PERFORMANCE ANALYSIS OF
A PISTON ENGINED AIRPLANE -
PIPER CHEROKEE PA-28-180

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Performance Analysis of a piston engined airplane - Piper Cherokee PA-28-180

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The report is intended to serve as an example of performance calculation of a typical piston engined airplane.

Problem statement: Obtain the following for the prescribed airplane:

- Information about the airplane.
- Drag Polar at cruising speed and during take-off condition.
- Engine Characteristics.
- Variation of stalling speed with altitude for flaps up and flaps down conditions.
- Variation of $V_{\text{max}}$ and $V_{\text{min}}$ with altitude.
- Variation of $R/C$ and $\gamma$ with speed and altitude. Variation of $V_{R/C_{\text{max}}}$ and $V_{\gamma_{\text{max}}}$ with altitude. Values of absolute ceiling and service ceiling.
- Variation of range and endurance with flight speed in constant velocity flights at cruising altitude. Speeds corresponding to $R_{\text{max}}$ and $E_{\text{max}}$ in these constant velocity flights. The range in constant $C_L$ flight at cruising altitude.
- Variation of minimum turn radius ($r_{\text{min}}$) and maximum rate of turn $\psi_{\text{max}}$ at selected altitudes and variation of $(V_{r_{\text{min}}})$ and $(V_{\psi_{\text{max}}})$ with altitude.
- Take-off and landing distances.
Figure 1 Three-view drawing of Piper Cherokee PA-28-180 (from Ref.2, 1995 edition)
Note: Dimensions are in inches, areas are in square feet and moments of inertia are in lb foot square.
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0.1 Information about the airplane

Airframe: Piper Cherokee PA-28-180
Type: Piston-engined propeller driven low speed recreational aircraft.
Manufacturer and country of origin: The Piper Aircraft Corporation, USA.

0.1.1 Overall Dimensions

Length: 23.29 feet (7.098 m)
Wing Span: 30 feet (9.0 m)
Height above ground: 7.291 feet (2.22 m)
Wheel base: 6.208 feet (1.892 m)
Wheel track: 10 feet (3.048 m)

0.1.2 Powerplant

Name: Lycoming O-360-A3A
Rating: 180BHP (135 kW) at 2700 rpm
Weight: 284 pounds (129kg)
Number: 1
Propeller: 74 inches (1.88 m) diameter, fixed pitch.

0.1.3 Weights

Maximum take-off weight: 2400 lbf (1090.9 kgf)
Empty weight: 1230 lbf (558 kgf)
Fuel capacity: 189 liters (181 liters usable)
Payload: 530.7 lbf (468.1 kgf)
Maximum wing loading: 15 lbf/ft$^2$ (73.2 kgf/m$^2$)
Maximum power loading: 13.3 lbf/Hp (8.0965 kgf/kW)
0.1.4 Wing Geometry

Planform Shape: Trapezoidal near root, rectangular afterwards and elliptical fillets at the tip.
Span: 30 feet (9 m)
Area: 160 $ft^2$ (14.86$m^2$)
Flap area: 14.9 $ft^2$ (1.384 $m^2$)
Aileron area: 10.8 $ft^2$ (1.003 $m^2$)
Airfoil: NACA - 65-215 series, $t/c = 15\%$, $C_{top} = 0.4$
Root Chord: 5.6766 feet (1.703 m) - Equivalent Trapezoidal wing
Tip Chord: 5.25 feet (1.575 m)
Mean Aerodynamic Chord: 5.498 feet (1.675 m)
Quarter chord Sweep: $1\frac{48}{4}$
Dihedral: 6$^\circ$
Twist: $-2^\circ$
Incidence: 4.62$^\circ$ at root, 2.62$^\circ$ at tip
Taper Ratio: 0.9248 (equivalent wing)
Aspect Ratio: 5.738
High lift devices: simple flaps having 3 different settings: 10$^\circ$, 25$^\circ$, and 40$^\circ$

0.1.5 Fuselage Geometry

Length: 22.5 feet (6.874 m)
Maximum fuselage cross section dimension (vertical): 3.89 feet (1.185 m)
Accommodation: 4 persons in two pairs, baggage compartment with 200 lbf (90.91 kgf) capacity. Starboard side door, removable rear seats that give 44 $ft^3$ (1.25 $m^3$) space. Can carry stretcher.

0.1.6 Horizontal Tail Geometry

Plan-form shape: Rectangular with elliptical fillets at tips.
Span: 10 feet (3.048 m)
Area: 24.4 $ft^2$ (2.267 $m^2$)
Root Chord = Tip chord = 2.5 feet (0.762 m)
Aspect Ratio: 4.0983
Airfoil: NACA 0012.
Wetted area: 45.198 $ft^2$ (4.199 $m^2$)
0.1.7 Vertical Tail Geometry

Span : 3.68 feet (1.121 m)
Area : 11.6 $ft^2$ (1.078 $m^2$)
Root Chord : 3.28 feet (1 m)
Tip chord : 1.492 feet (0.4547 m)
Mean Aerodynamic Chord : 2.386 feet (0.727) m
Quarter Chord Sweep : 21.8°
Taper Ratio : 0.4548
Aspect Ratio : 1.188
Airfoil : NACA 0010.

