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THE AERODYNAMICS OF A WIND-TUNNEL FAN

By Blake W. Corson, Jr.
Langley Memorial Aeronautical Laboratory

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SUMMARY

The vortex blade-element theory modified to apply to an axial fan working in a duct is reviewed. Thrust and power coefficients for a fan are identified with the corresponding coefficients for airplane propellers. The relation of pressure produced by the fan to the blade-element coefficients is developed. The distribution of axial velocity of fluid through a fan is assumed to be controlled by the fan itself. The radial distribution of tangential velocity imported by the fan to fluid moving through the fan is shown to be independent of the axial-velocity distribution.

A nondimensional coefficient, designated the rotation constant, is introduced. This constant is based solely upon design information. The use of the rotation constant in simplifying the design of a fan for a specific operating condition is demonstrated. Based on the use of the rotation constant, a graphical method is outlined by which the performance of a given fan in a given wind tunnel may be predicted and by which the distributions of axial velocity of the fluid through the fan under various operating conditions may be established.

INTRODUCTION

Comprehensive treatment of factors bearing on the operation of axial fans have been made by several investigators. Following customary procedure, each investigator assumed that the axial velocity of fluid through the fan was uniform. Ignoring viscous distortion of the velocity distribution, this assumption is logical for the design condition but may not, of necessity, hold for all operating conditions. Radially nonuniform axial velocity under some operating conditions is mentioned in reference 1. Performance estimation for a fan that must work over a

range of operating conditions can be made only by some procedure which takes account of the interrelation of axial velocity and torque distributions.

This paper deals only with the aerodynamics of the fan proper and demonstrates the manner in which the radial distribution of axial velocity of fluid through a fan is governed by the geometry of the fan itself. Although special reference is made to a wind-tunnel fan, the conclusions arrived at here apply to any single-stage fan working in a close duct. Some of the derivations published in the references are repeated here, but a consistent notation is used to coordinate the work of various authors. The vortex blade-element theory modified to apply to a fan working in a duct is developed. A direct procedure is indicated for design of the optimum fan for a given combination of mass flow, shaft speed, power, and fan diameter. A relation is established between elemental torque coefficient, axial velocity, radius, and a rotation constant. Based on this relation, a method is given by which performance charts for a fan may be prepared.

SYMBOLS

D diameter of fan

D_0 hub-housing diameter

r radius of any blade element under consideration

r_0 hub-housing radius

n shaft speed, revolutions per second

V axial velocity of fluid through fan

ω angular velocity imparted to fluid by fan

$w = \omega r$,   tangential velocity at radius r imparted to fluid by fan

V_f final velocity of air after leaving fan; vector sum of V and w

V_R velocity of fluid relative to blade element; vector sum of $(V + 2\pi r n - w/2)$

J   slip function   (V/nD),   significant only when based
     on uniform axial velocity

J'   slip function based on axial velocity at a radius   r

$\emptyset$ helix angle $= \tan^{-1} \dfrac{V}{(2\pi rn - w/2)}$

β blade angle

β_0 reference blade angle

α angle of attack $= \beta - \emptyset$

b   blade width, or chord

h   airfoil section thickness

B   number of blades

L   lift

C_L lift coefficient

D   drag

R   resultant force, vector sum of   L + D

γ $\tan^{-1} D/L$

ρ mass density of air, slugs per cubic foot

Q   torque

Q_c torque coefficient $\left(\dfrac{Q}{\rho V^2 D^3}\right)$

Q_c' elemental torque coefficient

$$B \int_{r_0/D}^{0.5} Q_c' \ d\left(\frac{r}{D}\right) = Q_c$$

C_Q torque coefficient $\left(\dfrac{Q}{\rho n^2 D^5}\right)$

C_Q' elemental torque coefficient

$$B \int_{r_o/D}^{0.5} C_Q' \ d \left(\frac{r}{D}\right) = C_Q$$

P   power input to fan

C_P power coefficient $\left(2\pi C_Q = \dfrac{P}{\rho n^3 D^5}\right)$

T   thrust, shaft tension

T_c thrust coefficient $\left(\dfrac{T}{\rho V^2 D^2}\right)$

T_c' elemental thrust coefficient

$$B \int_{r_o/D}^{0.5} T_c' \ d \left(\frac{r}{D}\right) = T_c$$

C_T thrust coefficient $\left(\dfrac{T}{\rho n^2 D^4}\right)$

C_T' elemental thrust coefficient

$$B \int_{r_o/D}^{0.5} C_T' \ d \left(\frac{r}{D}\right) = C_T$$

η_o profile efficiency

η_R rotational efficiency

η fan efficiency

M   mass flow through fan

τ rotation constant $\left(\dfrac{P}{(nD)^2 BM}\right)$

E   energy ratio of wind tunnel $\left(\dfrac{\frac{1}{2}\,\rho_T\,V_T{}^3\,A_T}{P}\right)$

λ fan wind-tunnel constant (defined in appendix D)

p   static-pressure increment across fan

q_w dynamic pressure due to rotational component of velocity downstream from fan

A   area

C, C_1, C_2, etc.   constants

Subscripts:

    F   fan

    T   throat

Primed symbols refer to blade elements.

BLADE-ELEMENT THEORY

    A vector diagram of forces and velocities acting on a blade element is shown in figure 1. The element is a section of a fan blade at radius r and is of area b $\times$ dr. The rotational interference velocity, designated a' ($2\pi rn$) in reference 2, is assumed in this derivation to be w/2, one-half of the final rotational velocity imparted to the annulus of air by B blade elements at radius r. This assumption is used in references 1 and 3. The inflow velocity, aV in reference 2, for a fan working in a duct is assumed to be zero. The torque, power, and thrust characteristics derived here are expressed as coefficients identical with the propeller characteristics defined in reference 2.

$$dL = \frac{1}{2}\,\rho\,V_R{}^2\,C_L\,b\,dr = \frac{1}{2}\,\rho\,\frac{V^2}{\sin^2\phi}\,C_L\,b\,dr$$

$$dR = \frac{dL}{\cos\gamma} = \rho\,V^2\,D^2\left[\frac{C_L\,(b/D)}{2\,\sin^2\phi\,\cos\gamma}\right]d\left(\frac{r}{D}\right)$$

Define

$$\frac{C_L \cdot \frac{b}{D}}{2 \sin^2 \phi \cos \gamma} = K$$

For one blade

$$dR = \rho V^2 D^2 K \, d\left(\frac{r}{D}\right)$$

For B blades,

$$\frac{dQ}{r} = dR \sin(\phi + \gamma) B$$

$$dQ = \rho V^2 D^3 B K \frac{r}{D} \sin(\phi + \gamma) \, d\left(\frac{r}{D}\right)$$

Define

$$Q_c{}' = K \frac{r}{D} \sin(\phi + \gamma) \qquad (1)$$

$$dQ = \rho V^2 D^3 B Q_c{}' \, d\left(\frac{r}{D}\right)$$

In a later analysis the axial velocity will be considered nonuniform. Since over-all thrust and torque are obtained by integration along the radius of the elemental thrusts and torques, velocity varying with radius must remain under the integral sign. It is better, therefore, to use coefficients that include the axial velocity.

