aspect ratio 8.39 over those of the same wing of aspect ratio 6.

Effect of dihedral.—The effects of the various dihedral arrangements on the performance criterions are

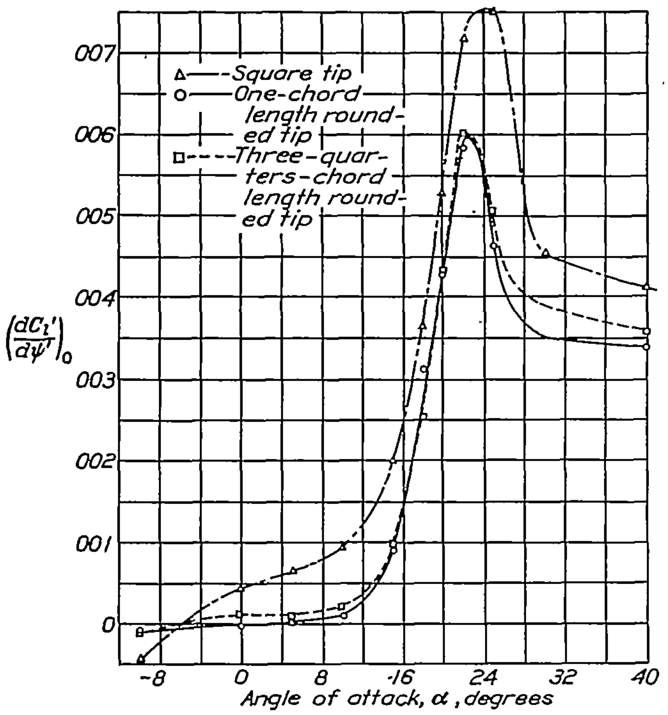

FIGURE 7.—Effect of wing tip plan form on rate of change of rolling-moment coefficient with angle of yaw. Maximum ordinate points on mean lines in one plane.

compared in table I. As the coefficients are based on the area of the wings with no dihedral, the maximum lift would be expected to be lower with dihedral because of the reduction of the projected area of the wing on a lateral plane. Such was the case in the actual tests; the reduction in maximum lift coefficients was proportional to the reduction in the projected area. The minimum drag coefficients, however, increased slightly in every case except with the Army tip with 93 percent semispan dihedral. The coefficients used would be expected to remain constant except for interference effects at the juncture of the portions of the wing forming the dihedral angle. In all cases the effect was no larger than about 5 percent. Since the maximum lift coefficient was decreased and the minimum drag coefficient was increased, the speed-range ratio was reduced about 10 percent for extreme dihedral angles except for the wing with Army tip and 93 percent semispan dihedral, for which the ratio remained practically constant. For dihedral angles normally used (5° or less), however, the effect is negligible. All the dihedrals reduced the rate-of-climb criterion by a slight amount.

AERODYNAMIC CHARACTERISTICS AFFECTING LATERAL STABILITY

In order to make a complete determination of the asymmetric motions and the lateral stability of an airplane, there are required nine resistance derivatives determined from the rate of change of rolling moment, of yawing moment, and of cross-wind force with cross-wind velocity, with rolling velocity, and with yawing velocity. The results are presented in such a manner that a direct determination is possible of four of the derivatives, namely, the three computed from the rate of change of rolling moment, of yawing moment, and of cross-wind force with cross-wind velocity, and the one computed from the rate of change of rolling moment with rolling velocity. Although these four factors are the important ones affected by the wing shapes tested, it is considered outside the scope of this report to make detailed lateral-stability calculations in which assumptions for the remaining factors would be required. Consequently, only the quantitative effects of the wing shapes on the four above-mentioned factors will be discussed.

Effect of plan form.—When a wing is yawed, the span load distribution is considerably changed and a rolling moment results that may become very large at high angles of attack. The particular changes that occur are clearly shown by means of pressure-distribution tests in reference 10. The particular plan form of the wing has an appreciable effect on the results as shown in figure 7 where the rates of change of rolling-moment coefficient with angle of yaw $\left(\frac{dC_l'}{d\psi'}\right)_0$ is given for three wings of the same elevation shape but having different plan forms. The effect is pronounced at an angle of attack of 10°, which corresponds to a lift coefficient of about 1.0, where the rate of change given by

