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## Notation

|                |   |
|----------------|---|
| a              | acceleration                            |
| $a_1$          | lift curve slope                        |
| b              | wing span                               |
| c, $\bar{c}$   | chord                                   |
| $C_{M0}$       | pitching moment coefficient             |
| $C_L$          | lift coefficient                        |
| $C_N$          | normal force coefficient                |
| $C_p$          | pressure coefficient                    |
| D              | drag                                    |
| F              | force                                   |
| g              | gravitational acceleration              |
| I              | inertial load                           |
| K              | gust alleviation factor                 |
| L              | lift                                    |
| m              | mass                                    |
| M              | Mach number, pitching moment            |
| n              | normal load factor                      |
| N              | number of cycles                        |
| r              | distance from CG                        |
| R              | radius                                  |
| S              | load, wing area                         |
| T              | thrust                                  |
| U              | gust velocity                           |
| V              | aircraft velocity                       |
| $V_A$          | optimum manoeuvre (corner) speed        |
| $V_B$          | design speed for maximum gust intensity |
| $V_D$          | design diving speed                     |
| $V_F$          | design flap speed                       |
| $V_H$ or $V_H$ | normal operating speed                  |
| $V_S$          | steady level flight stall speed         |
| w              | loading distribution                    |
| W              | weight                                  |

|                 |   |
|-----------------|---|
| $\alpha$        | angle of attack   |
| $\phi$          | bank angle  |
| $\lambda$       | taper ratio   |
| $\mu$           | gust alleviation coefficient                                      |
| $\theta$        | climb or descent angle (relative to horizontal), deflection angle |
| $\ddot{\theta}$ | pitching acceleration   |
| $\rho$          | air density   |

### **Subscripts**

|    |             |
|----|-------------|
| cf | centrifugal |
| cp | centripetal |
| e  | equivalent  |
| f  | forward     |
| g  | gust        |
| n  | normal      |
| r  | root        |
| t  | tip         |

### **Acronyms**

|      |                                    |
|------|------------------------------------|
| BM   | bending moment                     |
| CG   | centre of gravity                  |
| FAR  | Federal Airworthiness Requirements |
| FW   | fixed wing                         |
| GW   | guided weapon                      |
| JAR  | Joint Airworthiness Requirements   |
| MTOW | Maximum take-off weight            |
| RF   | reserve factor                     |
| RW   | rotary wing                        |
| SF   | shear force                        |
| UDL  | uniformly distributed load         |

## **1. Introduction**

Structural design is of critical importance to aircraft safety, costs and performance. The overall cost is related to the structural design in many complex ways, but typically aircraft end up costing between £200 and £500 per kg (with modern military aircraft such as the B-2 reportedly costing more per ounce than gold). In addition to its major and direct impact on cost, the aircraft structural weight affects performance. Every kg of aircraft structure means one less kg of fuel when the take-off weight is specified.

Before the structure can be designed, the loads that will be imposed on the aircraft must be determined. These notes deal with the general issue of aircraft loads and how they are predicted at the early stages of the design process.

Each part of the aircraft is subject to many different loads. In the final design of an aircraft structure, tens of thousands of loading conditions may be examined, of which several hundred may be critical for different parts of the aircraft. In addition to the obvious load categories, such as wing bending moments due to aerodynamic lift, many other loads must be considered. These include items such as inertia relief (the inertial forces that tend to reduce wing bending moments), landing loads, pressurization cycles on the fuselage, local high pressures on floors due to high-heeled shoes, etc.

These loads may be predicted using complex Navier-Stokes computations, wind tunnel tests, and other simulations. Static and dynamic load tests on structural components are also essentially carried out to ensure that the predicted strength can be achieved.

### 1.1 Airworthiness & Certification

Comprehensive design and certification documentation exists for all classes of aircraft. This includes significant amounts of data relating to structural strength and stiffness requirements.

#### Military Aircraft

- UK – Def Stan 00-970, Vol.1 – Fixed Wing, Vol.2 – Rotary Wing
- US - Mil Specs

#### Civil Aircraft

- Europe – JAR (Join Airworthiness Requirements)
- US – FAR (Federal Airworthiness Requirements)

#### Guided Weapons

General requirements/recommendations may be found in the restricted Def-Stan 08-5 document “Design Requirements for Weapon Systems (Guided Weapons, Torpedoes & Airborne Armament Stores)”. This includes, for example, the following limited structural design information:

$n_1 = 25$  for small missile.

proof factor = 1.125, ultimate factor = 1.33.

## 1.2 Failure Effects and Probabilities

The primary purpose of all the airworthiness certification documentation is to improve overall safety. This is often stated in terms of a varying effect severity and a probability of a failure incident.

Regarding civil aircraft, the effects may be defined as:

- minor effect – nuisance conditions, necessitating emergency procedure activation and imposed operating limitations;
- major effect – significant safety margin reductions, possible occupant injuries and difficult crew operations;
- hazardous effects – large reduction in safety margins, possible occupant fatalities and unduly high crew workloads;
- catastrophic effects – multiple occupant fatalities and likely loss of aircraft.

The probabilities are typically:

- probable - relating to minor effects and sub-divided into:
  - frequent (more often than once in 1000 hours) or
  - reasonably probable (several times in an aircraft’s life, between 1000 and 100,000 hours).
- improbable – covering major and hazardous effects and sub-divided into:
  - remote (major) – unlikely for one aircraft but may occur for fleet, once in  $10^7$  hours
  - extremely remote (hazardous) – unlikely at all,  $10^7$  to  $10^9$  hours.
- extremely remote (catastrophic) – no more than one in  $10^9$  hours.

## 2. Load Sources

There are two broad categories of load cases:

- Manoeuvring Cases

These result directly from the action of the aircraft's pilot (or possibly by the autopilot) and will occur as the aircraft performs its intended role.

- Environmental Cases

Due to the general environment in which the vehicle operates. Examples of causes include: atmospheric turbulence, pressure differential, kinetic heating, runway unevenness, etc.

### 2.1 Limit Loads & Load Spectrum

Detailed structural design cannot commence until a detailed knowledge of the structural loads is known. Two general sets are required for the study of both manoeuvre and environmental cases:

- Limit Loads

The maximum "one-off" loads expected to be experienced during normal operations. Most aircraft structural components will be designed to meet these. For rotary-wing aircraft, the design of the non-rotating structural components may be driven by these (e.g. the fuselage, tail boom, landing gear, etc.)

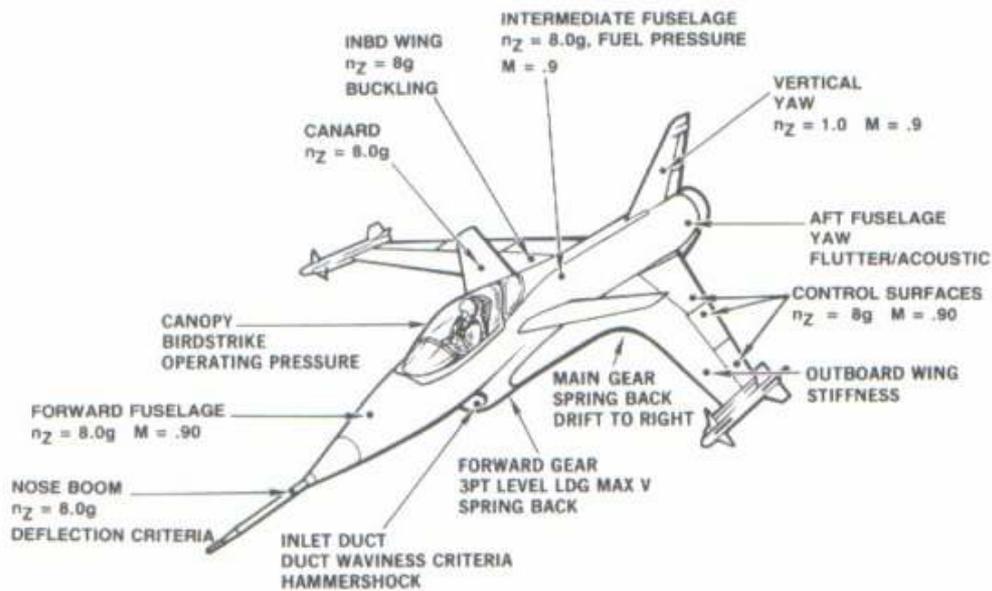
- Load Spectrum

This will comprise a variable set of loads, of magnitude less than the limit loads but with the number of applied cycles being of more critical importance instead. These will tend to be more important for components subjected to repeated cycling (e.g. those associated with helicopters and helicopter-mounted GW) and therefore prone to *fatigue* problems. On rotary-wing aircraft, this will include the main and tail rotors, gearboxes, shafting, rotating controls, etc.

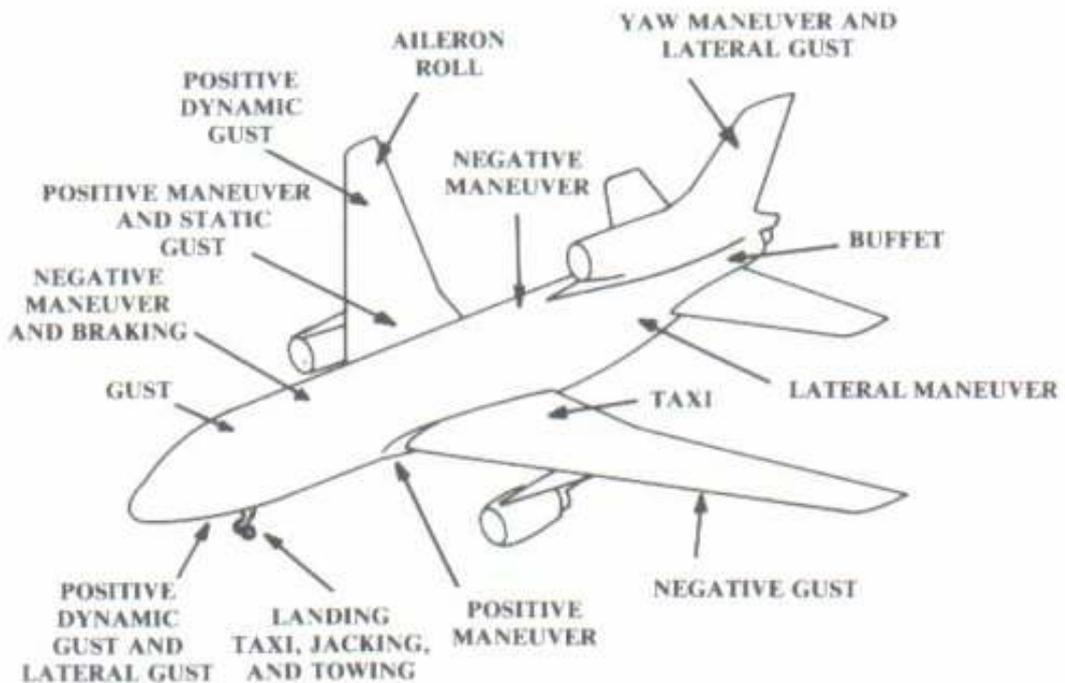
The main load classes for fixed-wing aircraft are:

- Symmetric flight loads (due to manoeuvres and environmental conditions).
- Asymmetric flight loads (due to manoeuvres and environmental conditions).
- Ground loads.
- Longitudinal acceleration and deceleration loads.

There will also be many subsidiary cases concerned with specific parts of airframe.



**Figure 2.1a Typical aircraft limit loads – fighter aircraft example**



**Figure 2.1b Typical aircraft limit loads – civil transport aircraft example**

## 2.2 Longax and Latax

Most loads are considered to be due to accelerations which are conventionally normalized with respect to gravitational acceleration  $g$  (with a standard value of  $9.80665 \text{ m/s}^2$ ). A *longitudinal*

*acceleration* of  $490.33 \text{ m/s}^2$  is therefore directly equivalent to a 50g acceleration (e.g. on a GW). This is sometimes abbreviated to the term *longax*.

Similarly, a *lateral acceleration* is commonly known as *latax*, e.g.  $245.2 \text{ m/s}^2$  equates to an acceleration 25g. It is more usually represented as a *normal load factor* ( $n$ ) = 25 in this particular example.