$$dQ = \rho n^2 D^5 B Q_c{}' \left(\frac{V}{nD}\right)^2 d\left(\frac{r}{D}\right)$$

$$= \rho n^2 D^5 B C_Q{}' \, d\left(\frac{r}{D}\right) \qquad (2)$$

$$Q = \rho n^2 D^5 B \int_{r_0/D}^{0.5} C_Q{}' \, d\left(\frac{r}{D}\right)$$

$$Q = \rho n^2 D^5 C_Q \qquad (3)$$

Similarly for thrust for one blade,

$$dT = dR \cos (\phi + \gamma)$$

For B blades,

$$dT = \rho V^2 D^2 B K \cos (\phi + \gamma) d \left(\frac{r}{D}\right)$$

Define

$$T_c' = K \cos (\phi + \gamma) \qquad (4)$$

$$dT = \rho V^2 D^2 B T_c' d \left(\frac{r}{D}\right) \qquad (5)$$

$$= \rho n^2 D^4 B T_c' \left(\frac{V}{nD}\right)^2 d \left(\frac{r}{D}\right)$$

$$= \rho n^2 D^4 B C_T' d \left(\frac{r}{D}\right) \qquad (6)$$

$$T = \rho n^2 D^4 B \int_{r_o/D}^{0.5} C_T' d \left(\frac{r}{D}\right)$$

$$T = \rho n^2 D^4 C_T \qquad (7)$$

The efficiency of a blade element,

$$\eta' = \frac{dT \, V}{2\pi n \, dQ}$$

$$\eta' = \frac{V}{2\pi r n \tan (\phi + \gamma)}$$

$$\eta' = \frac{1}{\tan (\phi + \gamma)} \left[\frac{V - \left(\frac{w/2}{2\pi r n}\right) V}{2\pi r n - \frac{w}{2}}\right]$$

$$\eta' = \frac{1}{\tan (\phi + \gamma)} \left(\frac{V}{2\pi r n - \frac{w}{2}}\right)\left(1 - \frac{w/2}{2\pi r n}\right)$$

$$\eta' = \frac{\tan \phi}{\tan (\phi + \gamma)} \left(1 - \frac{w/2}{2\pi rn}\right) \tag{8}$$

In appendix A the factor $\left(1 - \dfrac{w/2}{2\pi rn}\right)$ is shown to represent the rotational efficiency of the blade element. The ratio $\dfrac{\tan \phi}{\tan (\phi + \gamma)}$ represents the profile efficiency and is the expression arrived at by simple blade-element theory.

PRESSURE RELATIONS

Consider a fan unit working in a closely fitting cylindrical duct and moving a nonviscous incompressible fluid. Limit the length of the cylindrical portion of the duct to a short distance on each side of the fan and let the duct terminate in infinitely large cones expanding upstream and downstream. In order to avoid having to deal with rotation of the fluid, regard the fan as an actuator disk. The assumption of incompressible flow demands that there be no discontinuity of axial velocity at the fan. Useful energy imparted to the flow by the fan, therefore, must appear as increase in static pressure.

At a distance upstream where the velocity of flow is negligibly small, pressure energy in the fluid is entirely static pressure and is equal to the total pressure. Inasmuch as there can be no pressure gradient in motionless flow (if gravity is ignored) the total pressure is constant for all streamlines. By Bernoulli's theorem the total pressure is constant along any one streamline; hence, the total pressure is constant throughout the flow at all points upstream from the fan. The same reasoning can be applied to show that the total pressure is constant throughout the flow downstream from the fan. It then follows that the increment in total pressure across the fan must be constant over the entire fan disk and must appear only as an increase in static pressure.

The fact that the static-pressure increment across the fan is constant over the disk places no restrictions on the distribution of static pressure and axial velocity, which is governed by the relative amounts of work being done by various regions of the fan. If the tip portion of the fan is working harder than the portion near the hub,

an axial velocity greater than the mean is induced through
the region of the tip and is accompanied by corresponding-
ly lower static pressure in that region on both sides of
the propeller disk. The portion near the hub, doing rel-
atively less work, cannot maintain an axial velocity as
great as the mean; consequently, higher static pressures
on both sides of the propeller disk exist in this region.
The static-pressure increment across the fan, however, is
everywhere the same and the axial velocity through any
region is directly proportional to the relative amount of
work being done by that portion of the fan.

In the design of a fan, the distribution of axial
velocity must be assumed and is often regarded as uniform.
If diameter and shaft speed are known, the optimum fan can
be designed for the given value of J. Whenever the fan
is required to work at a value of J other than the de-
sign value, some portion of the fan is forced to do rela-
tively more work than another and a nonuniform axial-
velocity distribution results. Aerodynamic characteris-
tics of the fan, therefore, cannot be predicted by the
assumption of a uniform axial-velocity distribution under
all operating conditions.

The conclusion that a constant static-pressure incre-
ment exists over the entire propeller disk was based on
the assumption that no rotation was imparted to the fluid
as it passed through the fan. Consider a single fan, with-
out countervanes. If the fact is acknowledged that the
torque driving the fan reacts only on the fluid and that
the fan does impart rotation to the fluid, then account
must be taken of variation in the static-pressure incre-
ment across the fan disk, due to centrifugal pressure.
The axial velocity of a fluid particle (in incompressible
flow) is not changed during passage of the particle through
the fan. The absolute velocity of the fluid particle,
however, is increased because of the rotational velocity
imparted to it by the fan. Inseparable from this rotation
is an accompanying centrifugal pressure downstream from
the fan, which is manifest as static pressure. The cen-
trifugal pressure increases radially from hub housing to
duct wall and at any radius is equal to the integrated
centrifugal pressure increments from the hub housing to
that radius. It is now apparent that the fan imparts en-
ergy to the fluid as an increase both in static pressure
and in absolute dynamic pressure and that the static-
pressure increment being partly centrifugal cannot be con-
stant over the entire fan. Investigation of the distri-

bution of these pressure increments over the fan disk is
now in order. The flow pattern being assumed symmetrical
about the fan axis, investigation may be confined to a
plane through the axis in the region between the hub hous-
ing and duct wall. (See fig. 2.)

The only radial variation of the static-pressure in-
crement through the fan is the variation of the centrifu-
gal pressure; hence the static-pressure gradient is the
centrifugal-pressure gradient, which is

$$\frac{dp}{dr} = \rho \omega^2 r$$

$$dp = \rho \omega^2 r \, dr \qquad\qquad (9)$$

The blade-element theory, equation (5), shows the
pressure at radius r to be

$$p = \frac{dT}{2\pi r \, dr} = \frac{\rho \, V^2 D^2 B \, T_c' \, d\left(\frac{r}{D}\right)}{2\pi D^2 \, \frac{r}{D} \, d\left(\frac{r}{D}\right)}$$

$$p = \frac{\rho \, V^2 \, B \, T_c'}{2\pi \, \frac{r}{D}}$$

If equations (1) and (4) are combined and K is eliminated,
expressions for T_c' and p can be obtained

$$T_c' = \frac{Q_c'}{\frac{r}{D} \tan (\phi + \gamma)}$$

$$p = \frac{\rho \, V^2 B \, Q_c'}{2\pi \left(\frac{r}{D}\right)^2 \tan (\phi + \gamma)} \qquad\qquad (10)$$

Inasmuch as the gradient of pressure increment is
given as a function of the angular velocity imparted to
the fluid by the fan, it would be useful to know the pres-
sure increment in terms of the same parameter. The torque

required to impart angular velocity ω to an annulus of fluid at radius r is given by

$$dQ = 2\pi r \, dr \, \rho V r \omega r = 2\pi \rho D^3 \left(\frac{r}{D}\right)^2 V \omega r \, d\left(\frac{r}{D}\right)$$

By blade-element theory,

$$dQ = \rho \, V^2 \, D^3 \, B \, Q_c{}' \, d\left(\frac{r}{D}\right)$$

$$\rho \, V^2 \, D^3 \, B \, Q_c{}' \, d\left(\frac{r}{D}\right) = 2\pi \rho \, D^3 \left(\frac{r}{D}\right)^2 V \omega r \, d\left(\frac{r}{D}\right)$$

$$Q_c{}' \, V \, B = 2\pi \left(\frac{r}{D}\right)^2 \omega r$$

$$\omega r = \frac{Q_c{}' \, V \, B}{2\pi \left(\frac{r}{D}\right)^2} = w \tag{11}$$

If the values in equation (11) are substituted in equation (10), the pressure increment is expressed as a function of angular velocity and radius.

$$p = \frac{\rho \, V \, \omega \, r}{\tan(\phi + \gamma)}$$

$$p = \frac{\rho \, \omega \, r}{\tan(\phi + \gamma)} \left[\frac{V \left(2\pi r n - \frac{w}{2}\right)}{\left(2\pi r n - \frac{w}{2}\right)}\right]$$

$$p = \rho \, \omega \, r \left(2\pi r n - \frac{\omega r}{2}\right) \frac{\tan \phi}{\tan(\phi + \gamma)}$$

$$p = \rho \, \omega \, r^2 \left(2\pi n - \frac{\omega}{2}\right) \eta_o{}' \tag{12}$$

It is not feasible to express the profile efficiency as a function of either of the independent variables in equation (12). The profile efficiency may be assumed to be unity (that is, $\gamma = 0°$) without introducing much error. In order to determine the extent of its influence, however, profile efficiency will be retained and regarded as a constant.