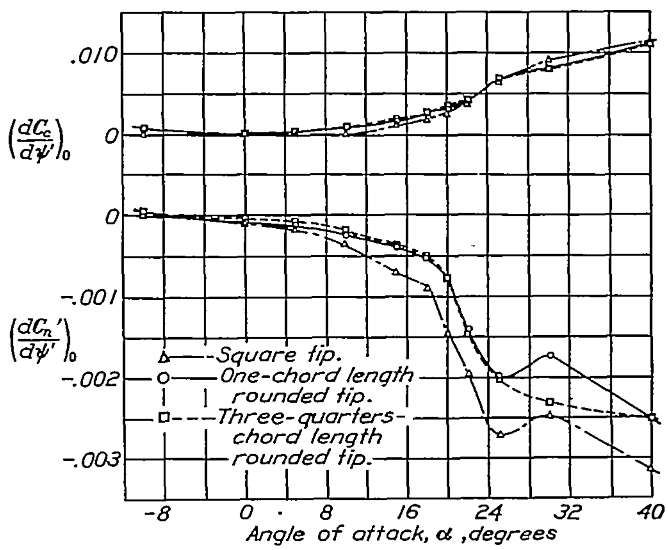

FIGURE 8.—Effect of wing tip plan form on rate of change of cross-wind force and yawing-moment coefficients with angle of yaw. Maximum ordinate points on mean lines in one plane.

the square tip was about four times that of the rounded tip. The effect of plan form on $\left(\frac{dC_n'}{d\psi'}\right)_0$ and $\left(\frac{dC_c}{d\psi'}\right)_0$ as shown in figure 8 is small compared with the values due to the fuselage and tail. At angles of attack near that for minimum speed, the values of $\left(\frac{dC_n'}{d\psi'}\right)_0$, which is a measure of the directional stability, were about twice as high for the rectangular wing as for the wings with rounded tips. The values of rates of change of rolling-moment coefficient with rolling $\left(\frac{dC_l'}{d\frac{p'b}{2V}}\right)_0$ as shown in figure 9 were reduced by rounding the tips in the order

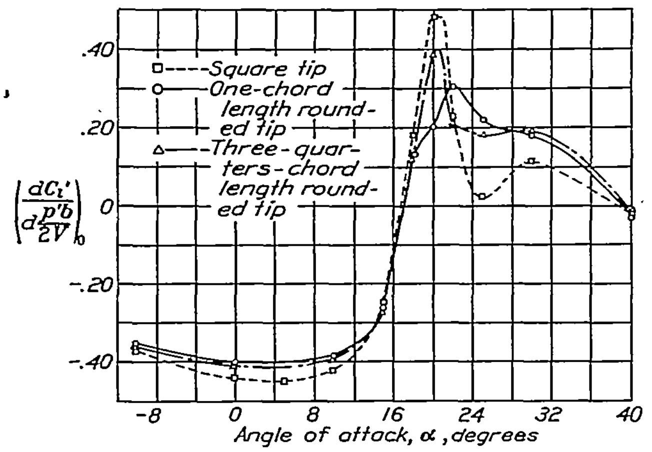

FIGURE 9.—Effect of wing tip plan form on rate of change of rolling-moment coefficient with rate of roll. Maximum ordinate points on mean lines in one plane.

that would be expected, the greatest values being for the rectangular wing.

Effect of elevation shape.—In figure 10 is shown the effect of elevation shape of a one-chord length rounded tip on $\left(\frac{dC_l'}{d\psi'}\right)_0$. Up to 15° angle of attack, changing

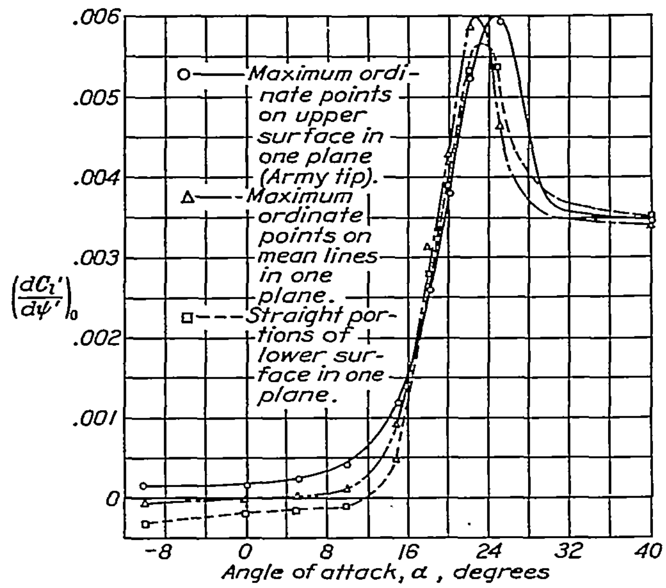

FIGURE 10.—Effect of elevation shape of a one-chord length rounded tip on rate of change of rolling-moment coefficient with angle of yaw.

the elevation shape resulted in a parallel shifting of the curves. Later tests showed that this shift could be expressed in terms of effective full-span dihedral; placing the maximum ordinate points on mean lines in one plane was equivalent to giving a wing with the Army tip (maximum ordinate points on upper surface in one plane) a negative dihedral angle of 1½°; whereas placing the straight portions of the lower surface in one plane