## 2.3 Load Factors

Aerospace structures are conventionally designed through the use of *design limit loads* multiplied by *load factors*.

### 2.3.1 *Design Limit Loads*

These are the maximum loads expected under normal operating conditions. FAR Part 25 (and most other regulations) specifies that there be no permanent deformation of the structure at limit load conditions.

### 2.3.2 *Load Factors*

Two sets of safety factors are generally applied to the limit loads:

#### *Proof Factor & Ultimate Factor*

##### Proof Factor

- Under proof loading, airframe should not distort permanently by more than a small specified amount (typically 0.1% permanent strain).
- Factor intended to ensure that structure always returns to original shape after design limit load has been applied.
- Design stress x proof factor must be less than material allowable proof stress.
- For military a/c, proof factor = 1.125
- For civil a/c, proof factor = 1.0

##### Ultimate Factor

- Factor intended to cover variation of material and structural properties, deterioration in service, inadequacy of load & stress analysis assumptions, flight outside envelope, etc.
- Under ultimate loading, airframe should not collapse.
- Design stress x ultimate factor must be less than material allowable ultimate stress.
- Usually ultimate factor = 1.5 for all a/c. In FAR Part 25 the ultimate safety factor is actually specified as 1.5. For some research or military aircraft the safety factor is as low as 1.20, while composite sailplane manufacturers may use 1.75. According to FAR 25,

the structure must be able to withstand the ultimate load for at least 3 seconds without failure.

### 2.3.3 Reserve Factors

It is common practice in stressing calculations to make use of *reserve factors (RF's)*, where:

$$\text{RF} = \text{allowable stress} / \text{factored design stress}$$

All calculated proof and ultimate RF's must therefore be  $> 1$  for all possible permutations of limit load conditions for each aircraft component. A successful structural design involves a great deal of iterations so that RF's are only just  $> 1$ ; this prevents over-design and eventually produces the optimum lightweight mass solution.

### **3. Structure Life – Design Philosophies**

A civil aircraft needs to be flown for many years in order to be economic and nowadays may accumulate over 40,000 flight hours. It is therefore necessary to ensure that the airframe can reach the required number of flying hours before the accumulation of damage renders the aircraft unsafe. The same is true, though to a considerably lesser degree, for military aircraft. Application of proof and ultimate factors to limit loads generally only adequate for short-lived aerostructures (e.g. GW).

Three main design philosophies are in current use:

#### **3.1 Safe Life & Life Factor**

An attempt is made to guarantee a certain length of life during which the aircraft component or whole aircraft is classified as safe. This concept depends on the assessment of the accumulation of damage from the sequence of loadings thought likely to arise during the life of the aircraft or component. At the time when the component is liable to become unsafe the component is scrapped, whatever its apparent condition. The method relies heavily upon statistical methods and gathered fatigue data. It has the disadvantage that it carries a weight penalty due to experimental test scatter and it is vulnerable to incorrect design assumptions, manufacturing errors and accidental in-service damage. The only significant advantage a safe-life structure has to an operator is that structural replacements can be planned on a tightly scheduled basis.

In most cases an accurate specification of the load and its resulting damage is very difficult and it then becomes necessary to introduce a “Life Factor”; this will result in a weight penalty. The Life Factor applied is usually somewhere between 3 and 6.

As an example, the Safe Life of the Hawk aircraft is only 6000 hours.

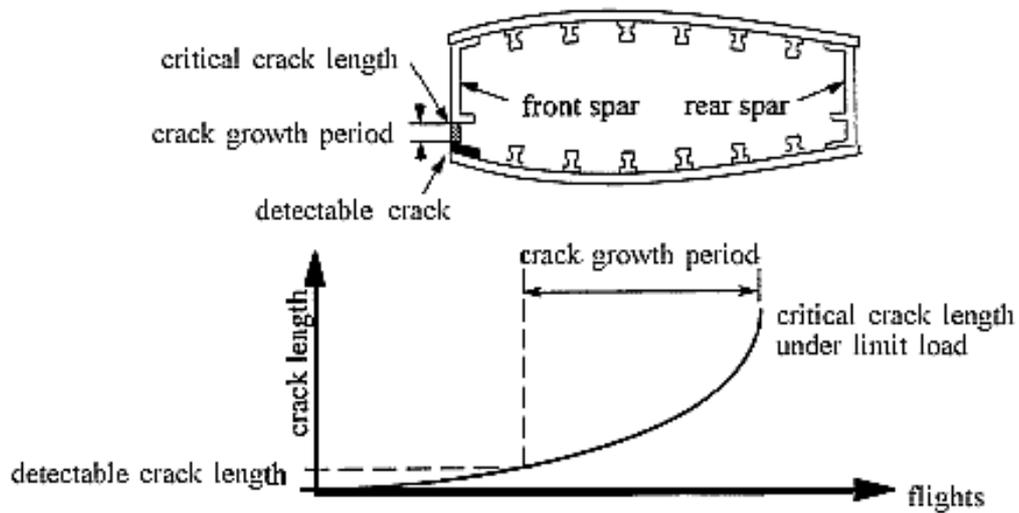
#### **3.2 Fail Safe Concept**

Under this concept it is accepted that the effects of the loads on the structure cannot always be accurately predicted; the possibility of failure is guarded against by providing alternative load paths. Thus, should a structural failure occur in a loaded member in service, its load is temporarily carried elsewhere in the structure, allowing the aircraft to complete its mission. The failure would have to be detected on a subsequent inspection and repaired.

The likelihood of numerous small failures and the time taken to carry out repairs make the fail safe philosophy less attractive for civil aircraft designs compared with military aircraft. Also, the possibility of a redundant structure is not always feasible, e.g. on the landing gear.

### 3.2.1 Damage Tolerant Design

As mentioned above, the fail safe concept is limited since many cracks are difficult to quickly detect and locate. This drawback may be overcome by ensuring that the resultant rate of crack growth is very slow or even completely arrested. This is the basis behind a damage tolerant design philosophy.



**Figure 3.1 Damage tolerant design philosophy example**

## 4. Symmetric Manoeuvre Loads

These will occur when the aircraft's pilot (or the autopilot) operates the longitudinal control surface (e.g. the elevator or canard) to cause aircraft to pitch nose-up or nose-down. This action may result in two distinct forms of acceleration:

- translational, which may be either *longitudinal* or *normal* to the flight path
- rotational

### 4.1 Normal Load Factor

The loads due to symmetric manoeuvres are most commonly analysed through use of the definition of a *normal load factor* ( $n$ ), whereby:

$$n = \text{Lift (L)} / \text{Weight (W)} \quad (4.1)$$

The load factor is more properly defined as the component of aerodynamic force perpendicular to the longitudinal axis divided by the aircraft weight. But, assuming that the angle of attack is not large,  $n = L/W$ .

The normal load factor is often misleadingly quoted in terms of gravitational accelerations (“g”), e.g. if  $n = 5$ ,  $L = 5 W$  and the aircraft (and its occupants) is said to be experiencing a load equivalent to 5 g's.

### 4.2 Unaccelerated Flight

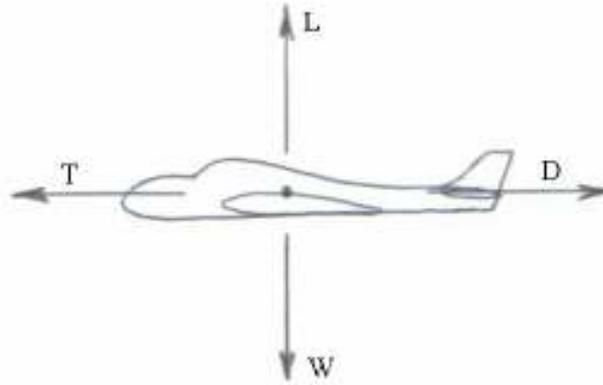
The analysis of these cases is simplified by:

- ignoring couples produced by misalignment of the lift, thrust, drag and weight vectors
- neglecting tailplane and fuselage lift

There are three particular cases to consider: (i) steady (straight & level, cruise), (ii) climbing, and (iii) descending.

#### 4.2.1 Unaccelerated Straight & Level (Cruise) Flight

This is the simplest case of all to study but is useful for providing a starting point.



**Figure 4.1 Unaccelerated straight & level flight**

A vertical and horizontal force balance gives:

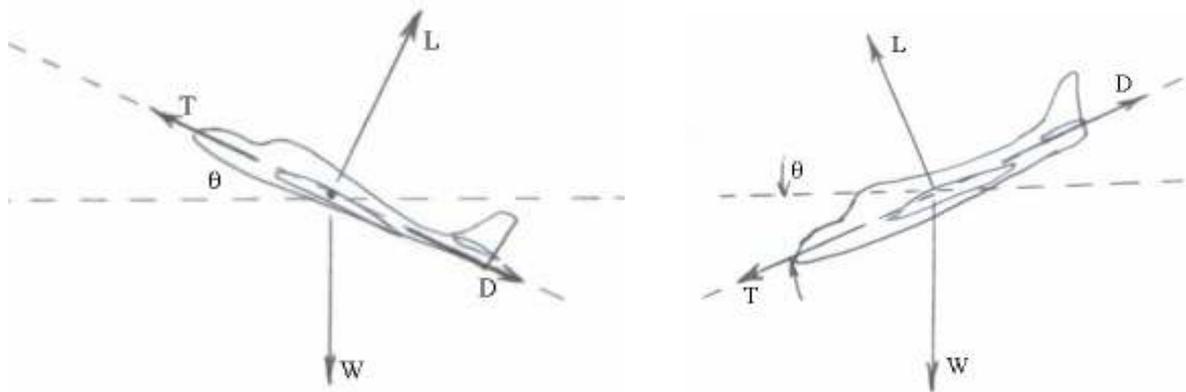
$$L = W, T = D, \therefore n = L / W = 1 \quad (4.2a)$$

**4.2.2 Unaccelerated climbing flight**

$$L = W \cos\theta, T = D + W \sin\theta, \therefore n = L / W = \cos\theta \quad (4.2b)$$

**4.2.3 Unaccelerated descending flight**

$$L = W \cos\theta, T = D - W \sin\theta, \therefore n = L / W = \cos\theta \quad (4.2c)$$



**Figure 4.2 (a) Unaccelerated climbing flight, (b) descending flight**

**4.3 Accelerated Flight due to Speed Change**

From Newton's 2nd & 3rd laws of motion, any body of mass ( $m$ ) subjected to an *acceleration* ( $a$ ) will possess an *inertial force* of magnitude ( $m a$ ), acting in the opposite direction to the acceleration.

*D'Alembert's principle* states that, if the inertial loads are included, then the whole system of loads and reactions can be considered as being in state of *quasi-static balance*.