0.1.8 Landing gear

Nose Wheel type with fairing, non-retractable.
Number of wheels : Nose 1, main 2, all same size.
Thickness 0.444 feet (0.135 m)
Diameter : 1.492 feet (0.4547 m)
Wheel base : 6.208 feet (1.892 m)
Wheel track : 10 feet (3.048 m)

0.1.9 Flight Condition

Altitude : 8000 ft (2438 m)
Mach number : 0.1972
Kinematic Viscosity : 0.17721 $\times 10^{-4} m^2/s$
Density : 0.9669 $kg/m^3$
Speed of Sound : 330.9 m/s
Flight Speed : 235 km/hr (65.27 m/s)
Weight of the Airplane : 2400 lbf (1090.91 kgf)

0.1.10 Other data

CG location : 6.81 feet (2.075 m) behind nose.
$I_x = 170$ slug $ft^2$ (230.4946 $kgm^2$)
$I_y = 1249$ slug $ft^2$ (1693.457 $kgm^2$)
$I_z = 1312$ slug $ft^2$ (1778.876 $kgm^2$)
0.1.11 Performance of PA-28-181* as given in Ref[3]

Maximum take-off weight: 2550lb (1356 kgf)
Powerplant rating: 180 BHP (135 kW)
Wing loading: 15lb/ft² (73.2kgs/m²)
Maximum level speed: 246 kmph
Cruising speed: 237 kmph
Stalling speed: 86 kmph, at flaps down condition
Maximum rate of climb: 203 m/min at sea level
Service ceiling: 4035 m
Take-off run: 350m
Take-off to 15m: 488m
Landing run: 280m
Landing distance from 15m: 427m
Range with allowance for taxi, take-off, climb, descent and 45 min reserves at 6000 feet (1830 m) at 55 power: 924 km

*Remark:

The performance calculations are being done for PA-28-180 as a large amount of data on airplane, engine and propeller are available in Ref [2]. However, information on actual performance of this airplane is not given there. Ref [3] (which is easily accessible) contains information about PA-28-181 which is only slightly different from PA-28-180
0.2 Estimation of Drag Polar

Following Ref[1] the drag polar is assumed to be of the form

\[ C_D = C_{D_o} + \frac{C_L^2}{\pi A_c} \]  

(1)

The quantity \( C_{D_o} \) is assumed to be given by

\[ C_{D_o} = C_{D_{oWB}} + C_{D_{oV}} + C_{D_{oH}} + C_{D_{oMisc}} \]  

(2)

where suffixes \( WB, V, H, Misc \) denote wing-body combination, vertical tail, horizontal tail and miscellaneous items respectively.

0.2.1 Estimation of \( C_{D_{oWB}} \)

\( C_{D_{oWB}} \) is obtained from

\[ C_{D_{oWB}} = C_{D_{oW}} + C_{D_{oB}} \frac{S_B}{S_{\text{ref}}} \]  

(3)

where the suffix B denotes fuselage and \( S_B \) is the maximum frontal area of fuselage.

\[ C_{D_{oW}} = C_{D_{fW}} + C_{D_{WW}} \]  

(4)

\( C_{D_{fW}} \) is the skin friction drag coefficient, obtained by first determining the skin friction coefficient of a flat plate and then correcting for the shape of the airfoil.

\( C_{D_{WW}} \) is the wave drag coefficient for the wing. In the present case, the Mach number is low and hence the wave drag is neglected.

The pressure drag coefficient for the wing \( C_{D_{oW}} \) is obtained from :

\[ C_{D_{oW}} = C_{f_w} \left[ 1 + L \left( \frac{t}{c} \right) \right] \frac{S_{\text{wet}}}{S_{\text{ref}}} \]  

(5)

The \( \text{Re} \) based on mean chord is \( 6.07586 \times 10^6 \).

Now, we find \( \text{Re}_{\text{cutoff}} \) corresponding to the roughness parameter \( \left( \frac{l}{k} \right) \).

Here, \( l = 5.498 \) feet and \( k = 0.0012 \) inches for mass production spray paint.

For this \( \frac{l}{k} \) of 54980, \( \text{Re}_{\text{cutoff}} = 4 \times 10^6 \).
We observe that $Re_{cutoff}$ is lower than that based on mean aerodynamic chord. Hence $C_f$ is found based on $Re_{cutoff}$, which, from Ref. [1] is found to be 0.0036.

From the airplane data, we have $S_{wet} = 313.77\text{ ft}^2 (29.14\text{ m}^2)$ and $S_{ref} = 160\text{ ft}^2 (14.86\text{ m}^2)$

Substituting the above values in eqn (5) yields

$$C_{Df_w} = 0.0036 \left[1 + 1.2 \times (0.15)\right] \times \frac{313.77}{160}$$

Thus $C_{Df_w}$ is found to be 0.00833 and so $C_{D_{OW}} = 0.00833$

### 0.2.2 Estimation of $C_{D_{OB}}$

$C_{D_{OB}}$ is the fuselage profile drag coefficient, and is given by

$$C_{D_{OB}} = C_{D_{fB}} + C_{D_{pB}}$$

### 0.2.3 Expressions for Drag Polar during cruise

where $C_{D_{fB}}$ is the skin friction drag coefficient, given by

$$C_{D_{fB}} = C_{fB} \frac{S_{WF}}{S_B}$$

where $S_{WF}$ is the fuselage wetted surface area $= 236.15\text{ ft}^2 (21.93\text{ m}^2)$ and $S_B$ is the frontal area of the fuselage $= 8.263\text{ ft}^2 (0.7675\text{ m}^2)$

Proceeding in the same manner as for the wing, $C_{D_{fB}}$ is found using $Re_{cutoff}$

$$Re_{length} = 2.49 \times 10^7$$

$$\frac{l}{k} = 2.25 \times 10^5 \Rightarrow Re_{cutoff} = 1.8 \times 10^7$$

$$\Rightarrow C_{fB} = 0.0028 \text{ from Ref [1].}$$

$$\Rightarrow C_{D_{fB}} = 0.0028 \times \frac{236.15}{8.263} = 0.08001$$
Now, $C_{D_{PB}}$ is given by

$$C_{D_{PB}} = C_{f_{B}} \left( \frac{60}{(l_f/d_f)^3} + 0.0025 \left( \frac{l_f}{d_f} \right)^4 \right) \times \frac{S_W}{S} \quad (8)$$

where $\frac{l_f}{d_f}$ is the fuselage fineness ratio.