The differential of pressure increment, from equation (12), is

$$dp = \eta_0{}' \; \rho \left[\omega r^2 \left(- \frac{d\omega}{2} \right) + \left(2\pi n - \frac{\omega}{2} \right) \left(2\omega r dr + r^2 d\omega \right) \right]$$

$$dp = \eta_0{}' \; \rho \left(-\frac{r^2 \omega d\omega}{2} + 4\pi n \omega r dr + 2\pi n r^2 \, d\omega - \omega^2 \, r dr - \frac{\omega r^2 d\omega}{2} \right)$$

$$dp = \eta_0{}' \; \rho \; r \left(- r\omega d\omega + 4\pi n \omega dr + 2\pi n r d\omega - \omega^2 dr \right) \tag{13}$$

If dp is eliminated between equations (9) and (13),

$$\omega^2 \, dr = \eta_0{}' \left(- \omega r d\omega + 4\pi n \omega dr + 2\pi n r d\omega - \omega^2 \, dr \right) \tag{14}$$

The steps followed in solving equation (14) are given in appendix B; the solution is stated in the following equation:

$$r^2 \; \frac{\omega}{2\pi n} = C \left(\frac{2\eta_0}{1 + \eta_0} - \frac{\omega}{2\pi n} \right)^{\frac{1-\eta_0}{1+\eta_0}} \tag{15}$$

Observation will show that, for reasonable values of ω,

n, and η_0, the function $\left(\dfrac{2\eta_0}{1 + \eta_0} - \dfrac{\omega}{2\pi n} \right)^{\frac{1-\eta_0}{1+\eta_0}}$ is always

nearly equal to unity. The error introduced by assuming perfect profile efficiency is much less than $1 - \eta_0$. The simplifying assumption that $\gamma = 0^\circ$ made in references 1 and 3 is found justifiable for equation (15) and will be

used here; therefore $r^2 \dfrac{\omega}{2\pi n}$ is a constant. The expression $2\pi n$ is a constant for all radii.

$$r^2 \omega = C_1 \tag{16}$$

$$w = \frac{C_2}{r/D} \tag{17}$$

Equation (17) shows that, as the fluid travels downstream from the fan, its tangential velocity is inversely proportional to the radius. This expression was derived by considering the interrelation of thrust, torque, tangential velocity, and pressure. Although, individually, these quantities are functions of the axial velocity of the fluid at the radius of the blade element, the final expression is independent of axial velocity. Equation (17) will be considered to hold for any radial distribution of axial velocity.

A useful relation between elemental torque coefficient and radius can be obtained by combining equations (11) and (17).

$$\frac{Q_c' \, V \, B}{2\pi \left(\frac{r}{D}\right)^2} = w = \frac{C_2}{r/D}$$

$$Q_c' \, V = \frac{2\pi C_2}{B} \frac{r}{D}$$

Multiplying through by $\dfrac{V}{(nD)^2}$ gives

$$Q_c' \left(\frac{V}{nD}\right)^2 = \frac{2\pi C_2}{B \, nD} \frac{r}{D} \frac{V}{nD} \tag{18}$$

From equation (17) it can be seen that the dimensions of the constant C_2 are in feet per second. The constant $\dfrac{2\pi C_2}{B \, nD}$ is therefore nondimensional. The rotation constant defined as $\tau = \dfrac{2\pi C_2}{B \, nD}$ holds for every operating section of the propeller at a given instant. Since V/nD for a blade element is J', equation (18) becomes

$$Q_Q' = \tau J' \frac{r}{D} \tag{19}$$

It is desirable to define the rotation constant in terms of basic wind-tunnel fan information. Thus,

$$P = 2\pi n \, Q = 2\pi n \, \rho \, n^2 \, D^5 \, B \int_{(r/D)_0}^{0.5} C_Q' \, d\left(\frac{r}{D}\right)$$

$$P = 2\pi\ \rho\ n^3\ D^5\ B \int_{(r/D)_0}^{0.5} \tau J' \frac{r}{D}\ d\left(\frac{r}{D}\right)$$

$$P = \tau\ B\ n^2\ D^2 \left[2\pi\ \rho\ D^2 \int_{(r/D)_0}^{0.5} V \frac{r}{D}\ d\left(\frac{r}{D}\right)\right]$$

$$2\pi\ \rho\ D^2 \int_{(r/D)_0}^{0.5} V \frac{r}{D}\ d\left(\frac{r}{D}\right) = M$$

$$\tau = \frac{P}{(nD)^2\ B\ M} \tag{20}$$

As might be expected, the rotation constant is also one of the many possible power coefficients.

By use of equation (19), other blade-element coefficients become

$$K = \frac{C_L\ \frac{b}{D}}{2\ \sin^2\ \phi\ \cos\ \gamma} = \frac{\tau}{J'\ \sin\ (\phi + \gamma)}$$

$$C_L\ b = \frac{2D\ \tau\ \sin^2\ \phi\ \cos\ \gamma}{J'\ \sin\ (\phi + \gamma)} \tag{21}$$

Usually $1 > \cos\ \gamma > 0.998$

$$C_T' = \frac{\tau\ J'}{\tan\ (\phi + \gamma)} \tag{22}$$

A point of interest, brought out in references 1 and 3, is that the increment of total pressure across the fan disk is constant at all radii. Centrifugal pressure at any r/D station is

$$dp = \rho\ D^2\ \omega^2 \frac{r}{D}\ d\left(\frac{r}{D}\right)$$

If equation (17) and the definition of τ are used,

$$w = \frac{T\,B\,n\,D}{2\pi\,\frac{r}{D}} \qquad\qquad (23)$$

$$\omega = \frac{T\,B\,n}{2\pi\left(\frac{r}{D}\right)^2}$$

$$dp = \rho\left(\frac{T\,B\,n\,D}{2\pi}\right)^2 \frac{d\left(\frac{r}{D}\right)}{\left(\frac{r}{D}\right)^3}$$

$$p = \frac{\rho}{2}\left(\frac{T\,B\,n\,D}{2\pi}\right)^2\left[\frac{1}{\left(\frac{r}{D}\right)_o^2} - \frac{1}{\left(\frac{r}{D}\right)^2}\right]$$

The accompanying absolute dynamic pressure increment is

$$q_w = \frac{\rho}{2}w^2 = \frac{\rho}{2}\left(\frac{T\,B\,n\,D}{2\pi}\right)^2 \frac{1}{\left(\frac{r}{D}\right)^2}$$

$$p + q_w = \frac{\rho}{2}\left(\frac{T\,B\,n\,D}{2\pi}\right)^2 \frac{1}{\left(\frac{r}{D}\right)_o^2} \qquad\qquad (24)$$

If countervanes are used, the increment in static pressure is constant over the entire fan.

The assumptions upon which the coefficient derivations are based do not hold in practice. Wind-tunnel fans work in air, a viscous compressible fluid. Owing to viscosity a boundary layer having a radial gradient of axial velocity exists. This distortion of the axial velocity pattern is independent of the influence of the fan and makes null the conclusion that the total pressure is constant for all streamlines. Because mass flow through every cross section of the system must be the same, the compressible air must decelerate slightly during passage through the fan into a region of higher pressure. The fact that profile efficiency is less than unity has already been discussed. None of these discrepancies is believed to be of sufficient consequence to invalidate the reasoning by which the derivations were obtained.

The defined coefficients are readily applicable to fan design and performance estimation.