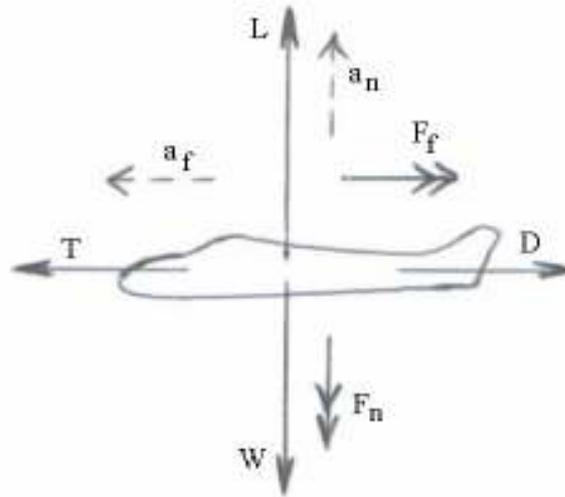
### 4.3.1 Accelerated Level Flight

From Figure 4.3, a quasi-static balance gives:

$$L = W + F_n = W + (W / g) a_n, \therefore L = W [1 + (a_n / g)]$$

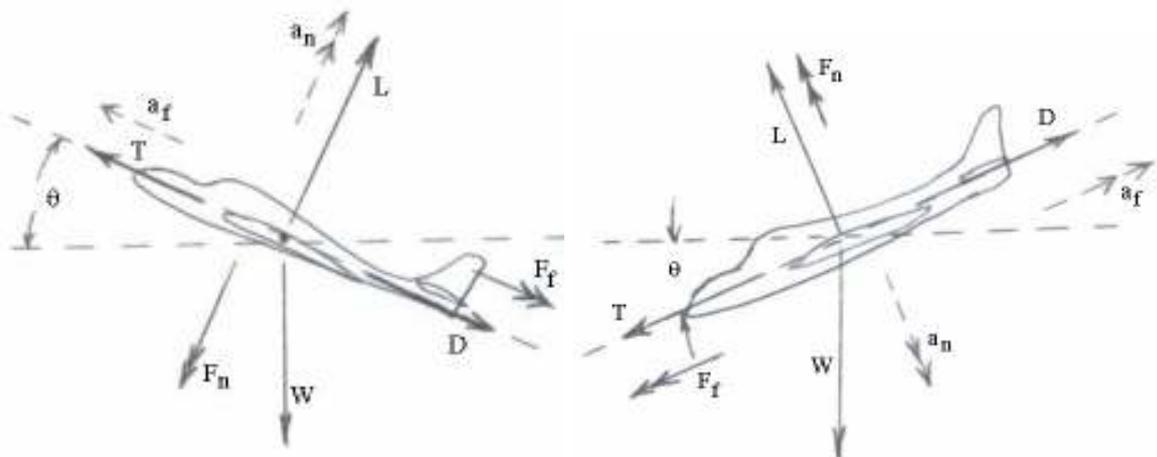
$$\therefore n = L / W = [1 + (a_n / g)] \quad (4.3a)$$

and also:  $T = D + F_f = D + (W a_f / g)$



**Figure 4.3 Accelerated level flight**

### 4.3.2 Accelerated Climbing Flight



**Figure 4.4 (a) Accelerated climbing flight, (b) descending flight**

A quasi-static force balance gives:  $L = W \cos\theta + F_n = W \cos\theta + (W / g) a_n$

$$\therefore L = W [\cos\theta + (a_n / g)], \therefore n = L / W = [\cos\theta + (a_n / g)] \quad (4.3b)$$

and also:  $T = D + W \sin\theta + F_f = D + W (\sin\theta + a_f / g)$

### 4.3.3 Accelerated Descending Flight

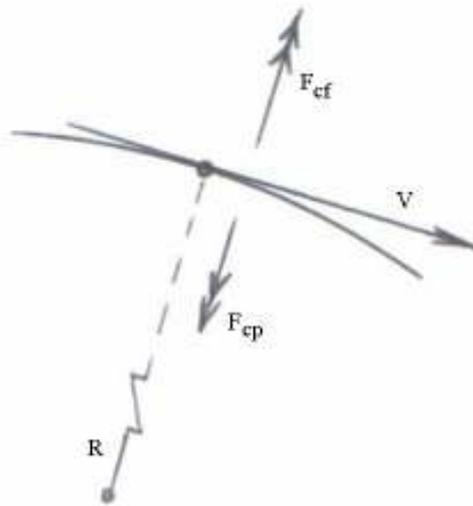
A quasi-static force balance gives:  $L + F_n = L + (W/g)a_n = W \cos\theta$

$$\therefore L = W [\cos\theta - (a_n/g)], \therefore n = L/W = [\cos\theta - (a_n/g)] \quad (4.3c)$$

and also:  $T + W \sin\theta + F_f = D, \therefore T = D - W (\sin\theta + a_f/g)$

### 4.4 Accelerations due to Direction Changes

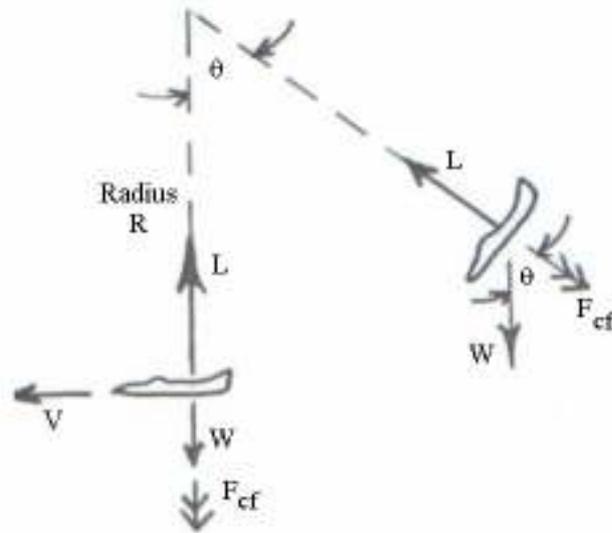
Since velocity is a vector quantity, accelerations can also arise due to direction changes at constant speed. Consider a body of mass ( $m$ ) in circular motion in a horizontal plane (Figure 4.5).



**Figure 4.5 Accelerated flight due to direction change**

The force maintaining the circular motion is known as the *centripetal force* ( $F_{cp}$ ); it acts towards the centre of the circle and is of magnitude  $(mV^2/R)$ . The equal and opposite reaction to this centripetal force is an inertial force known as the *centrifugal force* ( $F_{cf}$ ). This acts away from the centre of the circle and is also of magnitude  $(mV^2/R)$ . This should be included in any free body diagram force analysis for quasi-static balanced conditions.

#### 4.4.1 Pull Ups



**Figure 4.6 Pull-up manoeuvre**

Consider an aircraft pulling out of a dive in the vertical plane.

The centrifugal force,  $F_{cf} = mV^2/R = WV^2/(Rg)$

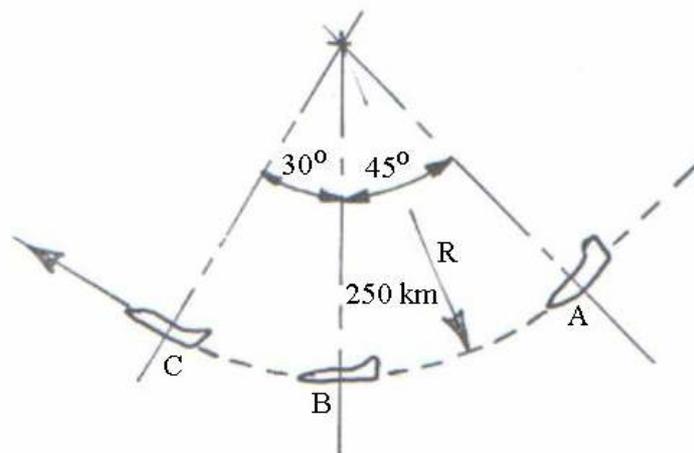
A quasi-static balance at (B) gives:  $L = W \cos\theta + W V^2 / (Rg) = W [\cos\theta + (V^2 / (Rg))]$

$$\therefore n = L / W = [\cos\theta + (V^2 / (Rg))] \quad (4.4)$$

The maximum load factor ( $n_{max}$ ) will occur when  $\theta = 0^\circ$  (i.e. at the bottom of the pull-up manoeuvre), when  $n_{max} = 1 + (V^2 / (Rg))$  (4.4a)

#### **Example 4.1**

An aircraft weighing 18 kN pulls out of a dive, as shown in Figure 4.7. Its speed is 250 km/hr at A & B but 160 km/hr at C. Calculate the lift and normal load factor at positions A, B & C.



**Figure 4.7 Pull-up manoeuvre – Example 4.1**

### Solution

250 km/r = 69.44 m/s, 160 km/hr = 44.44 m/s

At A:  $L = W [\cos 45^\circ + V^2/(gR)] = 48.123 \text{ kN}$ ,  $\therefore n = L/W = 48.123/18 = \mathbf{2.673}$

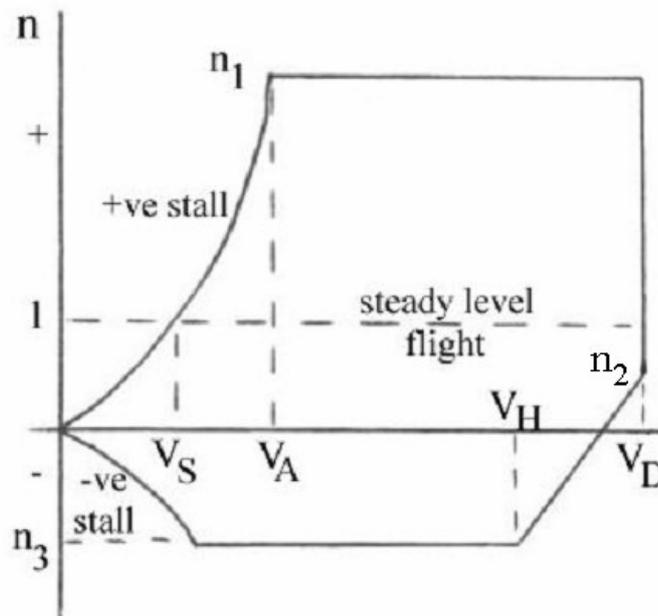
At B:  $L = W [1 + V^2/(gR)] = 53.395 \text{ kN}$ ,  $\therefore n = L/W = 53.395/18 = \mathbf{2.966}$

At C:  $L = W [\cos 30^\circ + V^2/(gR)] = 30.086 \text{ kN}$ ,  $\therefore n = L/W = 53.395/18 = \mathbf{1.671}$

### 4.5 Manoeuvre (n-V) Diagram or Flight Envelope

All aircraft have operational flight envelopes of permissible *load factor* ( $n$ ) against *airspeed* ( $V$ ), as defined by the appropriate operators, authorities and designers. The aircraft is thus structurally designed to withstand all possible flight conditions within this envelope. Any flight condition outside the envelope is potentially structurally unsound.

A typical *military* fixed-wing aircraft n-V diagram is given in Figure 4.8, as defined by DEF-Stan 00-970 Chapter 202.



**Figure 4.8 Typical military fixed-wing aircraft n-V diagram**

The speeds in Figure 4.8 are defined by:

$V_H$  = Normal Operating Speed

The maximum speed attainable in level flight at basic design mass with no external stores and maximum continuous cruise engine conditions

$V_D$  = Design Diving Speed =  $V_H \times$  factor given in specification, typically 1.25

Note that for aircraft designed for dive bombing and ground attack,  $V_H = V_D$

$V_A$  = Optimum Manoeuvre Speed (corner speed)

The minimum speed at which  $n_1$  can be achieved

$V_S$  = Steady Level Flight Stall Speed, i.e. conditions at which  $n = 1$

Note that all are plotted in terms of *equivalent airspeed units* ( $V_e$ ,  $V_{eas}$ ).

The boundaries of Figure 4.8 are defined as:

$n_1$  = maximum positive limit load factor (given in aircraft specification).

Typically:

$$n_2 = 1 - 0.3 n_1$$

$$n_3 = -0.6 (n_1 - 1)$$

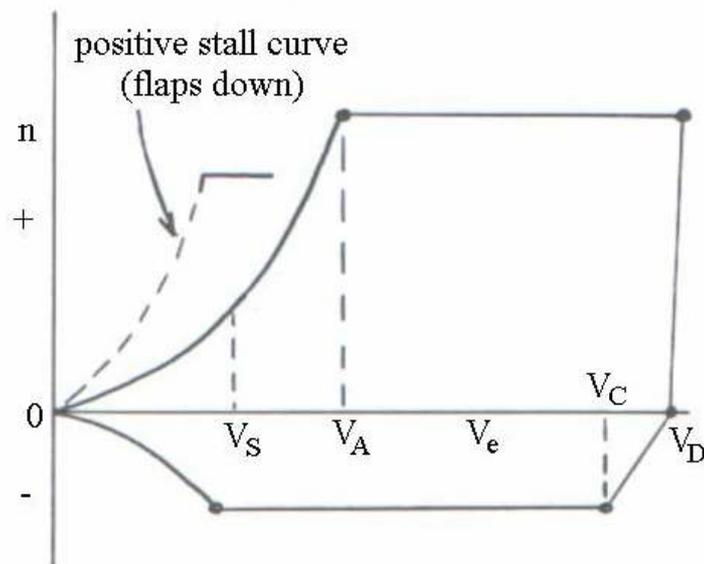
For the stall boundaries:

$$\text{Using } n = L/W = (0.5 \rho_0 V_e^2 S C_{L,max}) / W$$

$$\text{i.e. } n = (\rho_0 S C_{L,max} / 2W) V_e^2 \quad (4.5)$$

The negative stall boundary will be as for the positive case but with a different (lower)  $C_{L,max}$  value.