Substituting the values from aircraft dimensions, we obtain $C_{D_{PB}} = 0.005379$ and hence $C_{D_{OB}} = 0.085379$

Thus $C_{D_{WB}} = 0.01277$

### 0.2.4 Estimation of $C_{D_{HT}}$

The drag coefficient of the horizontal tail is given by

$$C_{D_{HT}} = C_{f_{p}} \left[ 1 + L \left( \frac{t}{c} \right) + 100 \left( \frac{t}{c} \right)^4 \right] R_{LS} \frac{S_{wet}}{S_{ref}} \quad (9)$$

The tail has NACA 0012 airfoil $\Rightarrow \frac{t}{c} = 0.12$

The wetted surface area of the horizontal tail is computed from the aircraft geometry. It is found that $S_{wet} = 45.199 ft^2(4.198 m^2)$ and $S_{ref} = 160 ft^2(14.86 m^2)$

$Re$ based on mean aerodynamic chord is $2.76 \times 10^6$

$\frac{l_k}{k} = 2.5 \times 10^4$

$Re_{cutoff}$ based on $\frac{l_k}{k}$ of $2.5 \times 10^4$ is $1.5 \times 10^6$

$\Rightarrow C_{f_{p}} = 0.004$ and $R_{LS} = 1.07$

Substituting the values in equation (9), we have $C_{D_{HT}} = 0.001408$

### 0.2.5 Estimation of $C_{D_{VT}}$

The drag coefficient of the vertical tail is given by

$$C_{D_{VT}} = C_{f_{p}} \left[ 1 + L \left( \frac{t}{c} \right) + 100 \left( \frac{t}{c} \right)^4 \right] R_{LS} \frac{S_{wet}}{S_{ref}} \quad (10)$$

The tail has NACA 0010 airfoil $\Rightarrow \frac{t}{c} = 0.10$

The wetted surface area of the vertical tail is computed from the aircraft geometry. It is found that $S_{wet} = 23.2 ft^2(2.155 m^2)$ and $S_{ref} = 160 ft^2(14.86 m^2)$

$Re$ based on mean aerodynamic chord is $2.63 \times 10^6$ $\frac{l_k}{k} = 2.38 \times 10^4$

$Re_{cutoff} = 1.5 \times 10^6$

$\Rightarrow C_{f_{p}} = 0.004$ and $R_{LS} = 1.07$

Substituting the values in equation (10), we have $C_{D_{VT}} = 0.0007$
0.2.6 Estimation of $C_{DLG}$ and $C_{DMisc}$

The landing gear drag coefficient can be obtained from Ref [1]. However, Ref [2] gives $C_{DLG} = 0.0044$ and $C_{DMisc} = 0.003875$.

0.2.7 Estimation of parasite drag coefficient $C_{DO}$

\[
C_{DO} = C_{DOWB} + C_{DOHT} + C_{DOVT} + C_{DOLG} + C_{DOMisc}
\]
\[\Rightarrow C_{DO} = 0.01277 + 0.001408 + 0.0007 + 0.003875 + 0.0044
\]
Thus $C_{DO} = 0.02315$, which is close to the value of 0.0244 given in Ref [2].

0.2.8 Estimation of induced drag coefficient $K$

The induced drag coefficient $K$ is evaluated from the expression

\[
K = \frac{1}{\pi Ae}
\]
where the Aspect ratio $A$ is 5.738 for the wing and $e$ is the ostwald efficiency factor, evaluated from the expression

\[
\frac{1}{e} = \frac{1}{e_{wing}} + \frac{1}{e_{fuselage}} + \frac{1}{e_{other}} \tag{11}
\]

From Ref [1], for an unswept wing, $e = 0.91$ and

\[
\frac{1}{e_{fus}} \times \left( \frac{S_f}{S_{ref}} \right) = 2
\]
\[\Rightarrow \frac{1}{e_{fus}} = 2 \times \frac{8.2638}{160} = 0.1033
\]

From Ref [1],

\[
\frac{1}{e_{other}} = 0.05
\]

Substituting the values in eqn 11, we obtain $e = 0.7973$ and $K = 0.0696$, which is very close to the value of 0.0697 given in Ref [2].
0.2.9 Expressions for Drag Polar during cruise
At cruise condition, the expression for the drag polar becomes

\[ C_D = 0.02315 + 0.0696C_L^2 \]  (12)

0.2.10 Expression for Drag Polar during take-off condition
To obtain drag polar under take-off condition, we take the flight velocity as \(1.2V_S\), where \(V_S\) is the stalling speed with flaps in take-off condition (\(\delta_f = 10^\circ\)). In the present case,

- \(C_{L_{\text{max}}}\) with \(10^\circ\) flap deflection is 1.42 from Ref [2]. Hence

\[ V_S = \sqrt{\frac{2 \times 10673.28}{1.42 \times 1.225 \times 14.86}} = 28.73 m/s \]

and so

\[ V_{TO} = 1.2 \times 28.73 = 34.47 m/s \]

- Reynold’s number based on mean aerodynamic chord of the wing in take-off condition = \(\frac{1.675 \times 34.47}{14.6 \times 10^{-6}} = 3.954 \times 10^6\)

- We notice that this Reynold’s number is very close to the cutoff Reynold’s number for the wing (\(4 \times 10^6\)) obtained in Section 0.2.1. Thus the value of \(C_f\) and other calculations will remain the same. Hence \(C_{D_0}\) for the airplane in take-off condition, without the flap, can be taken as 0.02315.