INFLUENCE OF WIND-TUNNEL DESIGN ON FAN DESIGN

The design of the optimum fan for a wind tunnel depends more upon the decisions of the wind-tunnel designer than upon those of the fan designer. When the driving power, volume flow, fan diameter, and hub-housing diameter have been chosen, the efficiency of the fan is fixed. Relation between driving power and volume flow is expressed by the "energy ratio" of the wind tunnel, which is the ratio of power in the air in a cross section of the test jet to power input to the fan.

$$E = \frac{\frac{1}{2} \rho_T A_T V_T^3}{P} \tag{25}$$

In order to estimate the power required to produce a given throat velocity, the designer must first estimate the energy ratio. The estimation of the energy ratio of a return flow wind tunnel is treated in reference 4. Although procedure based on drag estimation and expansion ratio may be followed to determine the energy ratio of a wind tunnel, estimation from experience is usually relied on.

For a given volume flow and blade-tip speed, the rotational efficiency can be shown to improve as the ratio of hub-housing diameter to fan diameter D_0/D increases. Rotational efficiency for the entire fan, assuming a uniform axial-velocity distribution, can be expressed as

$$\eta_R = 1 - \frac{P}{(\pi n D)^2 M}\left[\frac{\log \frac{1}{D_0/D}}{1 - \left(\frac{D_0}{D}\right)^2}\right] \tag{26}$$

The derivation of this expression is given in appendix C, and the function of D_0/D is plotted in figure 3.

Change in the ratio D_0/D may also be made to influence the profile efficiency of the fan. Profile efficiency for a single blade element, $\tan \phi/\tan (\phi + \gamma)$, is shown, in reference 2, to be greatest when

$$\phi = 45^{\circ} - \frac{\gamma}{2}$$

The value of γ can be estimated with very small error. If volume flow, fan diameter, and rotational speed are fixed, an arrangement similar to that shown in figure 4 will give high profile efficiency as well as high rotational efficiency. The abrupt increase in tunnel-wall diameter permits the use of the predetermined large-diameter fan at a location where the air velocity (assumed uniform) will at some mean radius give a desirably large helix angle. The hub housing is shaped to preserve the continuity of flow area for the correct rate of diffusion.

Limitations to the gain in efficiency obtainable by increasing D_O/D must be learned by experience. The divergence of the wind-tunnel wall ahead of the fan should not be so abrupt as to cause the flow to separate from the tunnel wall. If countervanes are to be used in conjunction with the fan, the rotational efficiency becomes of secondary interest because the rotational loss is regained except for the small profile loss in the countervanes themselves. Presumably the adverse effect of increased wetted area would, at some point, offset the benefit of increased efficiency. Influence of the hub-housing diameter ratio D_O/D is discussed in reference 1.

OPTIMUM FAN DESIGN

"Optimum fan" here implies a fan whose blade elements work at the highest lift-drag ratio of the airfoil section. A distribution of axial velocity undistorted by the fan is also implied. Design of a wind-tunnel fan is begun with the information furnished by the wind-tunnel designer: volume flow, air density, drive power, shaft speed, fan diameter, and hub-housing diameter. The number of blades used is assumed not to influence aerodynamic behavior of the fan and is to be determined chiefly from considerations of blade strength. Blade interference and number of blades are dealt with in reference 1 and blade strength is discussed in reference 5. Choice of number of blades completes the information needed to compute the rotation constant of the fan. From equation (20)

$$\tau = \frac{P}{(nD)^2 \ B \ M}$$

The first assumption to be made by the designer must
be with regard to the distribution of axial velocity
through the fan. An axial-velocity distribution can be es-
tablished by a judicious use of information contained in
references 6 and 7. This procedure involves computation
of the velocities in the boundary layer near a wall and in
the vicinity of a body of given shape and adjustment of
the velocities to give the correct volume flow. After much
work, however, the designer will still have only an approx-
imation. For most wind-tunnel-fan designs, the assumption
of uniform axial velocity reduces labor and, except in ex-
treme cases, does not introduce serious error. Determina-
tion of helix angle, blade shape, and blade twist is more
or less straightforward and is adapted to tabulation.

The helix angle for each radial station can be computed
from the definition.

$$\phi = \tan^{-1} \frac{V}{2\pi r n - \frac{w}{2}}$$

If equation (23) is substituted for w, ϕ becomes a
function of J' and τ.

$$\phi = \tan^{-1} \frac{V}{nD} \left[\frac{1}{2\pi \frac{r}{D} - \frac{\tau B}{4\pi \frac{r}{D}}} \right] = \tan^{-1} J' \left[\frac{1}{2\pi \frac{r}{D} - \frac{\tau B}{4\pi \frac{r}{D}}} \right] \qquad (27)$$

After selection of the blade airfoil profiles, blade shape
and orientation can be determined from equation (21).

$$C_L b = \frac{2D \tau \sin^2 \phi}{J' \sin (\phi + \gamma)}$$

Either lift coefficient or blade width may be arbitrarily
chosen. For the optimum fan, the lift coefficient is
chosen to correspond to the smallest value of the D/L
ratio, shown by airfoil characteristics, and the blade
width is computed. If the blade-width distribution is
fixed, as is sometimes desirable from strength considera-
tions, the lift coefficient can be computed by successive
approximations made with successively assumed values of γ.
Usually two approximations give the correct lift coeffi-

cient, ± 0.005, and the corresponding γ checks the assumed value within $\pm 0.05°$. Airfoil characteristics determine the angle of attack along with the lift coefficient and γ. Because there is usually only a small clearance between blade tip and duct wall, it is assumed that no tip vortices are shed from the blades of a fan working in a duct. On the basis of this assumption, flow about the blade profile is two-dimensional and airfoil characteristics for infinite aspect ratio are used in computing fan characteristics. The blade angle is the sum of the angle of attack and the helix angle.

Two expressions each are available for elemental torque and thrust coefficients:

$$C_Q' = \tau J' \frac{r}{D}$$

$$C_Q' = (J')^2 \left(\frac{C_L \frac{b}{D}}{2 \sin^2 \phi \cos \gamma} \right) \frac{r}{D} \sin (\phi + \gamma)$$

$$C_T' = \frac{\tau J'}{\tan (\phi + \gamma)}$$

$$C_T' = (J')^2 \frac{C_L \frac{b}{D}}{2 \sin^2 \phi \cos \gamma} \cos (\phi + \gamma)$$

Use of both expressions serves as a check on computations. Integration of the elemental coefficients gives the corresponding fan coefficients.

$$C_Q = B \int_{r_o/D}^{0.5} C_Q' \ d \left(\frac{r}{D} \right)$$

$$C_T = B \int_{r_o/D}^{0.5} C_T' \ d \left(\frac{r}{D} \right)$$

For any distribution of axial velocity, over-all fan efficiency is the ratio of the useful work to the work input.

$$\eta = \frac{\int V \, \Delta T}{\int 2\pi \, n \, \Delta Q}$$

The torque and thrust increments are given in equations (2) and (6).

$$\eta = \frac{\int_{r_0/D}^{0.5} V \, \rho \, n^2 \, D^4 \, B \, C_T{}' \, d\left(\frac{r}{D}\right)}{\int_{r_0/D}^{0.5} 2\pi \, n \, \rho \, n^2 \, D^5 \, B \, C_Q{}' \, d\left(\frac{r}{D}\right)}$$

$$\eta = \frac{B\int_{r_0/D}^{0.5} \frac{V}{nD} \, C_T{}' \, d\left(\frac{r}{D}\right)}{B\int_{r_0/D}^{0.5} 2\pi \, C_Q{}' \, d\left(\frac{r}{D}\right)} = \frac{B\int_{r_0/D}^{0.5} J' \, C_T{}' \, d\left(\frac{r}{D}\right)}{B\int_{r_0/D}^{0.5} 2\pi \, C_Q{}' \, d\left(\frac{r}{D}\right)}$$

$$\eta = \frac{B}{C_p} \int_{r_0/D}^{0.5} J' \, C_T{}' \, d\left(\frac{r}{D}\right) \qquad\qquad (28)$$

Determination of the thrust and the efficiency of a fan for a given power and the size and the shape of the blades completes the aerodynamic design.