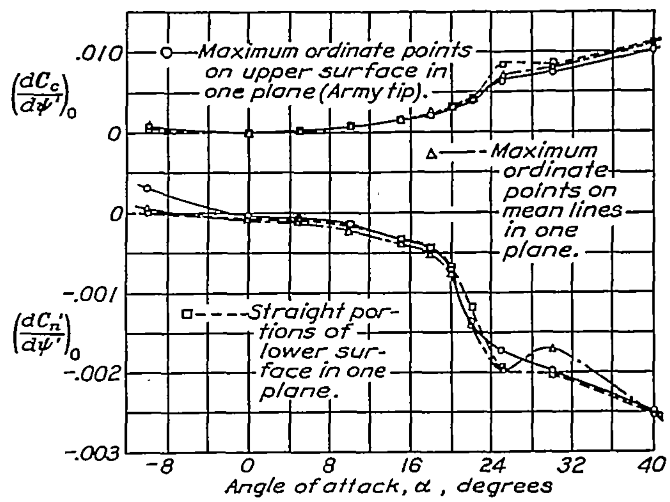

FIGURE 11.—Effect of elevation shape of a one-chord length rounded tip on rate of change of cross-wing force and yawing-moment coefficients with angle of yaw.

was equivalent to a negative dihedral angle of 2½°. Above 15° angle of attack, the elevation shape had practically no effect. Neither the values of $\left(\frac{dC_n'}{d\psi'}\right)_0$

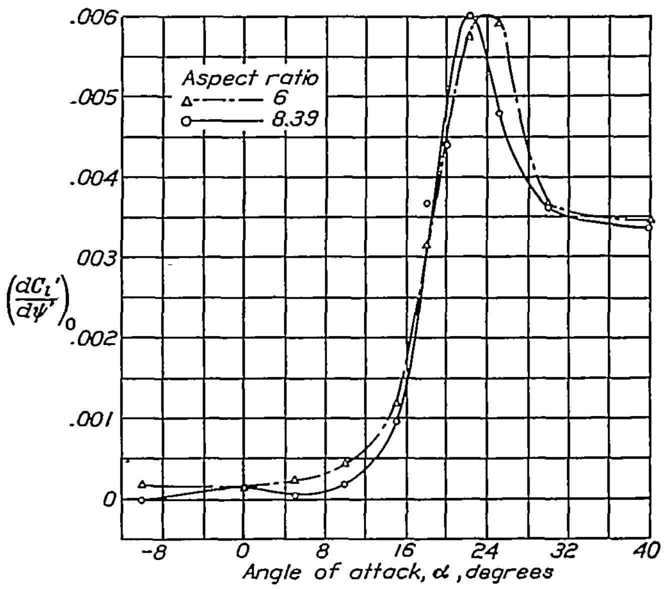

FIGURE 12.—Effect of aspect ratio of a wing with Army tip on rate of change of rolling-moment coefficient with angle of yaw. Maximum ordinate points on upper surface in one plane.

and $\left(\frac{dC_c'}{d\psi'}\right)_0$, given in figure 11, nor the values of $\left(\frac{dC_l'}{d\frac{p'b}{2V}}\right)_0$ were affected by elevation shape.

Effect of aspect ratio.—It had been previously found that increasing the aspect ratio of a wing decreased $\left(\frac{dC_l'}{d\psi'}\right)_0$. (See reference 11.) The results of the present tests (see fig. 12) show that the reduction in $\left(\frac{dC_l'}{d\psi'}\right)_0$ for the wing of aspect ratio 8.39 was equivalent to a negative dihedral angle of 1½° for the wing of aspect ratio 6. On the other hand, the values of $\left(\frac{dC_n'}{d\psi'}\right)_0$ and $\left(\frac{dC_c}{d\psi'}\right)_0$ were unaffected by aspect ratio.

Effect of dihedral.—As previously mentioned, most of the dihedral tests were made with the Army tip

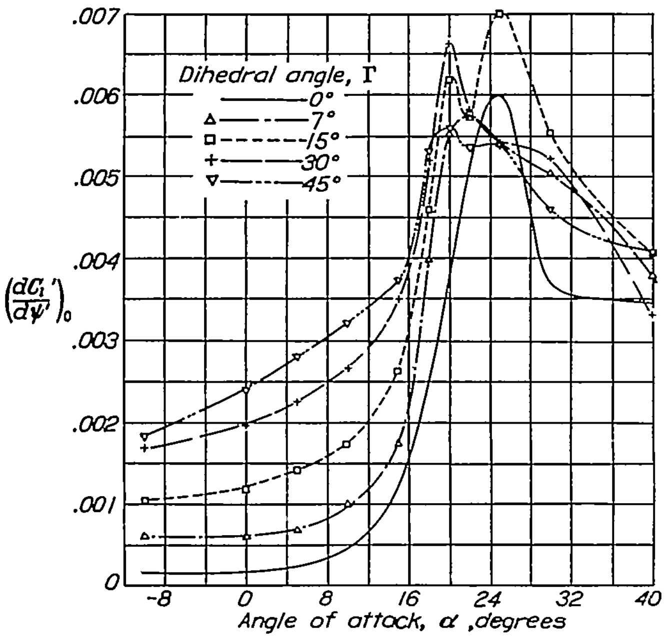

FIGURE 13.—Effect of dihedral of 25 percent semispan of wing with Army tip on rate of change of rolling-moment coefficient with angle of yaw.