- Similarly \(K\), without the flap, can be taken as 0.0696.

To apply correction to the drag polar for flap deflection, we proceed as follows

- The flap type is plain flap.

- From Fig. 1, the ratio of flap chord to wing chord is 0.16 and flap deflection is \(10^\circ\)

- The ratio of the area of the flapped portion of the wing to the wing plan-form area is 0.4827

- The ratio of the span of the flapped portion of the wing (including the fuselage width) to the total span is 0.597

- The ratio of the fuselage width to the wing span is 0.127, the wing aspect ratio is 5.738
Following Ref [1],
\[ \Delta C_{D_{flap}} = \Delta C_{D_p} + \Delta C_{D_i} + \Delta C_{D_{int}}, \]
where \( \Delta C_{D_p} \) = increase in profile drag coefficient due to flaps,
\( \Delta C_{D_i} \) = increase in induced drag coefficient due to flaps and
\( \Delta C_{D_{int}} \) = increase in interference drag due to flaps.

- Increment in \( C_{L_{max}} \) due to 10\(^o\) flap deflection (\( \Delta C_{L_{max}} \)), as noted earlier, is 0.09

Using these data and interpolating the curves given in Ref [1], we get:
\( \Delta C_{D_0} \) due to flap deflection = 0.0038

As per Ref [1], the increase in induced drag coefficient (\( \Delta C_{D_i} \)) due to flap deflection is \( \Delta K_f^2 \times \Delta C_{L_{max}}^2 \). Using Ref [1], \( \Delta K_f \) is estimated as 0.184

Consequently, \( \Delta C_{D_i} = 0.184^2 \times 0.09^2 = 0.00027 \)
The interference drag due to deflection of plain flaps is negligible. Thus the parasite drag coefficient in take-off condition is
\( C_{D_0} = 0.02315 + 0.0038 + 0.00027 = 0.02722 \)

Hence the drag polar in take-off condition is given by
\[ C_D = 0.02722 + 0.0696C_L^2 \quad (13) \]

Remarks:

- In the approach just presented, to estimate the drag polar in take-off condition, the change in the induced drag coefficient is included in the parasite drag coefficient. When the flap deflections are large, the change in the induced drag can be accounted for by reducing the value of the Ostwald efficiency factor (\( e \)) by 0.05 for take-off condition and 0.1 for landing condition (from Ref [1]). Equations 12 and 13 are the drag polars for cruise condition and take-off condition respectively.

- It may be pointed out that parabolic drag polar is not valid beyond \( C_{L_{max}} \). It is only approximate near \( C_L = 0 \) and \( C_L = C_{L_{max}} \).
0.3 Engine Characteristics

Model: Lycoming O-360-A3A.
Type: air-cooled, carbureted, four-cylinder, horizontally opposed piston engine.
Sea level power = 180 BHP (135 kW)
Propeller: 74 inches (1.88 m) diameter

The variations of power output and fuel consumption with altitude and rpm are shown in Fig.3.

For the present calculations, the values will be converted into SI units.

Figure 3 Characteristics of Lycoming O-360-A (from Ref.2, 1995 edition).
0.3.1 Variation of Engine BHP

The variation of engine BHP with altitude is assumed to be of the form

\[ BHP_a = BHP_{sealevel} (1.13\sigma - 0.13) \]

where \( \sigma \) is the density ratio = \( \rho / \rho_{SL} \)

The power outputs of the engine at select altitudes are given in Table 1.

<table>
<thead>
<tr>
<th>( h(m) )</th>
<th>( \sigma )</th>
<th>( BHP_a(kW) )</th>
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<tr>
<td>sea level</td>
<td>1.2256</td>
<td>135.000</td>
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<tr>
<td>500</td>
<td>0.9529</td>
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</tr>
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<td>5000</td>
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<td>74.163</td>
</tr>
</tbody>
</table>

Table 1: Variation of BHP with altitude

Note: The variation of engine BHP with flight speed is very slight and is generally neglected.

0.3.2 Thrust horsepower available

The available Thrust horsepower is obtained from \( BHP_a \times \eta_p \), where \( \eta_p \) is the propeller efficiency. \( \eta_p \) depends on the flight speed, rpm of the engine and the diameter of the propeller. This can be worked out at different speeds and altitudes using propeller charts. However, chapter six of Ref [2] gives an estimated curve of efficiency as a function of the advance ratio \( (J = \frac{\nu}{nD}) \) for the fixed pitch propeller used in the present airplane. This curve is shown in Figure 5, and according to Ref [2], is roughly valid for all altitudes and speeds.

For the purpose of calculating the airplane performance, an equation can be fitted to the \( \eta_p \) vs J curve shown in Fig.4. A fourth degree polynomial for \( \eta_p \) in terms of J is as follows
\[ \eta_p(J) = -2.071895 J^4 + 3.841567 J^3 - 3.6786 J^2 + 2.5586 J - 0.0051668 \] (14)

It is seen that the fit is very close. The dotted portions are extrapolations.

For the calculation of maximum speed, maximum rate of climb and maximum rate of turn, it is convenient to have maximum power available \( THP_a = \eta_p \times \text{BHP} \) as a function of velocity. The maximum power occurs at 2700 rpm (45 rps). Noting the propeller diameter as 1.88 m, the \( \eta_p \) vs J curve can be converted to \( \eta_p \) vs V curve (Fig5).