FAN-PERFORMANCE ESTIMATION

Performance estimation, as opposed to design, includes the determination of power absorbed, thrust, mass flow, and efficiency of a fan, the size and the shape of which are already known. Fan performance, as well as design, depends directly upon the energy ratio of the wind tunnel in which the fan works. The method of performance estimation outlined here is based on the following assumptions: (1) At any given time, all elements of a fan operate at the same value of the rotation constant; (2) the distribution of axial velocity through the fan is controlled by the fan itself; and (3) a different distribution exists for each value of the rotation constant. This procedure is as applicable to variable-pitch fans as to those having fixed blades, and in the case of either is laborious.

When information at hand consists only of the dimensions of the fan and the wind tunnel and the energy ratio of the wind tunnel, the analyst knows nothing of the fan's operation except that a certain thrust will accompany a certain mass flow. This fact is known from the wind-tunnel characteristics and leads to the following equation. (for derivation, see appendix D):

$$\sigma_T = \lambda \left(\frac{C_Q}{\tau}\right)^2 \tag{29}$$

where the fan wind-tunnel constant

$$\lambda = \frac{\eta_{design}}{E\, A_F\, B^2}\, 2\pi^2\, D^2 \left(\frac{\rho_F\, A_F}{\rho_T\, A_T}\right)^2$$

The design efficiency η_{design} in the foregoing expression is the fan efficiency that was assumed in computing the energy ratio.

The relation involving the thrust-torque-rotation constant, expressed by equation (29), coupled with the pertinent assumption that all blade elements operate at the same value of the rotation constant at a given instant furnishes means for a systematic analysis. The first step is to prepare, for each r/D station, charts of C_Q' and C_T'

plotted against J' (fig. 5). These charts show values
of C_Q', C_T', and J' for constant blade angles and for
constant values of the rotation constant T.

Basic design information will show whether the blade
angles are fixed or variable and will indicate the range
of J' and T over which the blade elements will be re-
quired to work. One set of computations fixes C_Q', C_T',
and the blade angle for a series of assumed J' values at
one value of T at one r/D station. For the given r/D
station, a reasonable value of T is chosen and several
J' values are selected, which bracket the working range
of the section. An outline of a sample computation pre-
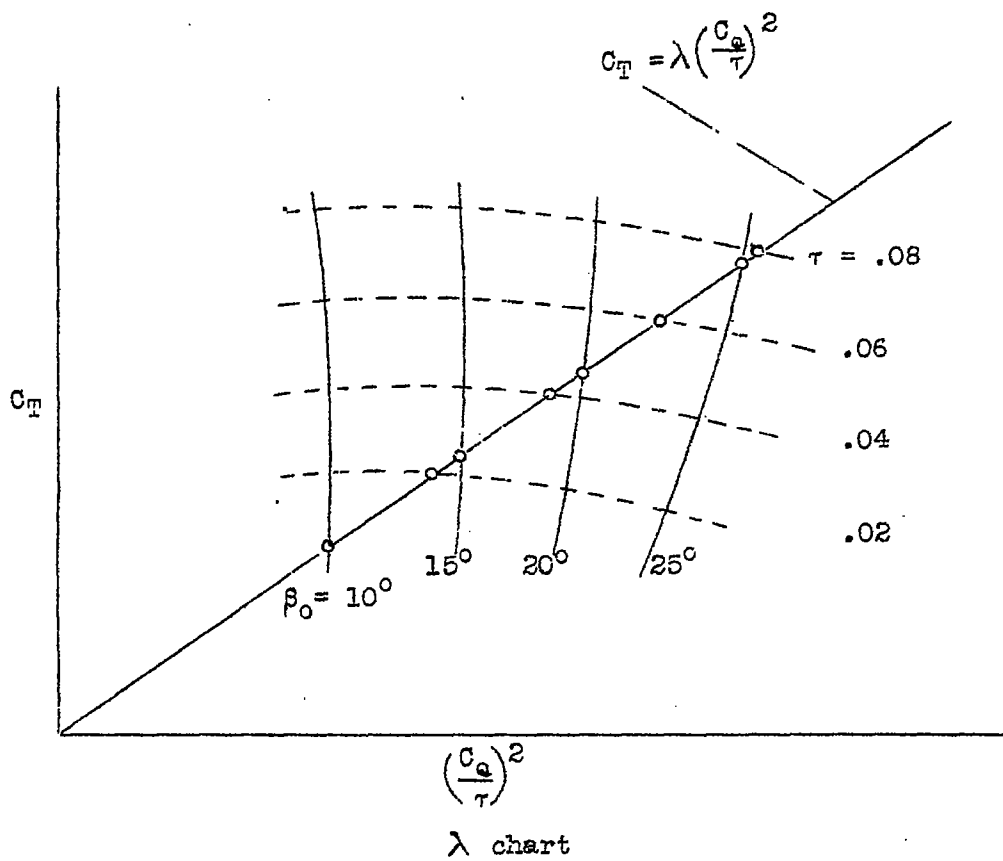paring C_Q' and C_T' charts follows.

$$\frac{r}{D} = K_1 \qquad b = \underline{\hspace{2cm}} \qquad \frac{h}{b} = \underline{\hspace{2cm}} \qquad \tau = K_2$$

$$\frac{1}{2\pi \dfrac{r}{D} - \dfrac{\tau B}{4\pi \dfrac{r}{D}}} = K_3 \qquad \tan \phi = K_3 J'$$

$$C_L = \frac{2D \ \tau \ \sin^2 \phi}{J' \ b \ \sin (\phi + \gamma)} = \frac{\mu}{\sin (\phi + \gamma)} \qquad C_Q' = \tau \frac{r}{D} J'$$

$$C_T' = \frac{\tau J'}{\tan (\phi + \gamma)}$$

J'	0.4	0.8	1.2	1.6
$\tan \phi$				
ϕ				
$\sin \phi$				
$\mu = \dfrac{2D \ \tau \ \sin^2 \phi}{J' \ b}$				
Assumed γ				
$(\phi + \gamma)$				
$\sin (\phi + \gamma)$				
$\tan (\phi + \gamma)$				
$C_L = \dfrac{\mu}{\sin (\phi + \gamma)}$				
α				
$\dfrac{D}{L} = \tan \gamma$				
γ (must check assumed value)				
C_Q'				
C_T'				
$\beta = \phi + \alpha$				

$$C_T = \lambda \left(\frac{C_Q}{\tau}\right)^2$$

$\tau = .08$

$.06$

$.04$

$.02$

C_T

$\beta_O = 10^\circ$ 15° 20° 25°

$\left(\dfrac{C_Q}{\tau}\right)^2$

λ chart

When values of $C_Q{}'$ and $C_T{}'$ are plotted, the corresponding values of the blade angle are noted. The foregoing computation must be repeated for enough values of τ to cover the working range of the fan. Curves for constant values of τ can be faired directly. Curves for constant blade angles must be faired by interpolating the values noted in plotting. Similar charts must be prepared for five or six r/D stations.

Consider performance estimation for a fixed-pitch fan. The blade angle at each r/D station will be known. For the proper blade angle at each r/D station, read from the $C_Q{}'$ and $C_T{}'$ charts values of $C_Q{}'$ and $C_T{}'$ corresponding to a single value of τ. The value of J' will vary from station to station if the distribution of axial velocity is not uniform. Integrate $C_Q{}'$ and $C_T{}'$ to get C_Q and C_T. Repeat the foregoing procedure for two other values of τ and plot C_T against $\left(\dfrac{C_Q}{\tau}\right)^2$. On the same chart plot equation (29). The intersection of the two curves, shown in the following diagram called the λ chart, gives C_Q, C_T and, by interpolation, gives the τ value under which the fan operates at the specified blade-angle setting.

The chart shown is for a variable-pitch fan. For a fixed-pitch fan only one β_0 curve exists.