although a few tests were made with the rectangular tip. The dihedral axis for the wing with the Army tip was located successively 25, 50, and 93 percent of the semispan from the tip of the wing. With the dihedral

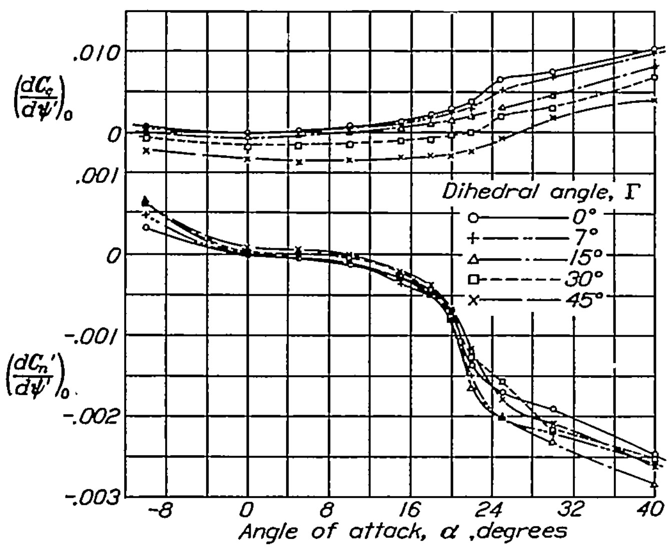

FIGURE 14.—Effect of dihedral of 25 percent semispan of wing with Army tip on rate of change of cross-wind force and yawing-moment coefficients with angle of yaw.

axis at the 25-percent point, the values of $\left(\frac{dC_l'}{d\psi'}\right)_0$ (fig. 13) are increased by dihedral up to angles of attack as high as 22°; with the dihedral axis at the 50-percent point (fig. 16), the values are increased up through 20° angle of attack; and with 93 percent of the semispan in use (fig. 18), the increase in $\left(\frac{dC_l'}{d\psi'}\right)_0$ due to dihedral is reduced to zero at 19° angle of attack. This action is explained by the manner in which a rectangular

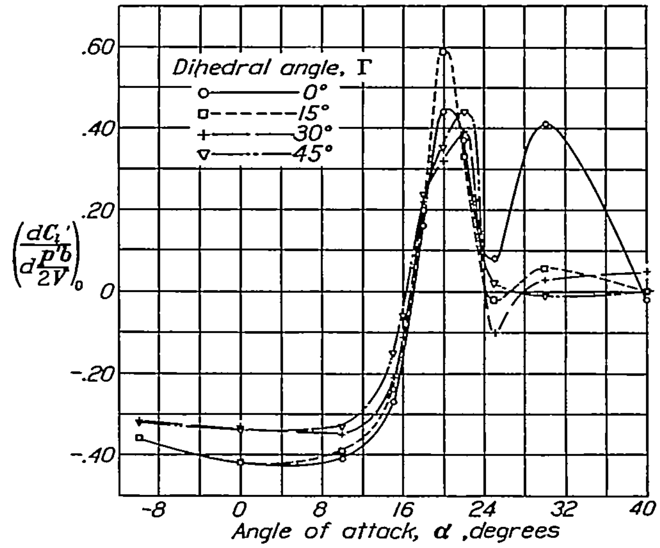

FIGURE 15.—Effect of dihedral of 25 percent semispan of wing with Army tip on rate of change of rolling-moment coefficient with rate of roll.

wing stalls. The burble starts at the center section and spreads toward the tips, the tips remaining unaffected for several degrees after the center-section flow breaks down. The effect of dihedral on $\left(\frac{dC_c}{d\psi'}\right)_0$