The expression for \( \eta_p \) in terms of velocity is as follows:

\[ \eta_p = -4.0447 \times 10^{-8} V^4 + 6.3445 \times 10^{-6} V^3 - 5.1398 \times 10^{-4} V^2 + 3.0244 \times 10^{-2} V - 0.0051668 \] (15)

Making use of the power available at different altitudes as given in Table.1 and the values of the propeller efficiency at different speeds given by equation 15, the maximum available thrust horsepower \( THP_a = \eta_p \times \text{BHP} \) can be obtained at different speeds and altitudes. These are plotted in Fig.6.
Figure 5 - Variation of propeller efficiency with velocity at 2700rpm

Figure 6 - Variation of THP with altitude
0.4 Steady Level Flight

0.4.1 Variation of stalling speed with altitude

In steady level flight, the equations of motion are

\[ T - D = 0 \]  
\[ L - W = 0 \]

Further,

\[ L = \frac{1}{2} \rho V^2 SC_L \implies W = \frac{1}{2} \rho V^2 SC_L \]  
\[ D = \frac{1}{2} \rho V^2 SC_D = T \]

\[ \Rightarrow V = \sqrt{\frac{2W}{\rho SC_L}} \]

Since \( C_L \) cannot exceed \( C_{L_{\text{max}}} \), there is a flight speed below which level flight is not possible. The flight speed at which \( C_L = C_{L_{\text{max}}} \) is called stalling speed and denoted by \( V_S \).

\[ V_S = \sqrt{\frac{2W}{\rho SC_{L_{\text{max}}}}} \]

Since density decreases with altitude, the stalling speed increases with height.
In the present case, \( W = 1088 \times 9.81 = 10673.28 \text{N} \)
\( S = 14.86 m^2 \)
As regards \( C_{L_{\text{max}}} \), Ref \[2\] gives the values of \( C_{L_{\text{max}}} \) as 1.33, 1.42, 1.70 and 1.86 for flap deflections of 0\(^o\), 10\(^o\), 25\(^o\) and 40\(^o\) respectively.
Using these data, the variation of stalling speeds with altitude are presented in Table.2 and plotted in Fig.8.

<table>
<thead>
<tr>
<th>( h ) (m)</th>
<th>( \sigma )</th>
<th>( V_s(\delta_f = 0^o) ) (m/s)</th>
<th>( V_s(\delta_f = 10^o) ) (m/s)</th>
<th>( V_s(\delta_f = 25^o) ) (m/s)</th>
<th>( V_s(\delta_f = 40^o) ) (m/s)</th>
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Table 2: Stalling speeds for various flap settings
Figure 8 - Variation of stalling speed with altitude for different flap settings
0.4.2 Variations of $V_{\text{max}}$ and $V_{\text{min}}$ with altitude

With a parabolic drag polar and engine output given by an analytical expression, the following procedure gives $V_{\text{max}}$ and $V_{\text{min}}$. Available power is denoted by $P_a$ and power required to overcome drag is denoted by $P_r$. At maximum speed in steady level flight, available power equals required power.

$$P_a = \text{BHP} \times \eta_p \quad (20)$$
$$P_r = D \times V = \frac{1}{2} \rho V^2 SC_D \quad (21)$$

The drag polar expresses $C_D$ in terms of $C_L$. Writing $C_L$ as $\frac{2W}{\rho SV^2}$ and substituting in the above equation we get

$$\text{BHP} \times \eta_p = \frac{1}{2} \rho V^2 SC_D + \frac{2KW^2}{\rho SV^2}$$

The propeller efficiency has already been expressed as a fourth order polynomial function of velocity and at a chosen altitude, BHP is constant with velocity. Their product $\eta_p \times \text{BHP}$ gives an analytical expression for power available. Substituting this expression on the left hand side of equation (21) and solving gives $V_{\text{max}}$ and $V_{\text{minpower}}$ at a chosen altitude. Repeating the procedure at different altitudes, we get $V_{\text{max}}$ and $V_{\text{min}}$ at various heights. It may be noted that

- The minimum speed so obtained corresponds that limited by power ($V_{\text{minpower}}$).
- If this minimum speed is less than the stalling speed, flight is not possible at $V_{\text{minpower}}$. The minimum velocity is thus higher of the stalling speed and ($V_{\text{minpower}}$).

The results are tabulated in Table 3 and plotted in Fig.9. It may be noted that at $h = 6338$ m, $V_{\text{max}}$ an $V_{\text{minpower}}$ are same. This altitude is the maximum height attainable by the airplane and will be referred later as absolute ceiling.

23
<table>
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<tr>
<th>altitude (m)</th>
<th>$\sigma$</th>
<th>$V_S(\delta_f = 0^\circ)$ (m/s)</th>
<th>$V_{min\text{power}}$ (m/s)</th>
<th>$V_{min}$ (m/s)</th>
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</table>

Table 3: $V_{max}$ and $V_{min}$ for various altitudes
Figure 9: Variation of maximum and minimum flight velocities with altitude for different flap settings. Note that minimum flight velocity is the stalling speed in this example.
0.5 Steady climb performance

In this flight, the C.G of the airplane moves along a straight line inclined to the horizontal at an angle $\gamma$. The velocity of flight is assumed to be constant during the climb.