The civilian fixed-wing  $n$ - $V$  diagram will be similar, in a typical form as shown in Figure 4.9, as defined by FAR or JAR. Here,  $V_C$  = normal operating (cruise) speed



**Figure 4.9 Typical civilian fixed-wing aircraft  $n$ - $V$  diagram**

| aircraft type |                         | $n_1$    | $n_3$    |
|---------------|-------------------------|----------|----------|
| military      | transports              | 3 to 4   | -1 to -2 |
|               | strategic bombers       | 3        | -1       |
|               | tactical bombers        | 4        | -2       |
|               | fighters                | 6.5 to 9 | -3 to -6 |
| civilian      | transports (< 1870 kg)  | 3.8      | -1.5     |
|               | transports (> 22000 kg) | 2.5      | -1       |
|               | utility                 | 4.4      | -1.8     |
|               | aerobatic               | 6        | -3       |
|               | homebuilts              | 5        | -2       |

**Table 4.1 Typical aircraft structural limit load factors**

In addition to  $V_c$ ,  $V_D$  and  $V_A$ , several other airspeeds are important in the definition of the aircraft's operating envelope.

$V_B$  is the design speed for maximum gust intensity. It is defined in one of two ways. Basically, it is the airspeed at which the required vertical gust produces maximum  $C_L$  on the aircraft. However,  $V_B$  need not be greater than  $V_{s1} * \sqrt{(n_g)}$  where  $n_g$  is the gust load factor at  $V_c$  and  $V_s$  is the stalling speed with flaps retracted.  $V_B$  also need not be more than  $V_c$ .

$V_c$  must be 43 knots greater than  $V_B$  except it need not exceed the maximum level flight airspeed at maximum continuous power. If  $V_D$  is limited by Mach number,  $V_c$  may be limited to a given Mach number (this is the usual case)..

$V_F$  is the design flap speed. It is a function of flap position and stalling speed and restricts the speeds at which flaps may be deployed. It must be  $> 1.6 V_{s1}$  with TO flaps and MTOW. It must also be  $> 1.8 V_{s1}$  with approach flaps and max landing weight and  $> 1.8 V_{s0}$  with landing flaps and max landing weight.

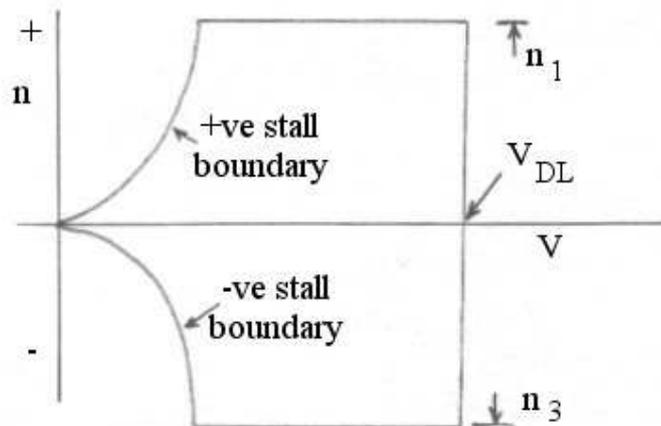
The maximum limit load values are not as important for RW a/c as for FW a/c. For FW aircraft there is often an over-riding need for high  $n_1$  capability for manoeuvrability reasons (e.g. for fighters). Some typical limit loads for RW aircraft are:

AH64 Apache requires  $n_1 = 1.75$ , Comanche performs sustained turns at  $n_1 = 2.5$ .

However, both civil and military airworthiness requirements dictate that the structure has to be designed to meet  $n_1 = 3.5$ .

The n-V diagram for an axisymmetric missile is typically as shown in Figure 4.10 and provides the design limit loads for many components. Here:

- $V_{DL}$  = design limit speed
- $n_1 = -n_3$  for axisymmetric GW (most usual), otherwise  $n_1 > n_3$

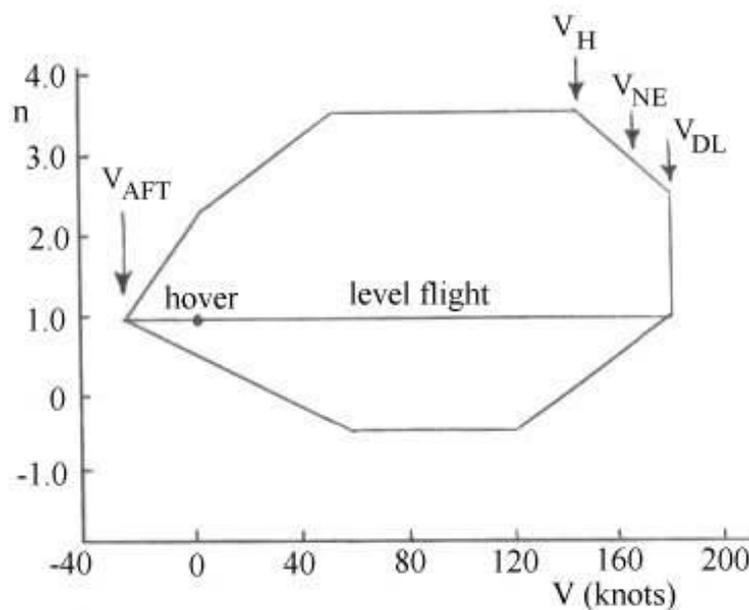


**Figure 4.10 Typical GW n-V diagram**

Flight outside the flight envelope is prevented on a GW by built-in accelerometers which override any control application. Typical  $n_1$  values are:

ASRAAM = 60?, standard tactical GW = 25

A typical n-V for a rotary-wing aircraft is shown in Figure 4.11. The envelope limits will generally be set by either the customer (for military aircraft) or by JAR/FAR (for civil aircraft).



**Figure 4.11 Typical rotary-wing aircraft n-V diagram**

The speed definitions in this case are:

$V_H$  - maximum horizontal speed, using normal engine rating.

$V_{DL}$  - design limit speed. Used to calculate design loads and also the speed that must be achieved during structural demonstration flights

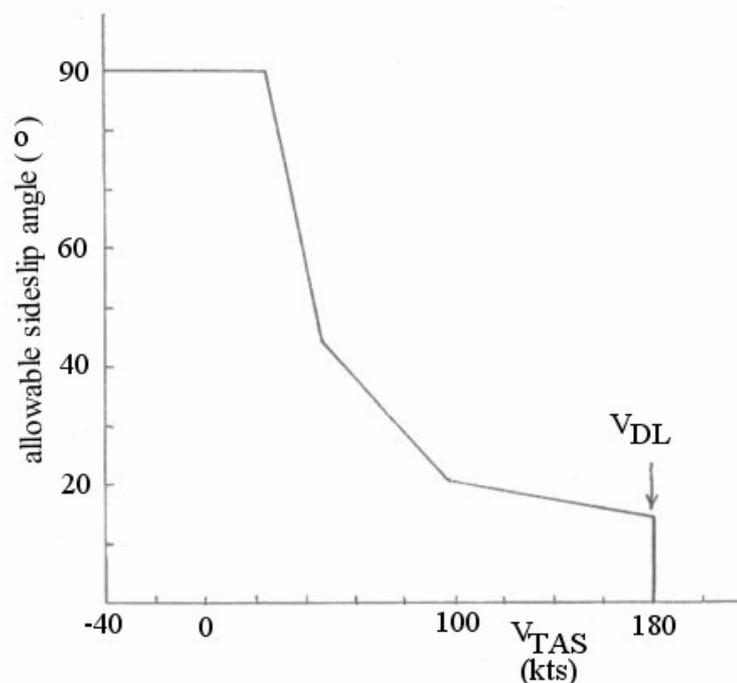
For military a/c,  $V_{DL} = 1.2 V_H$ .

$V_{NE}$  - never exceed (red line) speed

For civil a/c  $= 0.9 V_{DL}$  while for military a/c, it is defined by the procuring agency.

$V_{AFT}$  - maximum allowable rearwards speed. This is primarily safety, rather than structurally, driven (i.e. due to rearward visibility considerations).

Another flight envelope of relevance to rotary wing aircraft is the sideslip envelope (Figure 4.12). This defines all allowable sideslip angles for given aircraft speeds. Note that the allowable angle reduces as aircraft speed increases in order to reduce the pressure loads acting on the tail boom and tail rotor in particular.



**Figure 4.12 Typical helicopter sideslip envelope**

#### 4.6 Placard Diagrams

The placard diagram shows the structural design airspeeds as a function of altitude. These speeds are specified by the airworthiness regulations (JARs, FARs, etc.) in sometimes obscure ways and the following outlines a simpler description.

1. The structural design cruise Mach number,  $M_c$ , may be specified by the designer, but it need not exceed the maximum speed in level flight (at maximum continuous power). To maintain operational flexibility, this is generally the value chosen for  $M_c$ . For most transonic aircraft this value is about 6% above the typical cruise speed. So:

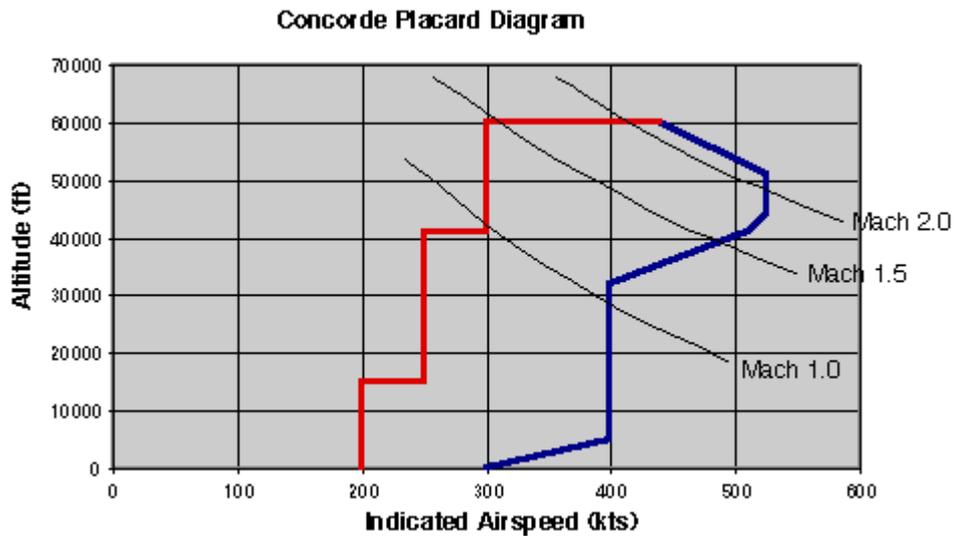
$$M_c = M_{mo}, \text{ the maximum operating Mach number, about } 1.06 M_{\text{cruise}}.$$

2. The design dive Mach number,  $M_D$  is approximately the maximum operating Mach number plus 7%. For civil aircraft, it is required to be  $1.25 M_c$  unless an analysis of a 20 sec. dive at 7.5 degrees followed by a 1.5 g pullout shows the maximum Mach to be less. It usually does, and  $M_D$  ends up only about 7% more than  $M_c$ .
3. The design dive speed,  $V_D$ , is about  $1.15 V_c$  for civil transport aircraft, based on a rational analysis (again 20 sec. at 7.5 degree dive). It must be  $1.25 V_c$  unless otherwise computed.

The chart is constructed by deciding on a structural design altitude. The altitude at the "knee" of the placard diagram is selected based on operational requirements. It is generally not necessary for an aircraft to withstand gusts at Mach 0.8 at sea level, but it is desirable for there to be no restriction on the Mach number at 30,000 ft altitude. Thus, this altitude is typically chosen to be between 25,000 and 28,000 ft. At this altitude and above, the  $V_c$ - $M_c$  line corresponds to  $M_c$ . Below this altitude,  $V_c$  corresponds to  $V_{ceqv}$  at the design altitude. That is, the line of true speed versus altitude is constructed as determined by  $M_c$  (note the kink at the edge of the stratosphere). The line is continued below the chosen altitude at constant dynamic pressure (constant equivalent airspeed).

The  $V_D$  line is about  $1.15 V_c$  until the Mach number reaches  $M_D$ .

For supersonic aircraft it is often not necessary to operate at high equivalent airspeeds at low altitudes. So, another kink in the placard diagram is sometimes introduced. Concorde, for example, is placarded to 400 kts IAS at subsonic speeds (below 32,000ft).