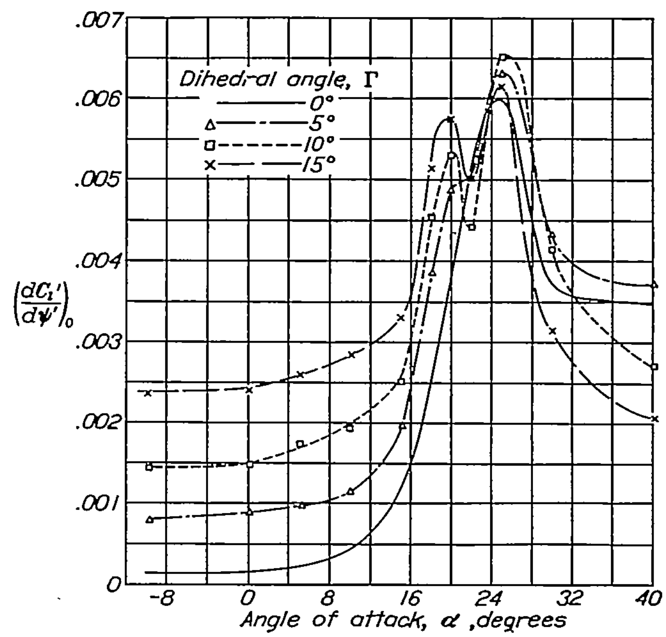

FIGURE 16.—Effect of dihedral of 50 percent semispan of wing with Army tip on rate of change of rolling-moment coefficient with angle of yaw.

and $\left(\frac{dC_n'}{d\psi'}\right)_0$ shown in figures 14, 17, and 19, although not large, is fairly consistent for the three dihedral axes used. The values of $\left(\frac{dC_c}{d\psi'}\right)_0$ were reduced for the wing alone as would be expected but the values of

$\left(\frac{dC_n'}{d\psi'}\right)_0$, the directional-stability criterion, became more positive. The values of $\left(\frac{dC_l'}{d\frac{p'b}{2V}}\right)_0$ were decreased somewhat when the dihedral axis was at 25-percent semispan (fig. 15) and the dihedral angle was greater

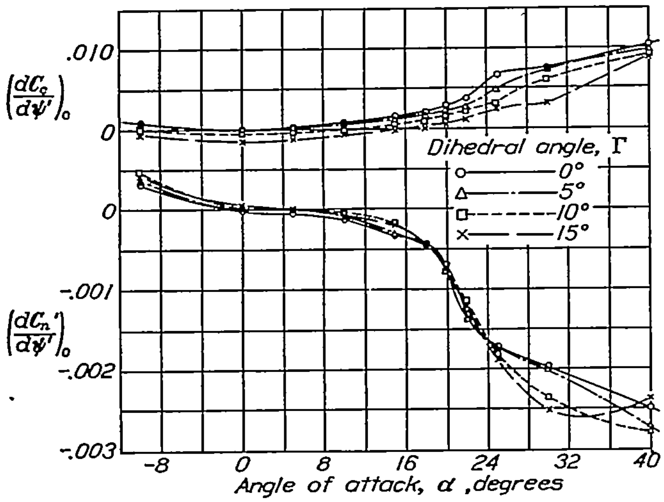

FIGURE 17.—Effect of dihedral of 60 percent semispan of wing with Army tip on rate of change of cross-wind force and yawing-moment coefficients with angle of yaw.

than 15°. For the other dihedral-axis locations, the values were hardly affected for the angles up to 15°. When the wing with square tips was given dihedral the values of $\left(\frac{dC_l'}{d\psi'}\right)_0$, $\left(\frac{dC_n'}{d\psi'}\right)_0$, $\left(\frac{dC_c}{d\psi'}\right)_0$, and $\left(\frac{dC_l'}{d\frac{p'b}{2V}}\right)_0$ were

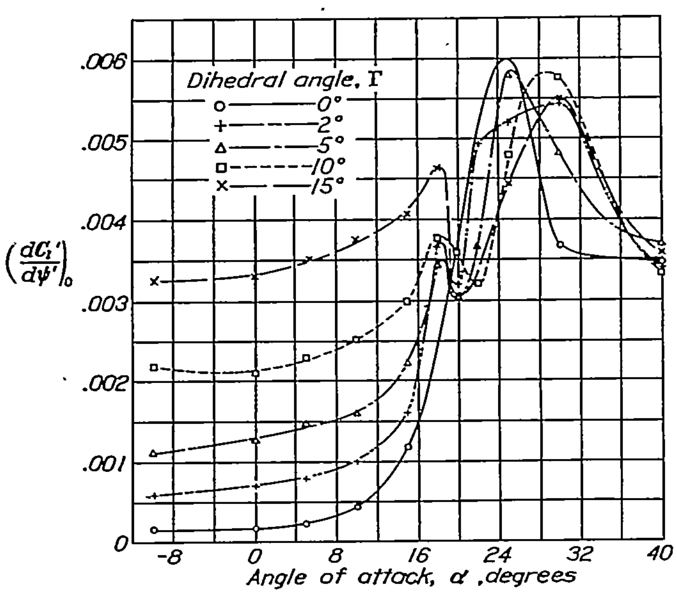

FIGURE 18.—Effect of dihedral of 93 percent semispan of wing with Army tip on rate of change of rolling-moment coefficient with angle of yaw.

affected in about the same manner as were the values for the wing with the Army tip, except that the values of $\left(\frac{dC_l'}{d\psi'}\right)_0$ due to dihedral reversed in effect at 18° angle of attack and considerably reduced the total $\left(\frac{dC_l'}{d\psi'}\right)_0$ for the wing. (See figs. 20 and 21.)