![Figure 10 - Forces on an aircraft in steady climb](image)

Since the flight is steady, acceleration is zero and the equations of motion can be written as:

$$T - D - W \sin \gamma = 0$$  \hspace{1cm} (22)

$$L - W \cos \gamma = 0$$  \hspace{1cm} (23)

Noting that $C_L = \frac{2L}{\rho SV^2} = \frac{2W \cos \gamma}{\rho SV^2}$, we get

$$C_D = C_{D_0} + K \left( \frac{2W \cos \gamma}{\rho SV^2} \right)^2$$
Also

\[ V_c = V \sin \gamma \]
\[ \cos \gamma = \sqrt{1 - \frac{V_c^2}{V^2}} \]

Using the above equations,

\[ A \left( \frac{V_c}{V} \right)^2 + B \left( \frac{V_c}{V} \right) + C = 0 \]  \hspace{1cm} (24)

Where \( A = \frac{kW^2}{2\rho V^2 S}, \) \( B = -W \) and \( C = T_{avail} - \frac{1}{2} \rho V^2 S C_{D0} - A \)

The available thrust horsepower is given by the expression:

\[ TV = \text{power available} = P_a = BH P_{sealed} (1.13\sigma - 0.13) \eta_p \]

Equation \ref{24} gives 2 values of \( V_c/V \). We choose the value which is less than 1.0. Consequently

\[ \gamma = \sin^{-1} \frac{V_c}{V} \]  \hspace{1cm} (25)
\[ V_c = V \sin \gamma \]  \hspace{1cm} (26)

Thrust can be obtained for a given velocity and height. Thus the quadratic in \( \frac{V_c}{V} \) can be solved and the angle of climb (\( \gamma \)) and rate of climb (\( V_c \)) can be found for various altitudes and flight speeds. These are plotted in Figures 11 and 12 respectively. The velocities at which maximum values of \( V_c \) and \( \gamma \) occur are taken for each altitude and presented in Figure 13. The plot of \( R/C_{max} \) vs altitude is shown in Figure 14.

**Remark:**

It is observed that the maximum rate of climb and maximum angle of climb decrease with altitude, but the velocity at which the rate of climb and angle of climb are maximum increase very slightly with height.

**Service Ceiling and Absolute Ceiling**

The altitude at which the maximum rate of climb becomes 50 metres/minute is called the service ceiling and the altitude at which the maximum rate of climb becomes zero is called the absolute ceiling of the airplane. These are obtained from Figure 14. It is observed that the absolute ceiling is 6342m and the service ceiling is 5322m. It may be pointed out that the absolute ceiling obtained from \( R/C_{max} \) consideration and that from \( V_{max} \) consideration are same (as they should be).
Figure 12 - Variation of rate of climb with flight speed at various altitudes.
Figure 13 - Variation of velocities of maximum climb rate and maximum climb angle with altitude.
Figure 14 - Variation of maximum rate of climb with altitude
0.6 Range and Endurance

0.6.1 Estimation of Range in constant velocity flight

It is convenient for the pilot to cruise at constant velocity. Hence we consider the range performance in constant velocity flights. In such a flight at a given altitude, the range of a piston-engine powered airplane is given by

\[ R = \frac{3600\eta_p}{BSFC\sqrt{k_1k_2}} \left[ \tan^{-1} \frac{W_1}{\sqrt{k_1/k_2}} - \tan^{-1} \frac{W_1}{\sqrt{k_1/k_2}} \right] \]  

(27)

where \( k_1 = \frac{1}{2} \rho V^2 S C_D \), \( k_2 = \frac{2K}{\rho S V^2} \) and \( W_1 \) and \( W_2 \) are the weights of the aircraft at the start and end of cruise. From this expression, we can obtain range as a function of velocity and hence find maximum range for constant velocity flight at cruising altitude. Taking \( W_1 \) as maximum take-off weight and the amount of usable fuel as 50 U.S. gallons (1331.78 N), we get \( W_1 = 10673.28 \text{N} \) and \( W_2 = 9341.5 \text{N} \).

Procedure for finding range in constant velocity flight

It must be noted that the efficiency of the propeller is a function of both the engine rpm as well as the flight velocity. Also, the engine power output is decided by the rpm, which can be adjusted by the pilot by means of the throttle control. However, the rpm setting again influences the fuel consumption and BSFC is thus indirectly affected. Thus it becomes necessary to use an iterative procedure to determine the range.

The variation of engine BHP and fuel consumption as functions of rpm are extrapolated down to 1200 rpm from the engine characteristics (Figure 3). These are shown in Figures 15 and 16. To work with convenient numbers, the rpm scale was factored down by 100. Quadratic curves were fitted for the plots in Figures 15 and 16.

Remark: During the calculations, it was noticed that the engine curves given in Figure 3 are limited to 2000 rpm. However, the power required at velocities close to minimum power condition is much lower than that at 2000 rpm. Hence the \( BHP_n \) and fuel consumption curves were extrapolated down to 1200 rpm. It may be added that Ref[5] has plotted power output and BSFC curves from 2700 to 1200 rpm for Pratt and Whitney Wasp Major engine. Hence the extrapolation to 1200 rpm performed in the present case seems justifiable.
Figure 15 - BHP vs 100s of rpm

Figure 16 - Variation of fuel consumption with engine rpm
The equations for variations of engine BHP and fuel consumption as functions of rpm are given below:

\[ BHP = -0.2123r^2 + 13.906r - 82.916 \]  \hspace{1cm} (28)

where BHP is in horsepower and \( r \) is \( \frac{N}{100} \).

\[ Fuel_{consumption} = 0.0059r^2 + 0.1124r + 3.3255 \]  \hspace{1cm} (29)

where the fuel consumption is in gallons per hour and \( r \) is \( \frac{N}{100} \).