Tabulate encircled points on λ chart and compute the efficiency and the mass flow. The form following is a simple one for this purpose:

β_0 (deg)	τ	C_T	C_Q	η	$\dfrac{M}{N}$
10.0					
14.1					
15.0					
18.2					

Further desirable information includes the efficiency, the axial-velocity distribution, and the mass flow. Axial-velocity distribution varies directly as J'. As the working T is now known, as well as the blade angle, values of C_T' and J' can be taken directly from the C_T' charts and the integrated product $J' C_T'$ can be used in equation (28) to determine the fan efficiency. Mass flow can be computed from equation (30), which is derived from the definition of the rotation constant.

$$M = \frac{2\pi \, \rho_F \, D^3}{B} \, n \, \frac{C_Q}{T} \tag{30}$$

Throat velocity must be computed from the mass flow. Inasmuch as fan diameter D is included in the primary information, all performance characteristics are now known in terms of shaft speed. n. The shaft speed depends entirely on the type of driving power rotating the fan. If the type of driving power is known, the thrust, the power, the efficiency, and the mass flow can be found immediately for one blade-angle setting.

If the wind-tunnel fan is of the variable-pitch type, analysis at each of a series of blade-angle settings will be necessary. The λ chart will then carry a curve for each blade-angle setting as shown. Because the distribution of axial velocity through the fan is assumed to be nonuniform and to vary with operating conditions, the slip function V/nD loses some of its meaning. Better parameters for expressing wind-tunnel fan characteristics are blade angle β or rotation constant T. Final information, including shaft speed, power, thrust, efficiency, and volume flow, again is dependent upon the type of driving power.

Langley Memorial Aeronautical Laboratory,
    National Advisory Committee for Aeronautics,
        Langley Field, Va., June 21, 1941.

Total pressure ratio across fan $= \dfrac{H_1}{H_0} = \left[1 + \dfrac{550\ hp\ \eta}{c_p\ m\ T_0} \right]^{r-1}$

where $H_0 =$ tot. press. ahead of fan
$H_1 =$ " " behind fain
$hp =$ input horsepower

$\eta =$ adiabatic efficiency of fan
$c_p =$ spec. heat at const. pressure
$T_0 =$ abs. temp. ahead of fan

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APPENDIX A

When fluid moves through a fan with axial velocity V at radius r, the force required to impart continuously a tangential velocity w to an annulus of fluid of thickness Δr is

$$\Delta F = (2\pi\ r\ \Delta\ r)\ \rho\ V\ w$$

If this force is supplied by the fan rotating at shaft speed n, power expended in rotating the annulus of fluid is

$$\Delta P = 2\pi\ r\ n\ \Delta F = 2\pi\ r\ n\ (2\pi\ r\ \rho\ V\ \Delta r)\ w$$

Rotational energy in the annulus of fluid leaving the fan in unit time is

$$\Delta E = (2\pi\ r\ \rho\ V\ \Delta\ r)\frac{w^2}{2}$$

which is equal to the power lost.

The rotational efficiency of the fan-blade element at radius r is

$$\eta_R' = 1 - \frac{\text{power loss}}{\text{power input}}$$

$$\eta_R' = 1 - \frac{(2\pi\ r\ \rho\ V\ \Delta\ r)\ \frac{w^2}{2}}{2\pi\ r\ n\ (2\pi\ r\ \rho\ V\ \Delta\ r)\ w}$$

$$\eta_R' = 1 - \frac{w/2}{2\pi\ r\ n}$$

APPENDIX B.

$$\omega^2 dr = \eta_0'\ (-\ \omega r d\omega + 4\pi n\omega dr + 2\pi n r d\omega - \omega^2 dr) \qquad (14)$$

$$(1 + \eta_0') \, \omega^2 dr = \eta_0' \, (- \omega r d\omega + 4\pi n \omega dr + 2\pi n r d\omega)$$

$$\frac{1 + \eta_0'}{\eta_0'} \, \omega^2 dr + \omega r d\omega - 2\pi n \, (2\omega dr + r d\omega) = 0$$

$$\frac{1}{\eta_0'} \, \omega^2 dr + 2\omega^2 dr - \omega^2 dr + \omega r d\omega - 2\pi n \, (2\omega dr + r d\omega) = 0$$

$$\frac{1 - \eta_0'}{\eta_0'} \, \omega^2 dr + \omega(2\omega dr + r d\omega) - 2\pi n (2\omega dr + r d\omega) = 0$$

$$\frac{1 - \eta_0'}{\eta_0'} \, \omega^2 dr + (2\omega dr + r d\omega) \, (\omega - 2\pi n) = 0$$

$$\frac{1 - \eta_0'}{\eta_0'} \left(\frac{\omega}{2\pi n}\right)^2 dr + \left(2 \, \frac{\omega}{2\pi n} \, dr + r \, \frac{d\omega}{2\pi n}\right) \left(\frac{\omega}{2\pi n} - 1\right) = 0$$

Let
$$\frac{1 - \eta_0'}{\eta_0'} = a$$

and
$$\frac{\omega}{2\pi n} = x$$

Then, $$a x^2 dr + (2x dr + r dx) \, (x - 1) = 0$$

$$\frac{a x^2}{x - 1} \, dr + 2x dr + r dx = 0$$

$$\left(\frac{a x^2}{x - 1} + 2x\right) dr + r dx = 0$$

$$\frac{dr}{r} + \frac{(x - 1) \, dx}{a x^2 + 2x(x - 1)} = 0$$

$$\frac{dr}{r} + \frac{dx}{a x + 2(x - 1)} - \frac{dx}{a x^2 + 2x^2 - 2x} = 0$$

$$\frac{dr}{r} + \frac{dx}{(a + 2)x - 2} - \frac{dx}{x \, [(a + 2)x - 2]} = 0$$

Let $$(a + 2) = b$$

Then,

$$\int \frac{dr}{r} + \int \frac{dx}{bx - 2} - \int \frac{dx}{x\,(bx - 2)} = \log C_3$$

$$\log r + \frac{1}{b}\int \frac{-\,dx}{\frac{2}{b} - x} + \frac{1}{b}\int \frac{dx}{x\left(\frac{2}{b} - x\right)} = \log C_4$$

$$\log r + \frac{1}{b}\log\left(\frac{2}{b} - x\right) + \frac{1}{2}\log x - \frac{1}{2}\log\left(\frac{2}{b} - x\right) = \log C_5$$

$$2\log r + \left(\frac{2}{b} - 1\right)\log\left(\frac{2}{b} - x\right) + \log x = \log C_6$$

$$\log r^2\,x - \log\left(\frac{2}{b} - x\right)^{1-\frac{2}{b}} = \log C_7$$

$$r^2\,x = C\left(\frac{2}{b} - x\right)^{1-\frac{2}{b}}$$

$$r^2\left(\frac{\omega}{2\pi n}\right) = C\left(\frac{2\eta_0}{1 + \eta_0} - \frac{\omega}{2\pi n}\right)^{\frac{1-\eta_0}{1+\eta_0}} \tag{15}$$

APPENDIX C

If a uniform distribution of axial velocity is assumed, the rotational efficiency of the entire fan is

$$\eta_R = \frac{\int \text{mass flow through annulus (at radius } r) \times \text{elemental } \eta_R}{M}$$

$$\eta_R = \frac{1}{M}\int 2\pi r \rho V\left(1 - \frac{w/2}{2\pi rn}\right)dr$$

$$\eta_R = \frac{1}{M}\int 2\pi r \rho V\,dr - \int \frac{\tau BD^2}{8\pi^2 r^2}\,2\pi \rho V r\,dr$$

$$\eta_R = \frac{1}{M}\left[M - \frac{TBD^2 \rho V}{4\pi}\int_{r_0/D}^{0.5} \frac{d\left(\frac{r}{D}\right)}{r/D}\right]$$

From equation (20)

$$P = T B (nD)^2 M$$

$$\eta_R = 1 - \frac{\rho V P}{4\pi n^2 M \left\{ \rho V\pi D^2\left[0.25 - \left(\frac{r_0}{D}\right)^2\right]\right\}}\log\frac{0.5}{r_0/D}$$

$$\eta_R = 1 - \frac{P}{(\pi nD)^2 M}\left[\frac{\log\frac{1}{D_0/D}}{1 - \left(\frac{D_0}{D}\right)^2}\right] \tag{26}$$