A more direct comparison of the effects of the dihedrals tested was made by computing the increments of

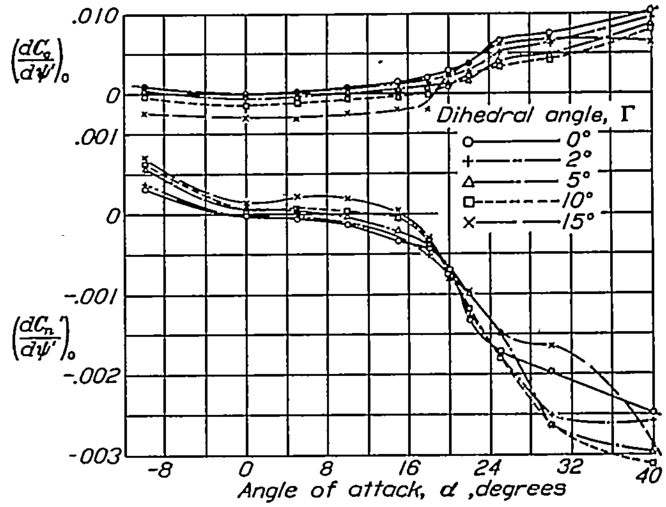

FIGURE 19.—Effect of dihedral of 93 percent semispan of wing with Army tip on rate of change of cross-wind force and yawing-moment coefficients with angle of yaw.

rates of change due to dihedral angle, $\Delta\left(\frac{dC_l'}{d\psi'}\right)_0$, $\Delta\left(\frac{dC_n'}{d\psi'}\right)_0$, and $\Delta\left(\frac{dC_c}{d\psi'}\right)_0$ for 0°, 10°, and 15° angles of attack and plotting them against dihedral angle in figure 22. It

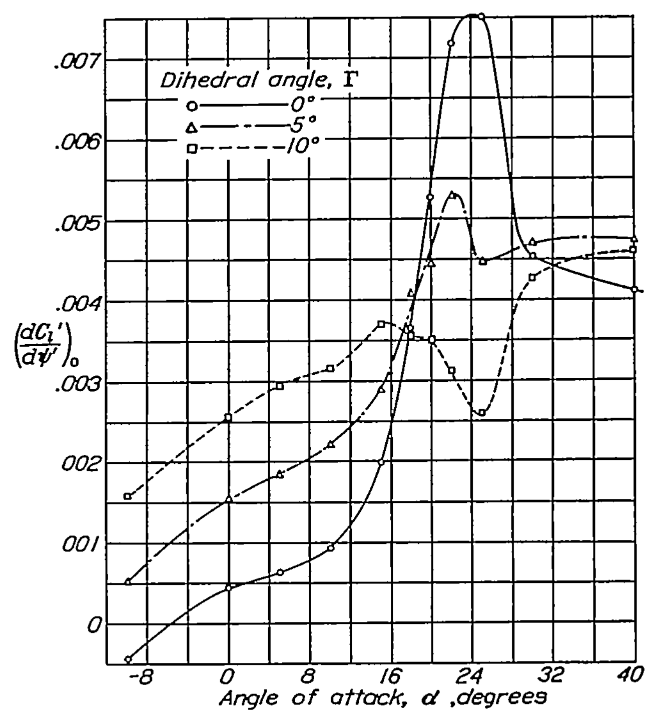

FIGURE 20.—Effect of dihedral of 93 percent semispan of a rectangular wing on rate of change of rolling-moment coefficient with angle of yaw.

may be seen that the increments with the rectangular wing are practically the same as for the wing with the Army tip when the same percentage of the semispan is used in both cases. This agreement is an indication that the effects due to a dihedral angle with practically the full semispan in use may be added directly to the basic 0° dihedral curves regardless of tip shape.