The following iterative procedure was used to find the range:

1. Choose a velocity starting with \( V_{min\_power} \).
2. Find \( C_L \) at steady level flight condition for the weights corresponding to the start and end of cruise.
3. Obtain \( C_D \) for these two values of \( C_L \) using drag polar.
4. Find the THP required for steady level flight condition at the start and end of cruise.
5. Assume a value of propeller efficiency \( \eta_p \) and find the average BHP required.
6. From the variation of BHP with engine rpm, find the rpm of the engine for this BHP output at cruising altitude. Since the quadratic equation gives two solutions, select the valid solution (i.e. rpm \( \leq 2700 \)).
7. For this rpm setting and flight speed, calculate J and the propeller efficiency from \( \eta_p \) vs J curve.
8. If the efficiency assumed in step 5 and efficiency obtained from step 7 are significantly different, repeat steps 5 to 8 till the two values of \( \eta_p \) are almost same. This determines the rpm needed for flight at the chosen flight velocity.
9. Now find the BSFC of the engine from variation of fuel consumption vs engine rpm and convert all quantities to SI Units.
10. One U.S.Gallon is taken as 3.78 litres an density of fuel is taken as 0.76 kg/m\(^3\).
11. Substitute for all terms in equation 27 and find the range.

12. Now repeat the procedure for different velocities till $V_{max}$ for the cruising altitude.

### 0.6.2 Endurance

Since the flight velocity is constant, the endurance of the aircraft for each flight velocity can be obtained from the expression $E = \frac{\text{Range}}{V}$.

The variations of endurance and range with velocity are given in table 4 and plotted in Figures 17 and 18 respectively.

<table>
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<th>Velocity (m/s)</th>
<th>$THP_r$ (kW)</th>
<th>$\eta_p$</th>
<th>rpm</th>
<th>$BHP_r$ (hp)</th>
<th>$\dot{m}_f$ (gal/hr)</th>
<th>BSFC (N/kW-hr)</th>
<th>R (km)</th>
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</table>

Table 4: Range and Endurance in constant velocity flights

Note: $\dot{m}_f$ is the fuel flow rate, R is the range and E is the endurance.
Figure 17 - Variation of endurance with velocity for constant velocity flights

Figure 18 - Variation of range with velocity for constant velocity flights
Remarks:

- It is seen that the maximum endurance is almost constant in the speed range of 33 to 40 m/s.

- The range calculated in the present computation is the Gross Still Air Range. This is found to be maximum (1474 km) at a speed of 64 m/s (230 kmph). It may be recalled that range depends on propeller efficiency, engine rpm, fuel consumption which in turn are all dependent on flight velocity. At a particular setting, these quantities are at their optimum values and hence give maximum range.

- The range quoted in Section 1.11 is 924 km. This range accounts for taxi, take-off, climb, descent and reserves for 45 min and can be called as safe range. This value is generally two-thirds of the GSAR. Noting that two-thirds of GSAR is 982 km, it is seen that the calculated value is in fair agreement with the value given in Ref[3].
0.7 Turning Flight Performance

In this section, the performance of the aircraft in a steady co-ordinated level turn is studied. The forces acting on the airplane are shown in Figure 19.

![Figure 19 - Forces on an airplane in turning flight](image)

The equations of motion in a steady, level, coordinated turn are:

\[
T - D = 0 \tag{30}
\]

\[
W - L \cos \phi = 0 \tag{31}
\]

\[
L \sin \phi = \frac{W V^2}{g R} \tag{32}
\]

These equations give

\[
r = \frac{V^2}{g \tan \phi} \tag{33}
\]

\[
\dot{\psi} = \frac{V}{r} = \frac{g \tan \phi}{V} \tag{34}
\]

Load Factor \( n = \frac{L}{W} = \frac{1}{\cos \phi} \tag{35} \)
In the following calculations, $C_{L_{\text{max}}} = 1.33$ and $n_{\text{max}} = 3.5$ are assumed where the load factor $(n)$ is given by $n = L/W$. The following procedure is then used to obtain $r_{\text{min}}$ and $\dot{\psi}_{\text{max}}$.

1. A flight speed and altitude are chosen and the lift coefficient in level flight lift $C_{L_{L}}$ is obtained as:

$$C_{L_{L}} = \frac{2(W/S)}{\rho V^2}$$

2. If $C_{L_{\text{max}}}/C_{L_{L}} < n_{\text{max}}$, where $n_{\text{max}}$ is the maximum load factor for which the aircraft is designed, then the turn is limited by $C_{L_{\text{max}}}$ and $C_{L_{T_1}} = C_{L_{\text{max}}}$. However if $C_{L_{\text{max}}}/C_{L_{L}} > n_{\text{max}}$, then the turn is limited by $n_{\text{max}}$, and $C_{L_{T_1}} = n_{\text{max}}C_{L_{L}}$.

3. From the drag polar, $C_{D_{T_1}}$ is obtained corresponding to $C_{L_{T_1}}$. Then

$$D_{T_1} = \frac{1}{2} \rho V^2 S C_{D_{T_1}}$$

If $D_{T_1} > T_a$, where $T_a$ is the available thrust at that speed and altitude, then the turn is limited by the engine output. The maximum permissible value of $C_D$ in this case is found from

$$C_{D} = \frac{2T_a}{\rho V^2 S}$$

From the drag polar, the value of $C_{L_T}$ is calculated as

$$C_{L_T} = \sqrt{\frac{C_{D} - C_{D_{O}}}{K}}$$

However if $D_{T_1} < T_a$, then the turn is not limited by the engine output and the value of $C_{L_{T_1}}$ calculated in step (2) is taken as $C_{L_{T}}$.

4. Once $C_{L_T}$ is known, the load factor $n$ which satisfies the three constraints of $C_{L_{\text{max}}}$, $n_{\text{max}}$ and $T_a$ is given by:

$$n = \frac{C_{L_T}}{C_{L_{L}}}$$

5. Knowing $n$, the values of turn radius $(r)$ and rate of turn $(\dot{\psi})$ can be calculated from equations 33, 34 and 35.
6. The above steps are then repeated for various speeds at the same altitude and then the procedure is repeated at various altitudes.