APPENDIX D

The resistance of the interior of a wind tunnel to air flow through the tunnel is assumed here to increase as the square of the throat velocity. The power in a cross section at the throat is

$$\frac{1}{2}\rho_T V_T^2 A_T V_T$$

The energy ratio is

$$E = \frac{\frac{1}{2}\rho_T V_T^3 A_T}{P}$$

In order to indicate the effectiveness of the wind-tunnel tube as a diffuser, the fan efficiency η_{design} with which the drive power is applied, must be stated. This same value of efficiency was assumed in computing the energy ratio of the wind tunnel.

$$E \times \text{shaft power} = \tfrac{1}{2} \rho_T V_T{}^3 A_T = \tfrac{1}{2} M V_T{}^2$$

$$E \times \frac{\text{thrust power}}{\eta_{\text{design}}} = \frac{M V_T{}^2}{2}$$

$$V_T{}^2 = \left(\frac{\rho_F A_F}{\rho_T A_T} \times \text{mean velocity through fan} \right)^2$$

$$V_T{}^2 = \left[\frac{\rho_F A_F}{\rho_T A_T} \frac{2\pi D^2}{A_F} \int_{(r/D)_0}^{0.5} \frac{r}{D} V \, d\left(\frac{r}{D}\right) \right]^2$$

$$V_T{}^2 = \left[\frac{\rho_F A_F}{\rho_T A_T} \frac{2\pi n D^3}{T B A_F} B \int_{(r/D)_0}^{0.5} \tau \frac{r}{D} J' \, d\left(\frac{r}{D}\right) \right]^2$$

$$V_T{}^2 = \left[\frac{\rho_F A_F}{\rho_T A_T} \frac{2\pi n D^3}{T B A_F} B \int_{(r/D)_0}^{0.5} C_Q' \, d\left(\frac{r}{D}\right) \right]^2$$

$$V_T{}^2 = \left(\frac{\rho_F A_F}{\rho_T A_T} \frac{2\pi n D^3}{A_F B} \frac{C_Q}{\tau} \right)^2$$

$$\text{Thrust power} = \frac{\eta_{\text{design}} M}{2E} V_T{}^2$$

$$\text{Thrust power} = \frac{T}{A_F} \times \text{volume flow through fan} = \frac{T M}{\rho_F A_F}$$

$$\rho_F n^2 D^4 C_T \frac{M}{\rho_F A_F} = \frac{\eta_{\text{design}} M}{2E} V_T{}^2$$

$$n^2 D^4 c_T = \frac{\eta_{design}}{2E} A_F \left(\frac{\rho_F}{\rho_T} \frac{A_F}{A_T} \frac{2\pi n D^3}{A_F B} \frac{c_Q}{T}\right)^2$$

$$c_T = \frac{\eta_{design}}{E} \frac{2\pi^2 D^2}{A_F B^2} \left(\frac{\rho_F}{\rho_T} \frac{A_F}{A_T}\right)^2 \left(\frac{c_Q}{T}\right)^2$$
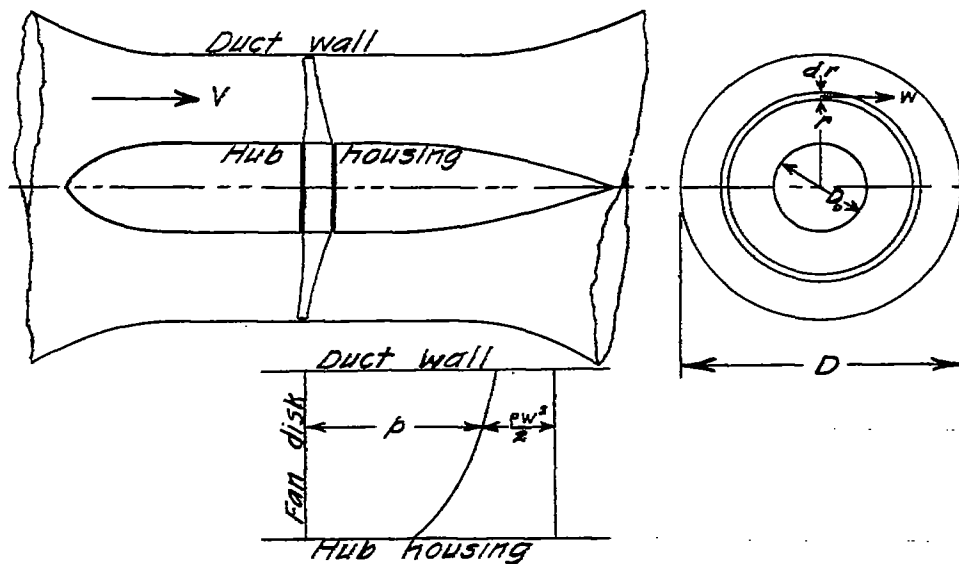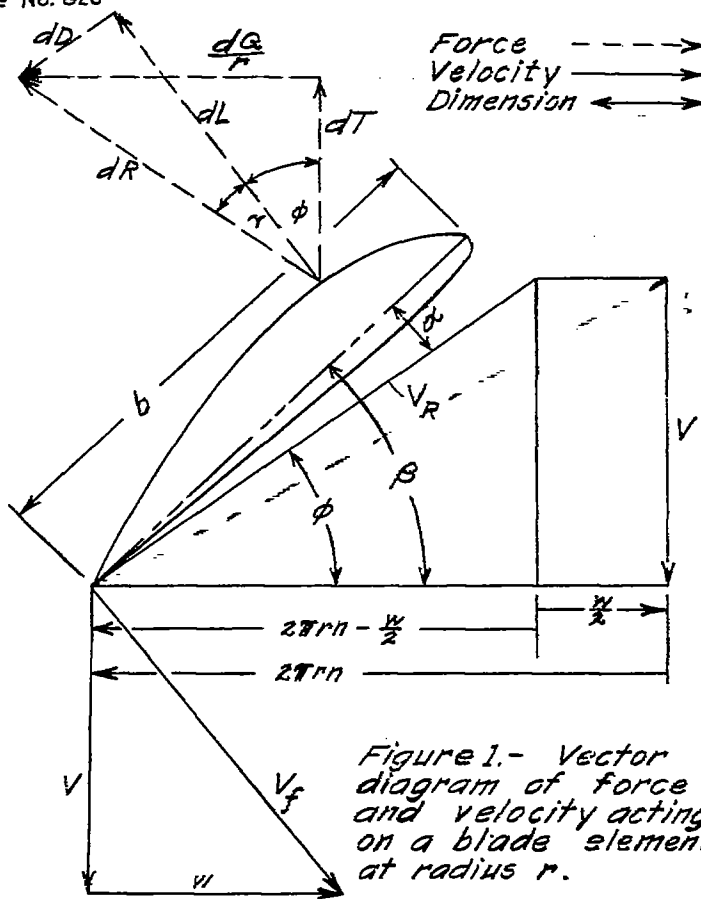
Let

$$\lambda = \frac{2\eta_{design} \pi^2 D^2}{E A_F B^2} \left(\frac{\rho_F}{\rho_T} \frac{A_F}{A_T}\right)^2$$

$$c_T = \lambda \left(\frac{c_Q}{T}\right)^2$$

REFERENCES

1. Keller, Curt, Marks, Lionel S., and Weske, John R.: The Theory and Performance of Axial Flow Fans. McGraw-Hill Book Co., Inc., 1937.

2. Weick, Fred E.: Aircraft Propeller Design. McGraw-Hill Book Co., Inc., 1930.

3. Glauert, H.: Fans. Vol. IV, div. L, sec. 6, ch. XI of Aerodynamic Theory, W. F. Durand, ed., Julius Springer (Berlin), 1935, pp. 338-341.

4. Wattendorf, Frank L. The Efficiency of Return Flow Wind Tunnels. Reps. Nos. 4 & 5, Sci. Rep. Nat. Tsing Hua Univ. (Peiping), ser. A, vol. III, July 1936, pp. 377-402.

5. Collar, A. R.: The Design of Wind Tunnel Fans. R.& M. No. 1889, British A.R.C., 1940.

6. von Kármán, Th.: Turbulence and Skin Friction. Jour. Aero. Sci., vol. I, no. 1, Jan. 1934, pp. 1-20.