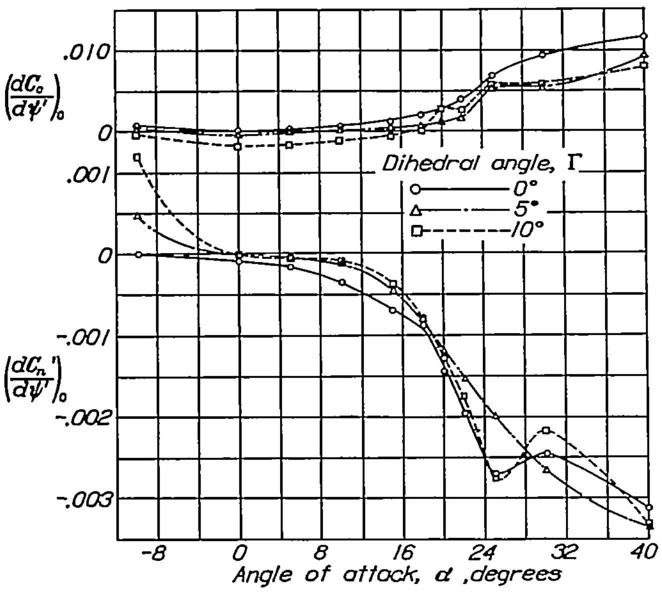

FIGURE 21.—Effect of dihedral of 93 percent semispan of rectangular wing on rate of change of cross-wind force and yawing-moment coefficients with angle of yaw.

The increments $\Delta\left(\frac{dC_l'}{d\psi'}\right)_0$ and $\Delta\left(\frac{dC_n'}{d\psi'}\right)_0$ lend themselves to further evaluation since they vary as a straight line with the angle of dihedral Γ up to an angle of 15°.

An average curve has been drawn through the sets of points and the equations of the curves have been determined to be

$$\frac{d}{d\Gamma}\Delta\left(\frac{dC_l'}{d\psi'}\right)_0=0.000333K-0.000118K^{2.35}$$

$$\frac{d}{d\Gamma}\Delta\left(\frac{dC_n'}{d\psi'}\right)_0=0.000024K^{1.5}$$

where K is the fraction of the semispan in use.

These equations may be used with sufficient accuracy in determining the effect of dihedral of any fraction of the semispan of a rectangular wing of Clark Y or similar section having an aspect ratio of 6 with the Army tip. These values, of course, must be added to the basic values for the wing with 0° dihedral. If the entire semispan has dihedral, K becomes 1.0, $\frac{d}{d\Gamma}\Delta\left(\frac{dC_l'}{d\psi'}\right)_0=0.000215$, and $\frac{d}{d\Gamma}\Delta\left(\frac{dC_n'}{d\psi'}\right)_0=0.000024$. Blenk (reference 6) found corresponding values of 0.000204 and 0.000030 for a rectangular wing with square tips.

Comparison of calculated and experimental results.—When a wing with a dihedral angle Γ is yawed through an angle ψ', the new wing chord along the wind direction is

$$c'=\frac{c}{\cos\psi'}$$

and the spanwise displacement of the trailing edge relative to the leading edge of the chord line is