**Figure 4.13 Placard Diagram for Concorde**

4.7 Accelerated Flight with Pitching Accelerations

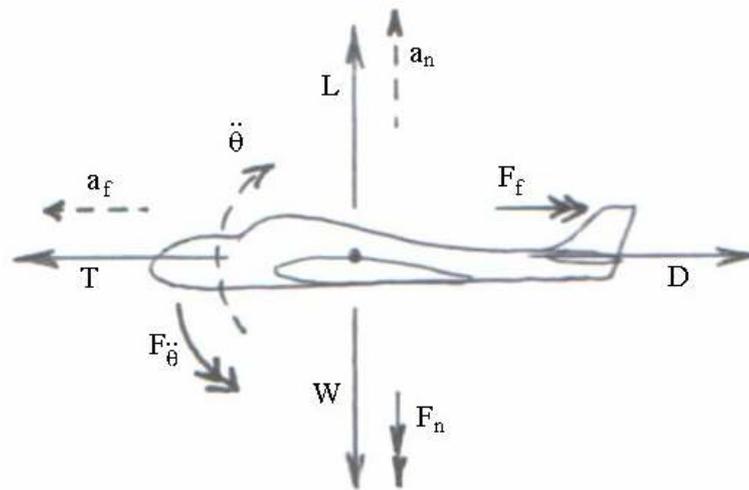
4.7.1 *Level Accelerated Flight with a Pitching Acceleration*

Aircraft pitching acceleration =  $\ddot{\theta}$ ,  $\therefore$  resultant normal acceleration =  $r\ddot{\theta}$

(where r is distance from aircraft's CG position)

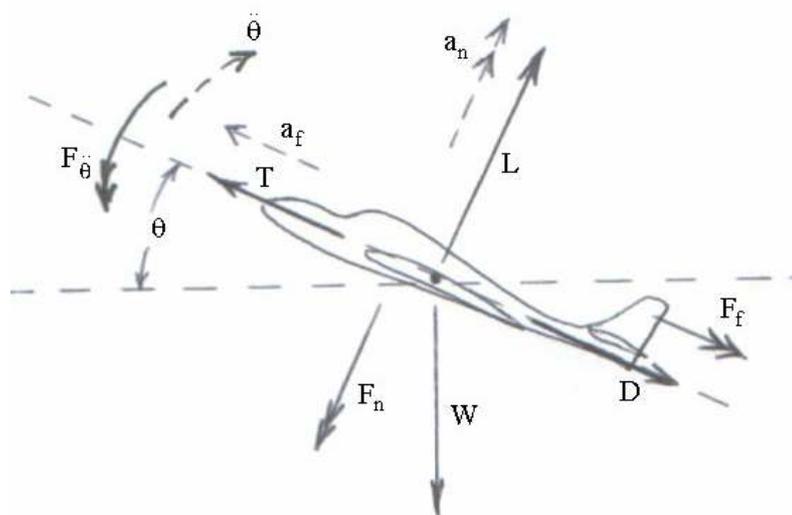
$$\therefore \text{normal inertial force} = F_{\ddot{\theta}} = mr\ddot{\theta} = (W/g)r\ddot{\theta} \quad (4.6)$$

(i.e. it is dependent upon position relative to the aircraft CG)



**Figure 4.14 Level accelerated flight with pitching acceleration**

#### 4.7.2 Climbing Accelerated Flight with a Pitching Acceleration



**Figure 4.15 Climbing accelerated flight with pitching acceleration**

A quasi-static force balance yields:

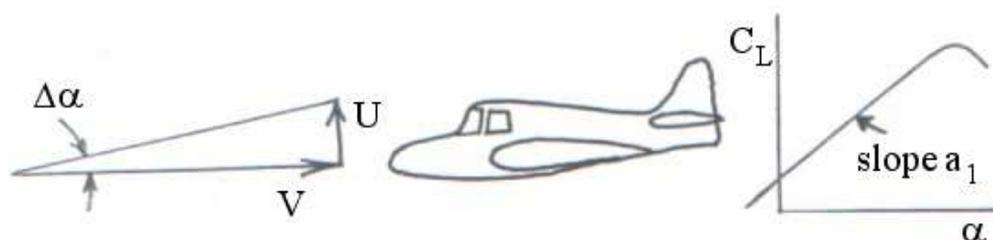
$$\therefore L = W \cos \theta + F_n + F_{\ddot{\theta}}, \quad \therefore L = W \left( \cos \theta + \frac{a_n}{g} + \frac{r\ddot{\theta}}{g} \right),$$

and 
$$\therefore n = \left( \cos \theta + \frac{a_n}{g} + \frac{r\ddot{\theta}}{g} \right) \quad (4.7)$$

also 
$$\therefore T = D + W \left( \sin \theta + \frac{a_f}{g} \right)$$

#### 4.8 Symmetric Environmental Loads – Gusts

These can easily exceed the manoeuvre loads in some cases, especially for slow & light aircraft. Consider an a/c in steady level flight at an airspeed  $V$  and experiencing an instantaneous (sharp-edged) gust  $U$  in a vertical upwards direction.



**Figure 4.16 Sharp-edged gust analysis**

The aircraft is initially at an angle of attack of  $\alpha$  with a lift coefficient  $C_L$  and a lift curve slope of  $a_1$ . Assuming an instant effect of the gust on the entire a/c with no resultant trim change: change in angle of attack ( $\Delta\alpha$ ) =  $\tan^{-1}(U/V)$

For small values of  $\Delta\alpha$ ,  $\Delta\alpha = (U/V)$

$$\therefore \Delta C_L = (U/V) a_1$$

$$\therefore \Delta L = \frac{1}{2} \rho V^2 S \Delta C_L = \frac{1}{2} \rho V^2 S (U/V) a_1 = \frac{1}{2} \rho V U S a_1$$

$$\therefore \Delta n_g = \Delta L / W = \rho V U a_1 / 2 (W / S) \quad (4.8a)$$

Or, since  $L = W = \frac{1}{2} \rho V^2 S C_L$

$$\Delta n_g = \rho V U a_1 S / (2 W) = \rho V U a_1 S / [2 (\frac{1}{2} \rho V^2 S C_L)]$$

$$\text{i.e.} \quad \Delta n_g = U a_1 / (V C_L) \quad (4.8b)$$

#### 4.8.1 Gust Alleviation Factors

In reality, gusts follow cosine-like intensity as the aircraft flies through it, thus reducing its overall impact. Accordingly, a *gust alleviation factor* (K) is applied to the “design” gust speeds:

$$\text{For subsonic aircraft:} \quad K = 0.88 \mu / (5.3 + \mu) \quad (4.9a)$$

$$\text{For supersonic aircraft:} \quad K = \mu^{1.03} / (6.95 + \mu^{1.03}) \quad (4.9b)$$

Where the coefficient

$$\mu = 2 (W / S) / (\rho g \bar{c} a_1) \quad (4.9c)$$

accounts for small aircraft being more susceptible to gusts than large aircraft and also includes an altitude dependence term.

#### 4.8.2 Design Gust Values

The following (all in EAS), or similar, are usually used:

- At  $V_D$ ,  $U = 7.6$  m/s
- At  $V_H$  or  $V_C$ ,  $U = 15.2$  m/s
- At “rough air speed” ( $V_G$  or  $V_A$ ),  $U = 20$  m/s

Positive and negative gust load factors are then calculated from equations (4.8a) or (4.8b) (including gust alleviation factors) and superimposed onto n-V diagram to establish overall critical conditions.

#### **Example 4.2**

Construct a gust n-V diagram for a BAe 125 executive jet aircraft for an altitude of 10.5 km (where  $\rho / \rho_0 = 0.3163$ ). For the aircraft:

Gross weight = 59 kN; gross wing area = 33 m<sup>2</sup>; rough airspeed (V<sub>G</sub>) = 400 km/hr (TAS); cruise airspeed (V<sub>C</sub>) = 720 km/hr (TAS); dive airspeed (V<sub>D</sub>) = 830 km/hr (TAS); C<sub>L,max</sub> = 1.8, lift curve slope = 4 per rad; K = 0.73.

| airspeed (EAS) | gust (EAS) 0 to 6 km altitude | gust (EAS) > 15 km altitude |
|----------------|-------------------------------|-----------------------------|
| V <sub>G</sub> | ± 20                          | ± 12                        |
| V <sub>C</sub> | ± 15                          | ± 8                         |
| V <sub>D</sub> | ± 8                           | ± 4                         |

Firstly, calculate the aircraft equivalent airspeeds in m/s:

$$V_G = 400 \times (1000 / 3600) \times \sqrt{0.3163} = 62.49 \text{ m/s}$$

$$V_C = 720 \times (1000 / 3600) \times \sqrt{0.3163} = 112.48 \text{ m/s}$$

$$V_D = 830 \times (1000 / 3600) \times \sqrt{0.3163} = 129.67 \text{ m/s}$$

For 10.5 km gusts, linearly interpolate between 6 and 15 km:

$$\text{At } V_G: \quad U = \frac{1}{2} (20 + 12) = \pm 16 \text{ m/s}$$

$$\text{At } V_C: \quad U = \frac{1}{2} (15 + 8) = \pm 11.5 \text{ m/s}$$

$$\text{At } V_D: \quad U = \frac{1}{2} (8 + 4) = \pm 6 \text{ m/s}$$

For positive stall curve:

$$\begin{aligned} - \text{ using } n = L/W &= \frac{1}{2} \rho_0 V_e^2 S C_{L,max} / W \\ &= \frac{1}{2} \times 1.225 \times 33 \times 1.8 \times V_e^2 / 59000 \\ &= 6.167 \times 10^{-4} V_e^2 \end{aligned}$$

|                |       |       |       |       |       |       |
|----------------|-------|-------|-------|-------|-------|-------|
| V <sub>e</sub> | 10    | 20    | 30    | 40    | 50    | 60    |
| n              | 0.062 | 0.247 | 0.555 | 0.987 | 1.542 | 2.220 |

For gust lines: using equation (4.8a):

$$\Delta n_g = \rho V_e U_e a_l K / 2 (W/S) = (\frac{1}{2} \times 1.225 \times 4 \times 0.73 \times 33 / 59000) V_e U_e = 1.003 \times 10^{-3} V_e U_e$$

$$\text{At } V_G = 62.49 \text{ m/s:}$$

$$\Delta n_g = 1.003 \times 10^{-3} \times 62.49 \times 16 = \pm 1.0$$

$$\therefore n = 1 \pm 1.0 = 0, +2.0$$

At  $V_C = 112.48$  m/s:

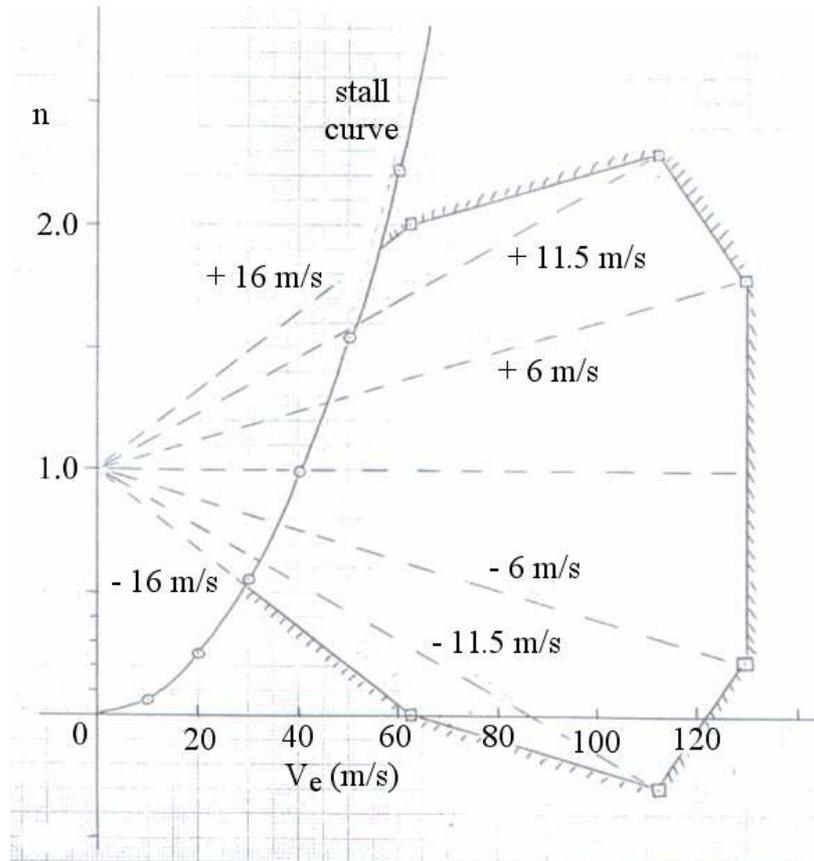
$$\Delta n_g = 1.003 \times 10^{-3} \times 112.48 \times 11.5 = \pm 1.29$$

$$\therefore n = 1 \pm 1.29 = -0.29, +2.29$$

At  $V_D = 129.67$  m/s:

$$\Delta n_g = 1.003 \times 10^{-3} \times 129.67 \times 6 = \pm 0.78$$

$$\therefore n = 1 \pm 0.78 = +0.22, +1.78$$

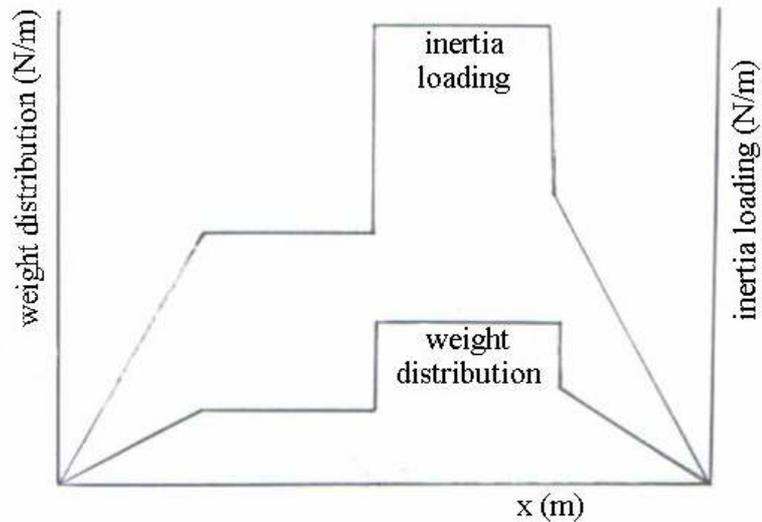


**Figure 4.17 Gust loading n-V diagram for Example 4.2**

## 5. Inertial Loading, Shear Force & Bending Moment Diagrams

### 5.1 Inertial Loading

This is the product of the relevant local normal load factor (n) and the local value of the weight distribution. An example for a constant n = 3 case is given in Figure 5.1.

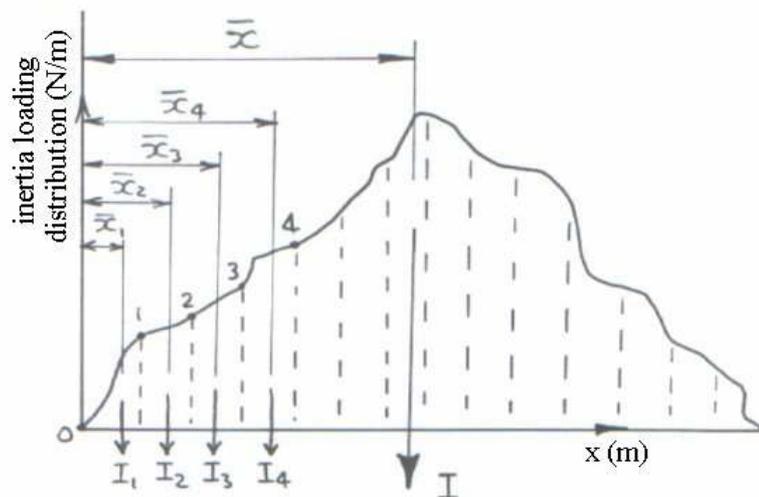


**Figure 5.1 Inertial loading example**

The total inertial load (I) acting on the structure may be obtained directly from the loading distribution by integrating it to give:

$$I = \sum_1^n I_n \quad (5.1)$$

where  $I_1$  = area under curve 0-1, etc on Figure 5.2

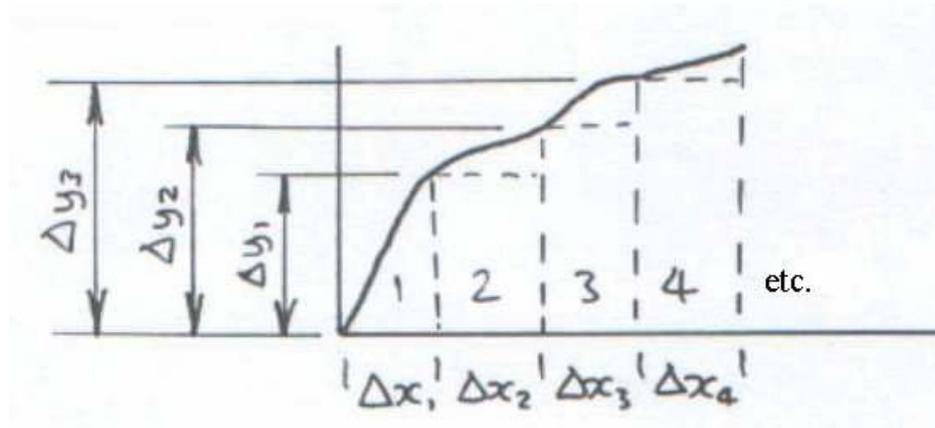


**Figure 5.2 Total inertia load from inertial loading**

The position at which it acts ( $\bar{x}$ ) may also be found from:

$$\bar{I}\bar{x} = \sum_1^n I_n \bar{x}_n \quad (5.2)$$

Figure 5.3 shows a graphical procedure for estimating the value of the total inertia load:



**Figure 5.3 Total inertia load estimation procedure**

- Split the total area under the curve into as many increments as required (the more the better for accuracy).
- Treat each increment as either a triangle or square.
- Calculate the individual element area and centroid position, i.e.

$$\bullet \quad I_1 = \left(\frac{1}{2}\right) \Delta x_1 \Delta y_1, \quad \bar{x}_1 = \left(\frac{2}{3}\right) \Delta x_1$$

$$\bullet \quad I_2 = I_{2,R} + I_{2,T} = (\Delta x_2 \Delta y_1) \left(\frac{1}{2}\right) \Delta x_2 (\Delta y_2 - \Delta y_1), \quad \bar{x}_{2,R} = \Delta x_1 + \left(\frac{1}{2}\right) \Delta x_2,$$

$$\bar{x}_{2,T} = \Delta x_1 + \left(\frac{2}{3}\right) \Delta x_2$$

## 5.2 Shear Force Diagram

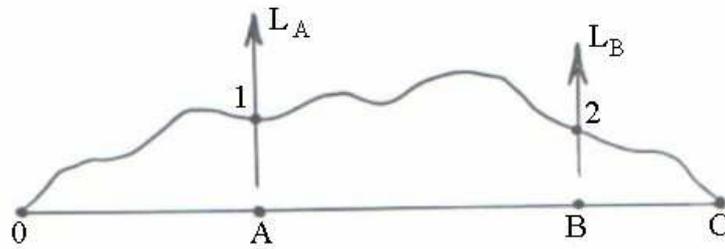
The *shear force diagram* may then be obtained directly from the loading distribution by:

$$\text{shear force (SF)} = \int_0^x I_l dx \quad (5.3)$$

i.e. the shear force at any point = area under the load distribution curve up to that point.

A sign convention is required and it is usually taken that the shear force is positive when it acts upwards when moving from left to right across the beam or body.

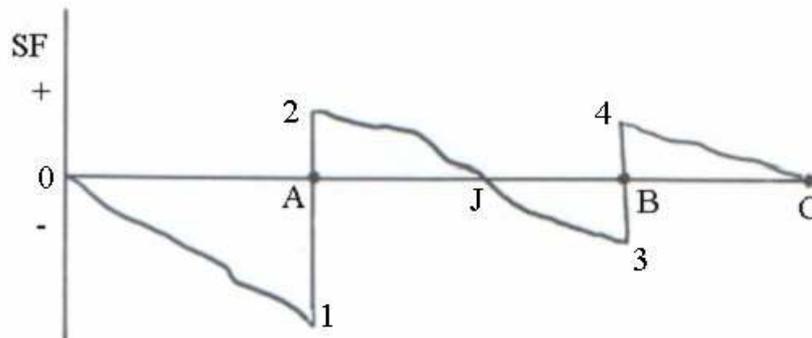
As an example, consider the loading distribution given in Figure 5.4, showing a distributed load plus two additional point loads.



**Figure 5.4 Shear force loading calculation example – load distribution**

- SF at 0 and C = 0
- SF at A = - (area under curve 0-1)
- SF at A = - (area under curve 0-1) +  $L_A$
- SF at B = - (area under curve 0-1-2) +  $L_A$
- SF at B = - (area under curve 0-1-2) +  $L_A$  +  $L_B$

This results in a shear force diagram as shown in Figure 5.5:



**Figure 5.5 Shear force calculation**

Note that steps in the shear force diagram are only present at the point load locations.

### 5.3 Bending Moment Diagram

The beam or body bending moment = shear force x distance

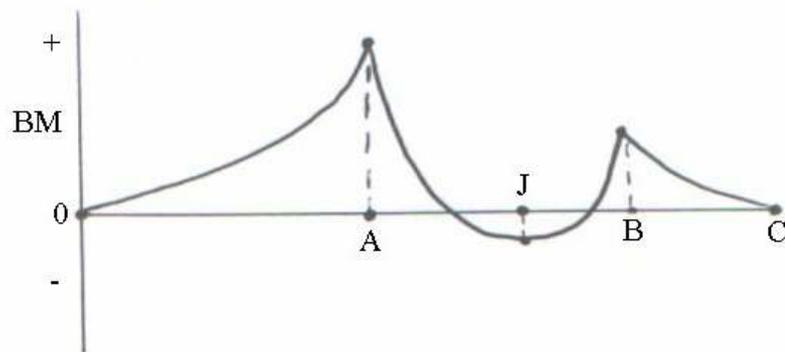
$$\text{i.e. bending moment (BM)} = \int_0^x (\text{SF}) dx \quad (5.4)$$

The BM at any point must therefore equal the area under the SF diagram to one side of that point. This can be to either side since the shear forces and bending moments must balance out

for equilibrium. A sign convention is again necessary and is arbitrary, usually taken as positive for an anti-clockwise moment (moving left to right).

As an example, using the preceding loading diagrams in Figures 5.4 and 5.5:

- BM at 0 = 0
- BM at A = + (area under SF curve 0-1)
- BM at J = + (area under SF curve 0-1) - (area under SF curve 2-J)
- BM at B = + (area under SF curve 0-1) - (area under SF curve 2-J) + (area under SF curve J-3)
- BM at C = 0



**Figure 5.6 Bending moment calculation**

Note that:

- Maxima and minima occur when  $SF = 0$  (since  $SF = d(BM)/dx$ ).
- The BM diagram may be drawn directly from the loading diagram if required.
- Steps only occur if point moments or couples act (e.g. pitching moment about wing aerodynamic centre).