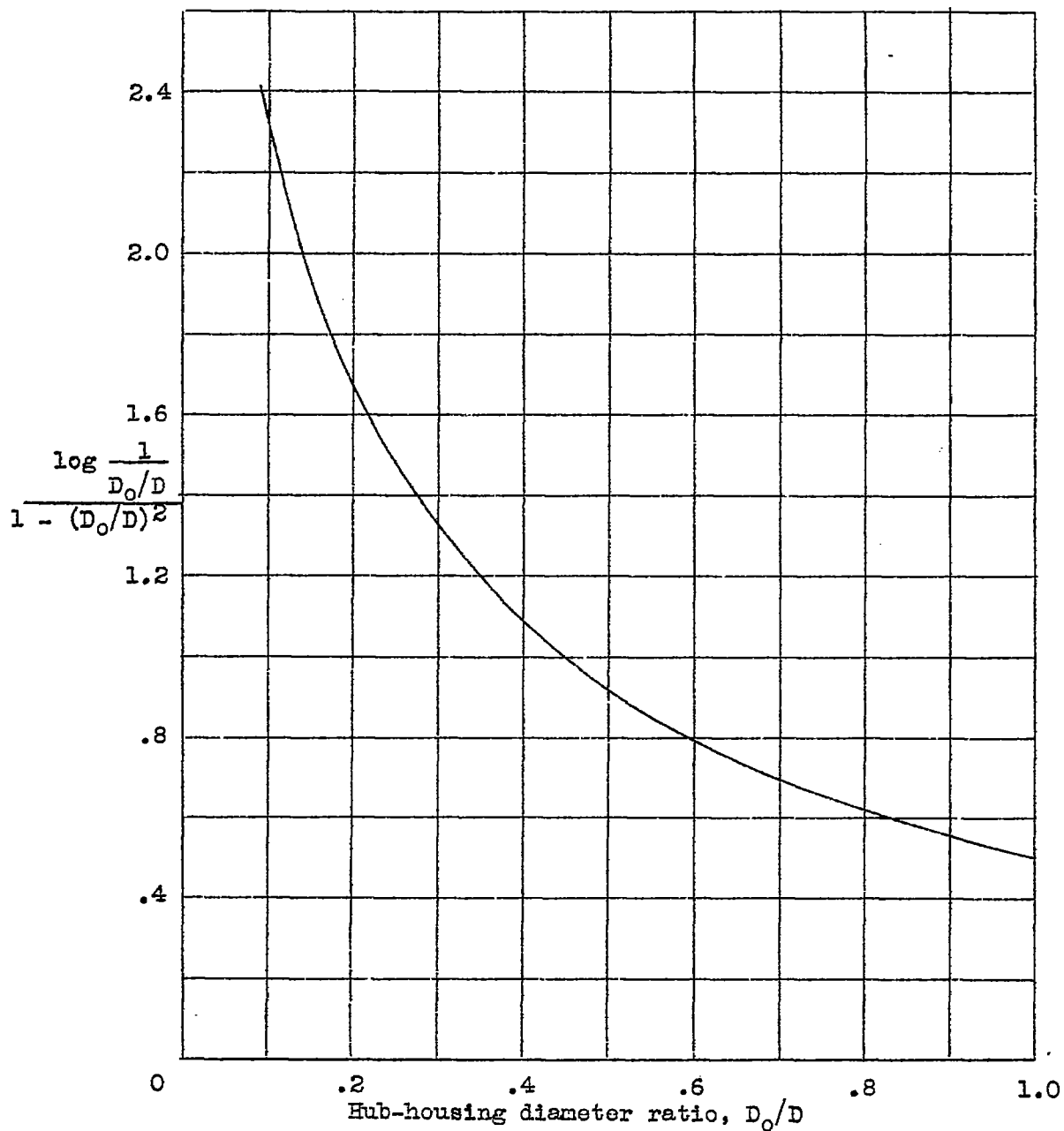7. Zahm, A. F.: Flow and Drag Formulas for Simple Quadrics. Rep. No. 253, NACA, 1927.

Force   -----→
Velocity   ——→
Dimension   ←——→

$$\frac{V}{2}\left(\frac{V}{2\pi rn}\right)$$

$$\sin\phi = \frac{1}{2\pi n \frac{r}{V}}$$

$$2\pi rn - \frac{w}{2}$$

$$2\pi rn$$

Figure 1.– Vector diagram of force and velocity acting on a blade element at radius r.

Duct wall

→ V

Hub   housing

Duct wall

Fan disk

p

$\frac{\rho w^2}{2}$

Hub housing

Figure 2.- Static–pressure increment produced by a single-stage fan rotor.

Figure 3.- Hub-housing ratio function for use in equation (26).

Tunnel wall

Counter vane

Diffuser

Air Flow

Fan

Hub housing

Hub housing

Fan

Counter vane
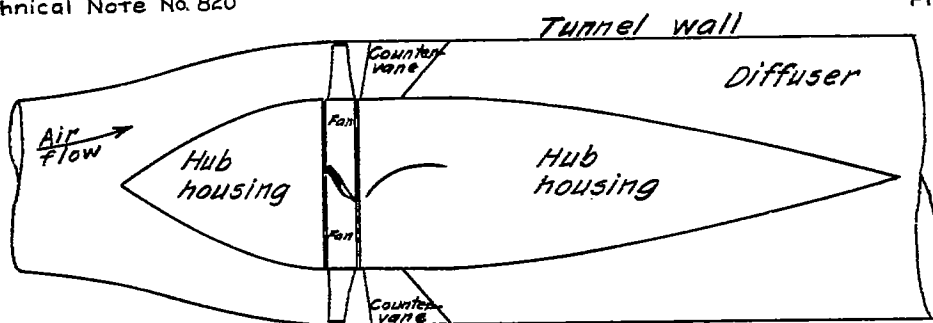
Figure 4.- Wind tunnel fan installation having a large D_o/D ratio.

$\frac{c}{D} = 0.25$

C_Q'

$T = .10$

.08

.06

.04

.02

$\beta = 5°$ 10° 15° 20° 25° 30°

C_T'

$T = .10$

.08

.06

.04

.02

$\beta = 5°$ 10° 15° 20° 25° 30°

J'
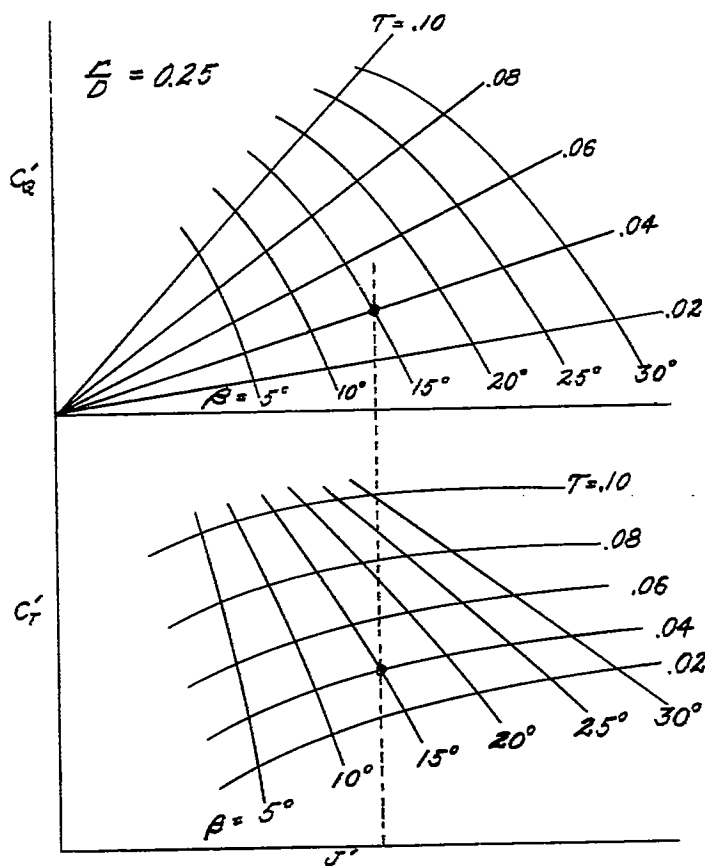
Figure 5.- Diagram of C_Q' & C_T' chart.