$$l=c'\sin\psi'$$

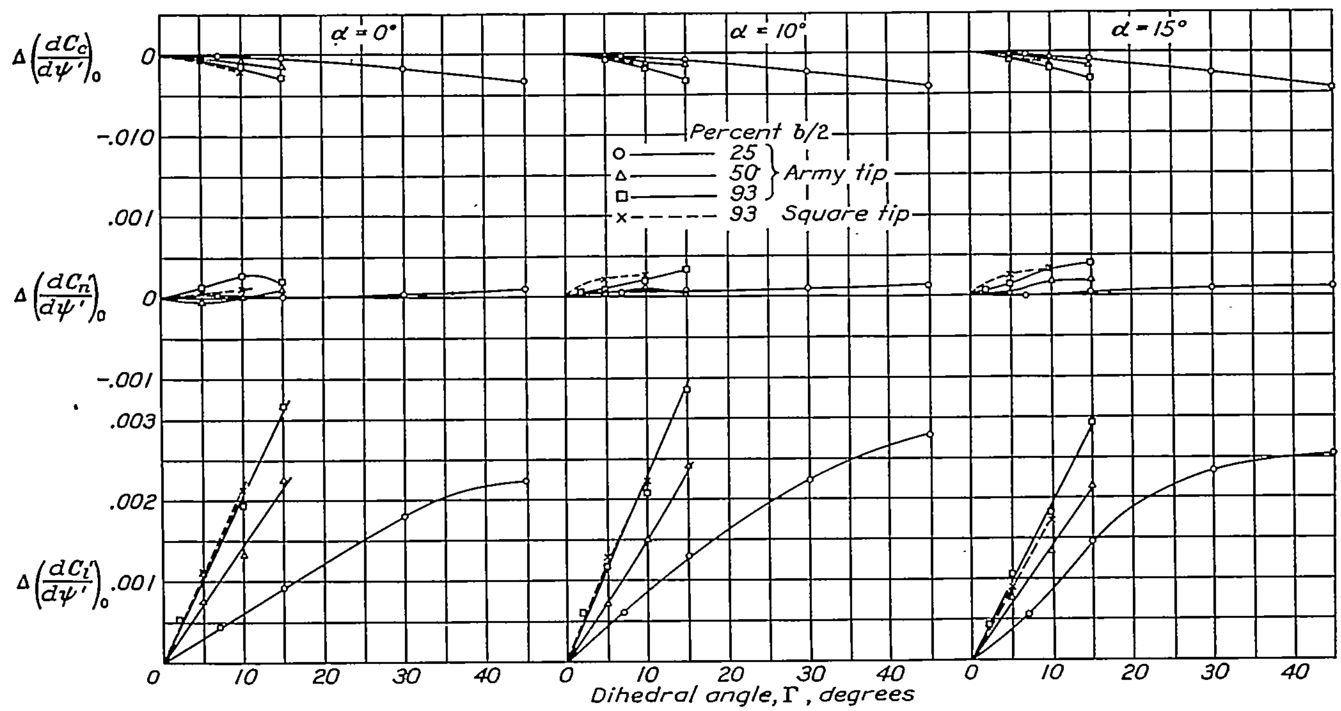

FIGURE 22.—Effect of dihedral on rates of change of rolling-moment, yawing-moment, and cross-wind force coefficients with angle of yaw.

Consequently, the values of $\frac{d}{d\Gamma}\Delta\left(\frac{dC_l'}{d\psi'}\right)_0$ and $\frac{d}{d\Gamma}\Delta\left(\frac{dC_n'}{d\psi'}\right)_0$ were determined from figure 22 for the wing with the Army tip and have been plotted against fraction of wing semispan in figure 23 for the three angles of attack.

Then, owing to the dihedral angle, the trailing edge of the new chord of the rearward wing is higher than its leading edge by an amount

$$h=l\sin\Gamma$$

which may be written

$$h = c' \sin \psi' \sin \Gamma$$

This value makes the angle of attack of c' less than the angle of attack of c so that

$$\sin \Delta\alpha' = \frac{h}{c'} = \sin \psi' \sin \Gamma$$

Now for small angles the sine equals the angle in radians, so that

$$\Delta\alpha' = \psi'\Gamma \text{ (all in radians)}$$

Likewise, on the forward wing the angle of attack is increased by

$$\Delta\alpha' = \psi'\Gamma$$

Wieselsberger has shown (reference 12) that the rolling moment due to an unsymmetrical span load distri-

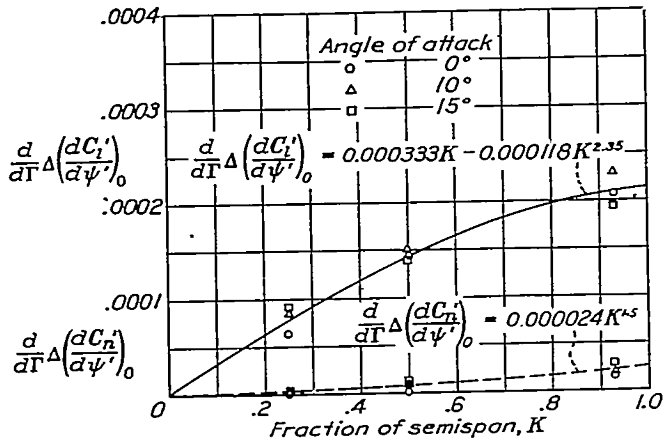

FIGURE 23.—Effect of dihedral span on rate of change with dihedral angle of the rate of change of rolling moment with angle of yaw.

bution resulting from equal changes in angle of attack of opposite sign on the two halves of a rectangular wing may be expressed by

$$M_x' = \frac{\rho V^2}{2} b^3 \zeta \alpha_q$$

where ζ is a factor dependent on the aspect ratio and the slope of the lift curve for a wing of infinite aspect ratio and α_q is the change in angle of attack.

Expressing this equation in coefficient form results in

$$C_l' = \zeta \alpha_q \left(\frac{b^2}{S}\right)$$

Substituting for α_q its equal $\frac{\psi'\Gamma}{57.3^2}$, for $\left(\frac{b^2}{S}\right)$ the value of 6, and for ζ the value 0.127 from Wieselsberger and Asano (reference 13) for a change in angle of attack over 93 percent of the span,

$$C_l' = 0.127 \frac{\psi'\Gamma}{57.3^2} \times 6$$

Then differentiating with respect to ψ' and Γ,

$$\frac{d}{d\Gamma}\left(\frac{dC_l'}{d\psi'}\right)_0 = 0.000232$$

which is, within the limits of accuracy, equal to the value found from the experiments reported herein.

CONCLUSIONS

1. The rate of change of rolling moment with angle of yaw was greatly affected by wing-tip shape.
2. Agreement was obtained between computed and experimental values of the rate of change of rolling moment with angle of yaw due to dihedral of a rectangular wing.
3. The dihedral effect was maintained to a higher angle of attack with dihedral of only the outer one-fourth of each semispan than when the entire semispan had dihedral.

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LANGLEY FIELD, VA., *August 27, 1935.*

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