### **Beam Loading Example 5.1**

The beam shown in Figure 5.7 is cantilevered at 0

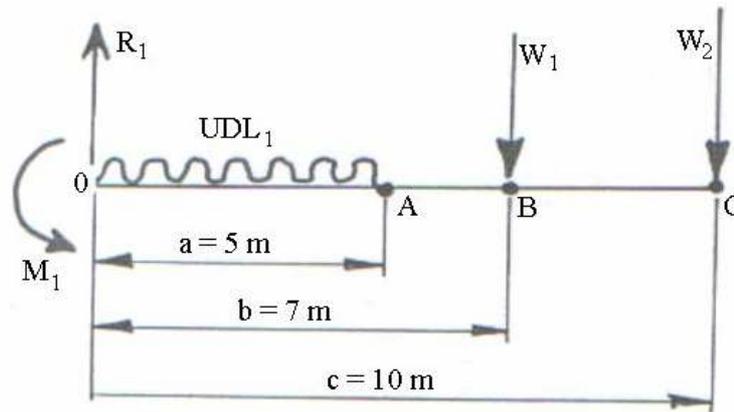
- $UDL_1 = 10 \text{ kN/m}$ ,  $W_1 = 20 \text{ kN}$ ,  $W_2 = 30 \text{ kN}$

### **Solution**

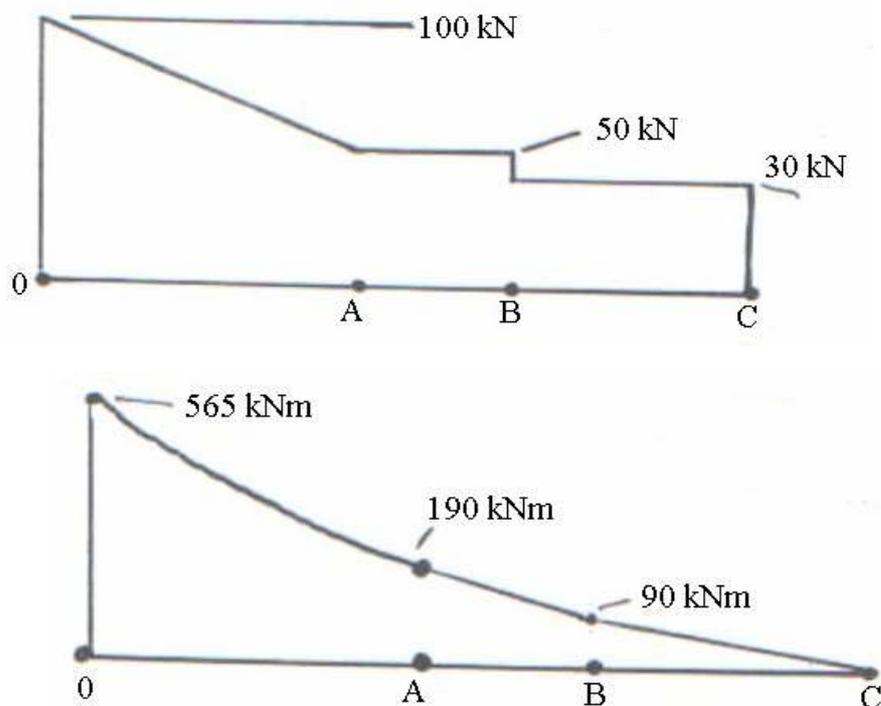
The first task is to find values for  $M_1$  and  $R_1$

- $R_1 = UDL_1 \times a + W_1 + W_2 = 100 \text{ kN}$
- $M_1 = (UDL_1 \times a \times a/2) + (W_1 \times b) + (W_2 \times c) = 565 \text{ kNm}$

The resultant shear force and bending moment diagrams are plotted in Figure 5.8.



**Figure 5.7 Beam loading Example 5.1**



**Figure 5.8 SF & BM diagrams for Example 5.1**

**Example 5.2**

Consider the following combat aircraft, as shown in Figure 5.9:

Wing gross planform area =  $21 \text{ m}^2$ ; wing geometric chord =  $2.7 \text{ m}$

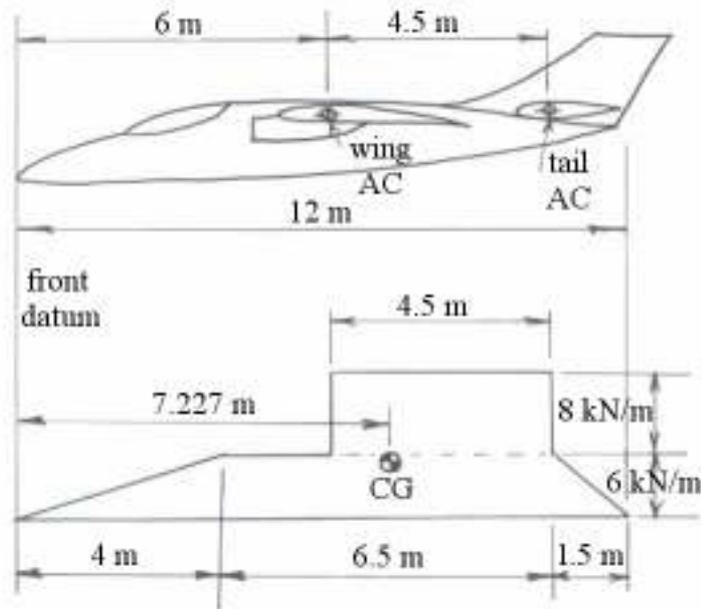
Wing pitching moment coefficient =  $-0.05$

At a particular moment in a symmetrical manoeuvre, the following conditions exist:

flight path angle to horizontal =  $+20^\circ$ ; equivalent airspeed =  $500 \text{ km/hr}$ ;

acceleration normal to flight path =  $+15 \text{ m/s}^2$ ; pitching acceleration =  $+1.6 \text{ rad/s}^2$

(nose-up)



**Figure 5.9 Aircraft geometry for Example 5.2**

Calculate the variation of normal load factor along fuselage. Obtain the inertia loading distribution normal to fuselage axis. Calculate the aerodynamic loads at the wing & tail aerodynamic centres. Finally, calculate and plot the SF diagram.

**Solution**

Using eq. (4.7):

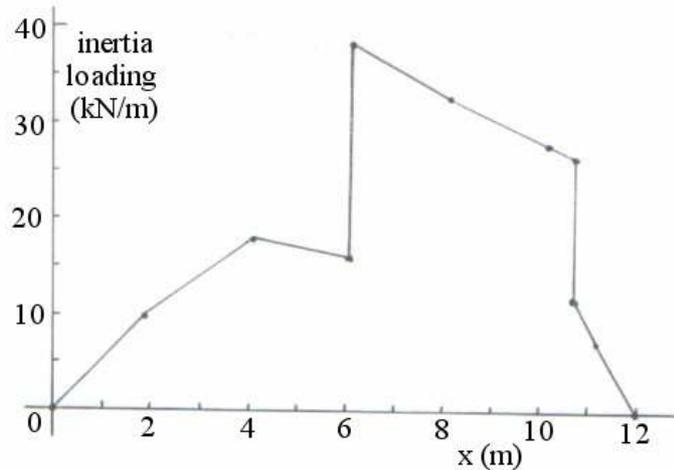
$$n = \cos 20 + (15 + 1.6 r) / 9.81 = 2.4687 + 0.1631 r$$

(where x = distance from front (nose) datum, r = distance forward of CG)

|       |       |       |       |       |        |        |        |        |        |
|-------|-------|-------|-------|-------|--------|--------|--------|--------|--------|
| x (m) | 0     | 2     | 4     | 6     | 8      | 10     | 10.5   | 11     | 12     |
| r (m) | 7.277 | 5.277 | 3.277 | 1.277 | -0.773 | -2.773 | -3.273 | -3.773 | -4.773 |
| n     | 3.647 | 3.321 | 2.995 | 2.669 | 2.343  | 2.016  | 1.935  | 1.853  | 1.690  |

Inertia loading = weight distribution x local n value

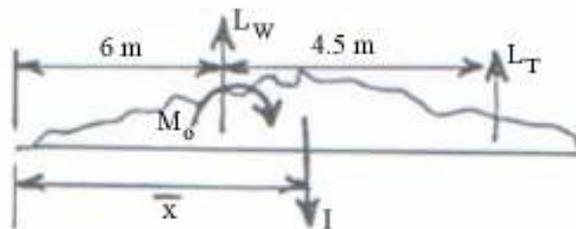
|          |       |       |       |              |       |       |              |       |       |
|----------|-------|-------|-------|--------------|-------|-------|--------------|-------|-------|
| x (m)    | 0     | 2     | 4     | 6            | 8     | 10    | 10.5         | 11    | 12    |
| n        | 3.647 | 3.321 | 2.995 | 2.669        | 2.343 | 2.016 | 1.935        | 1.853 | 1.690 |
| w (kN/m) | 0     | 3     | 6     | 6, 14        | 14    | 14    | 14, 6        | 4     | 0     |
| I (kN/m) | 0     | 9.963 | 17.97 | 16.01, 37.37 | 32.80 | 28.22 | 27.09, 11.61 | 7.412 | 0     |



**Figure 5.10 Aircraft loading diagram for Example 5.2**

For wing pitching moment:

$$M_o = 0.5 \times 1.225 \times (500/3.6)^2 \times 21 \times 2.7 \times -0.05 = -33.5 \text{ kNm}$$



Summing vertical forces:

$$L_W + L_T = I \quad (1)$$

(where  $I$  = total inertial force)

Moment balance about nose:

$$(L_W \times 6) + (L_T \times 10.5) - M_o = I$$

(where  $\bar{x}$  = distance from nose where  $I$  acts)

$$\therefore 6 L_W + 10.5 L_T = I - 33.5 \quad (2)$$

We need values for the inertia load ( $I$ ) and the position at which it acts ( $\bar{x}$ ). These may be obtained by breaking down the inertia loading distribution into a number ( $= m$ ) of smaller increments. Then:

- calculate the average inertia load due to each increment ( $I_m$ ), comprising contributions due to rectangular and triangular elements.
- calculate positions at which each incremental load acts ( $\bar{x}_m$ ).

| m | x (m)      | loading (kN/m) | $\Delta x$ (m) | $I_m$ (kN)              | $\bar{x}_m$ (m) | $I_m \bar{x}_m$ (kNm)      |
|---|------------|----------------|----------------|-------------------------|-----------------|----------------------------|
| 1 | 0<br>2     | 0<br>9.963     | 2              | 9.96                    | 1.33            | 13.28                      |
| 2 | 2<br>4     | 9.963<br>17.97 | 2              | 19.93 +<br>8.01 = 27.94 | 3<br>3.33       | 59.79 +<br>26.67 = 86.46   |
| 3 | 4<br>6     | 37.37<br>32.80 | 2              | 32.02 +<br>1.96 = 33.98 | 5<br>4.67       | 160.1 +<br>9.15 = 169.25   |
| 4 | 6<br>8     | 37.37<br>32.80 | 2              | 65.6 +<br>4.57 = 70.17  | 7<br>6.67       | 459.2 +<br>30.48 = 489.68  |
| 5 | 8<br>10    | 32.80<br>28.22 | 2              | 56.44 +<br>4.58 = 61.02 | 9<br>8.67       | 507.96 +<br>39.71 = 547.67 |
| 6 | 10<br>10.5 | 28.22<br>27.09 | 0.5            | 13.55 +<br>0.28 = 13.83 | 10.25<br>10.17  | 138.89 +<br>2.85 = 141.74  |
| 7 | 10.5<br>12 | 11.61<br>0     | 1.5            | 8.71                    | 11              | 95.78                      |

$$I = \sum_{m=1}^{m=7} I_m = 225.61 \text{ kN}, \quad E\bar{x} = \sum_{m=1}^{m=7} I_m \bar{x}_m = 1543.86 \text{ kNm} \therefore \bar{x} = \mathbf{6.84 \text{ m}}$$

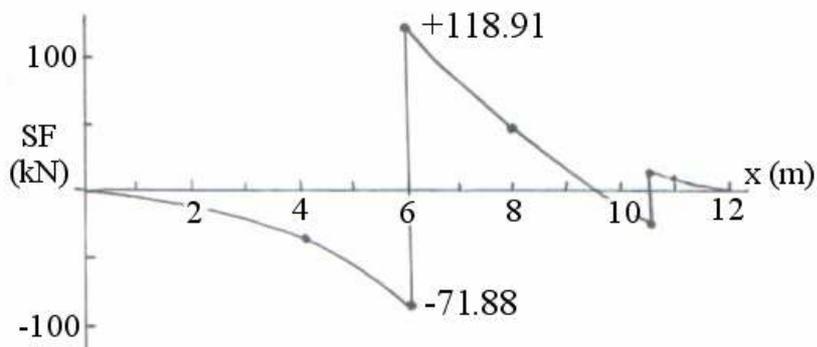
Substituting into (1) & (2):

$$L_W = 225.61 - L_T$$

$$\text{and } 6 L_W + 10.5 L_T = 1543.86 - 33.5 = 1510.36$$

$$\therefore L_T = \mathbf{34.82 \text{ kN}}, L_W = \mathbf{190.79 \text{ kN}}$$

| x (m)    | $\Delta SF$ (kN) | point load (kN) | SF (kN)           |
|----------|------------------|-----------------|-------------------|
| 0, 2     | - 9.96           |                 | 0, - 9.96         |
| 2, 4     | - 27.94          |                 | - 37.9            |
| 4, 6     | - 33.98          | + 190.79        | - 71.88           |
| 6, 8     | - 70.17          |                 | + 118.91, + 48.74 |
| 8, 10    | - 61.02          |                 | - 12.28           |
| 10, 10.5 | - 13.83          | + 34.82         | - 26.11           |
| 10.5, 12 | - 8.71           |                 | + 8.71, 0         |