The variations of turn rate and turn radius are shown in Figures 20 and 21 respectively. $V_{\psi_{\text{max}}}$ and $V_{r_{\text{min}}}$ are plotted against altitude in Figures 22 and 23.
Figure 20 - Variation of rate of turn with velocity at various altitudes
Figure 21 - Variation of radius of turn with velocity at various altitudes
Figure 22 - Variation of V(maximum rate of turn) with altitude

Figure 23 - Variation of V(minimum turn radius) with altitude
0.8 Take-off and Landing distance estimates

Take-off flight can be divided into three phases: take-off run, transition and climb.

0.8.1 Take-off run: distance estimate

Writing equations of motion for run phase of take off

\[ T - D - \mu R = \frac{W \, dV}{g \, dt} \]  

(36)

where the normal reaction \( R = W - L \). Thus acceleration can be written as

\[ \frac{dV}{dt} = \frac{g}{W} \times [T - D - \mu (W - L)] \]

writing \( \frac{dV}{dt} \) as \( \frac{dV}{ds} \times \frac{ds}{dt} \), we obtain

\[ ds = \frac{W}{g} \frac{V \, dV}{T - D - \mu (W - L)} \]

Now, at sea level, BHP = constant = 135\( kW \) at 2700 rpm. Thrust is given by \( BHP \times \eta_p/V \).
The distance covered during the run phase of take-off can be expressed as
\[
s_1 = \int_0^{V_{TO}} \frac{WV}{gF} dV \tag{37}
\]
where \( F \) is the accelerating force given by
\[
F = \frac{BHP \cdot \eta_p}{V} - D - \mu (W - L)
\]
\( \eta_p \) is a function of velocity and so the best way to estimate \( s_1 \) is by doing numerical integration, keeping the intervals \((dV)\) as small as possible. A \( dV \) of 2m/s was chosen for this particular exercise. \( V_{TO} = 1.2V_S \), where \( V_S \) is stall speed, and is calculated from
\[
V_S = \sqrt{\frac{2W}{\rho S C_{L_{max}}}}
\]
During take-off, flap deflection \((\delta_f)\) is 10° \( \Rightarrow C_{L_{max}} = 1.42 \)
We assume that the coefficient of friction is 0.02
Here, the take-off weight \( W = 10673.28 \) N
\( S = \) wing plan-form area = 14.86m²
Density \( \rho_{sl} = 1.225 \) kg/m³
Thus we obtain \( V_S = 28.73 \) m/s and \( V_{TO} = 34.4775 \) m/s

Substituting the above values in equation 37 and performing numerical integration, we obtain \( s_1 = 286 \) m

### 0.8.2 Distance covered in transition

The entire power of the engine is assumed to be used to overcome drag and to accelerate to a velocity \( V_2 \) given by \( V_2 = 1.1V_{TO} \). The height attained during transition is ignored. Writing the equations of motion, we obtain
\[
T - D = \frac{W}{g} \times \frac{dV}{dt} = \frac{W}{g} \times \left( \frac{V_2^2 - V_{TO}^2}{2s_2} \right) \quad (From \quad V_2^2 - V_1^2 = 2as)
\]

Thus
\[
s_2 = \frac{W}{g} \times \left( \frac{V_2^2 - V_{TO}^2}{T - D} \right)
\]

Where \( T \) and \( D \) are evaluated at a mean speed between \( V_2 \) and \( V_{TO} \)
\[
V_2 = 1.1 \times 34.4775 = 37.9253 \text{ m/s}
\]
Substituting the values in the above expression, we obtain
\( s_2 = 95.28 \) m
0.8.3 Distance covered during climb phase

The plane is assumed to climb to screen height (15m) at an angle of climb $\gamma$, where the climb angle $\gamma$ is given by

$$\gamma = \left( \frac{T - D}{W} \right)$$

For the climb phase, $T$ and $D$ are evaluated at $V_2$

The climb angle $\gamma$ was found to be 7.5 degrees.

Ignoring vertical height covered in transition, we get

$$s_3 = \frac{\text{screen ht}}{\gamma}$$

Substituting the values in the expression given above, we obtain $s_3 = 113.4$ m

Total takeoff distance is given by

$$s = s_1 + s_2 + s_3 = 286 + 95.28 + 113.4 = 494.7$$ m

which is close to the value of 488 m given in Section 1.11.

0.8.4 Landing distance estimate

The landing distance can be calculated in a manner similar to that for takeoff. However, due to uncertainty associated with piloting techniques during landing, the following formula is used:

$$s_{\text{land}} = \frac{V_a^2}{2a}$$

where $V_a = 1.3 \times V_S$ in landing configuration

Taking the airplane weight as that during take-off but $C_{L\text{max}}$ for landing flap setting (1.86), the stalling speed is 25.1 m/s. Hence approach speed is 32.6 m/s. Taking $a = -1.22 \text{m/s}^2$ for a simple braking system yields

$$s_{\text{land}} = 436 \text{ m}$$

which is close to the value of 426 m given in Section 1.11.
Concluding remarks

1. Performance of a piston-engined airplane has been estimated for stalling speed, maximum speed, minimum speed, steady climb, range, endurance, turning, take-off and landing.

2. A reasonable agreement has been observed between the calculated performance and the actual performance of the airplane.

3. Figure 25 presents the variations, with altitude, of the characteristic velocities corresponding to
   - stalling speed $V_s$
   - maximum speed $V_{max}$
   - minimum speed as dictated by power $V_{min_{power}}$
   - maximum rate of climb $V_{R/C_{max}}$
   - maximum angle of climb $V_{\gamma_{max}}$
   - maximum rate of turn $V_{\phi_{max}}$
   - minimum radius of turn $V_{r_{min}}$

0.9 Acknowledgements

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Figure 25: Variations of characteristic velocities with altitude.
Bibliography