**Figure 5.11 SF diagram for Example 5.2**

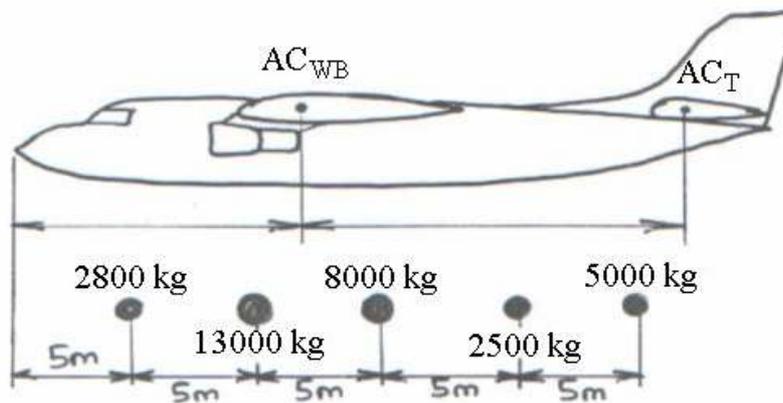
**Example 5.3**

An aircraft in steady level flight at  $V_e = 550$  km/hr is subjected to an upwards acceleration of  $15 \text{ m/s}^2$  and a nose-up pitching acceleration of  $1.2 \text{ rad/s}^2$ . Calculate the variation of the load factor  $n$  along the fuselage length and plot the SF and BM diagrams.

**Aircraft Data**

Wing: area =  $145 \text{ m}^2$ , mean chord =  $5 \text{ m}$ ,  $C_{M0} = -0.07$

Tailplane: area =  $50 \text{ m}^2$ , mean chord =  $3 \text{ m}$ ,  $C_{M0} = 0$



**Figure 5.12 Aircraft loading Example 5.3**

**Solution**

The first task is to calculate the CG location.

| x (m) | m (kg) | moment about x = 0 |
|-------|--------|--------------------|
| 5     | 2,800  | 14,000             |

|    |                 |                  |
|----|-----------------|------------------|
| 10 | 13,000          | 130,000          |
| 15 | 8,000           | 120,000          |
| 20 | 2,500           | 50,000           |
| 25 | 5,000           | 125,000          |
|    | $\Sigma$ 31,300 | $\Sigma$ 439,000 |

$$\therefore x_{CG} = 439,000 / 31,300 = 14.026 \text{ m from nose}$$

For variation in normal load factor (n), use equation (4.7):  $\therefore n = (\cos \theta + \frac{a_n}{g} + \frac{r\ddot{\theta}}{g})$

where r = distance forward of CG

Using  $a_n = 15 \text{ m/s}^2$ ,  $\ddot{\theta} = 1.2 \text{ rad/s}^2$

$$\therefore n = 2.529 + 0.122 r = 2.529 + 0.122 (14.026 - x)$$

Next calculate inertia forces:

| x (m) | n     | m (kg) | I (kN)          | Ix (kNm)         |
|-------|-------|--------|-----------------|------------------|
| 5     | 3.630 | 2,800  | 99.71           | 498.5            |
| 10    | 3.020 | 13,000 | 385.14          | 3,851.4          |
| 15    | 2.410 | 8,000  | 189.14          | 2,837.1          |
| 20    | 1.800 | 2,500  | 44.15           | 882.9            |
| 25    | 1.190 | 5,000  | 58.38           | 1,459.5          |
|       |       |        | $\Sigma$ 776.52 | $\Sigma$ 9,529.4 |

Next calculate pitching moments.

$$\text{Using: } M = \frac{1}{2} \rho V^2 S c C_{M_0}$$

$$\text{For wing: } M_0 = 0.5 \times 1.225 \times 152.782 \times 145 \times 5 \times -0.07 = -725.54 \text{ kNm}$$

$$\text{For tailplane: } M_0 = 0$$

Next calculate lift from wing ( $L_W$ ) and tailplane ( $L_T$ )

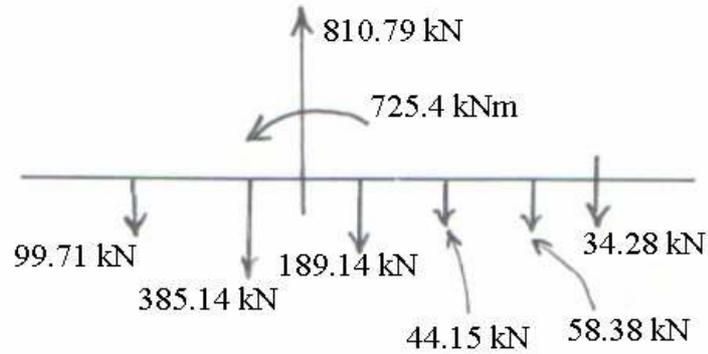
$$\text{Resolving forces vertically: } L_W + L_T = 776.52 \text{ _____ (1)}$$

$$\text{Taking moments about nose: } (L_W \times 12) + (L_T \times 27) + 725.54 = 9529.4 \text{ _____ (2)}$$

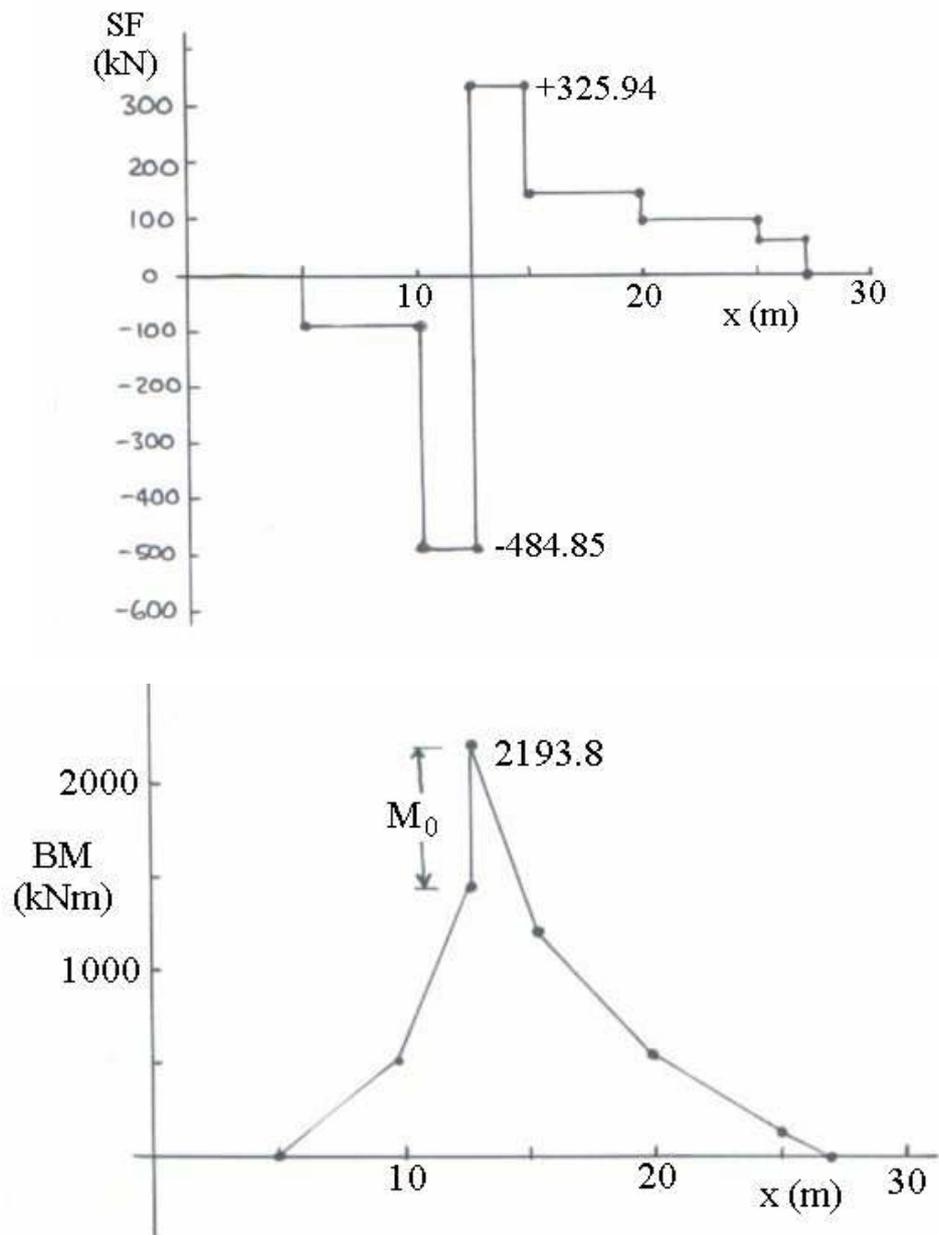
Solving simultaneous equations gives:

$$L_w = 810.79 \text{ kN}, L_T = -34.28 \text{ kN}$$

Next calculate SF and BM values from following resultant loading diagram:



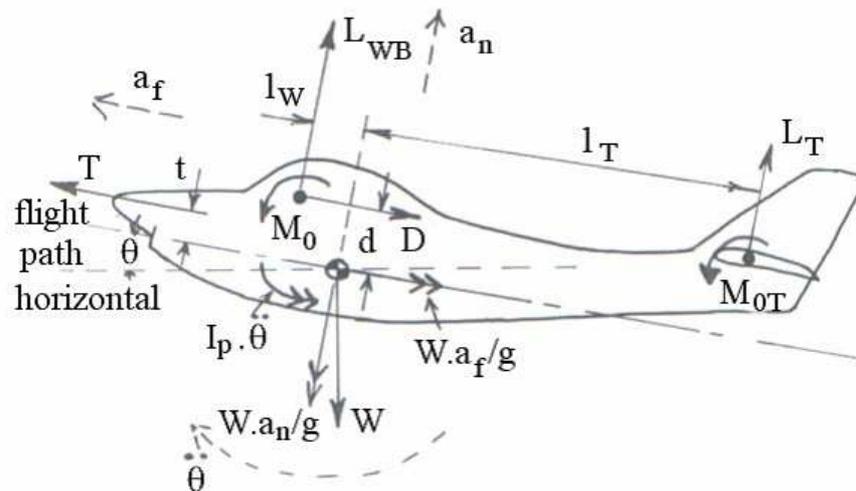
| x (m) | SF (kN)          | BM (kNm)         |
|-------|------------------|------------------|
| 0     | 0                | 0                |
| 5     | 0, -99.71        | 0                |
| 10    | -99.71, -484.85  | +498.6           |
| 12    | -484.85, +325.94 | +1468.3, +2193.8 |
| 15    | +325.94, +136.80 | +1216.0          |
| 20    | +136.80, +92.66  | +532.0           |
| 25    | +92.66,          | +68.72           |
| 27    | +34.28, 0        | 0                |
| 30    | 0                | 0                |



**Figure 5.13 SF & BM diagrams for Example 5.2**

## 6. Wing and Tail Loads

These may be obtained from a general balance of the forces acting on the airframe for any general manoeuvre.



**Figure 6.1 Force balance for wing and tail loads**

Assuming that the thrust  $T$  acts along the flight path, summing forces normal and parallel to flight path and summing moments about CG gives:

$$L_{WB} + L_T - W \cos \theta - W a_n / g = 0 \quad (6.1a)$$

$$T - D - W \sin \theta - W a_f / g = 0 \quad (6.1b)$$

$$L_{WB}l_W - L_Tl_T - M_0 - M_{0T} - I_p \ddot{\theta} + D d - T t = 0 \quad (6.1c)$$

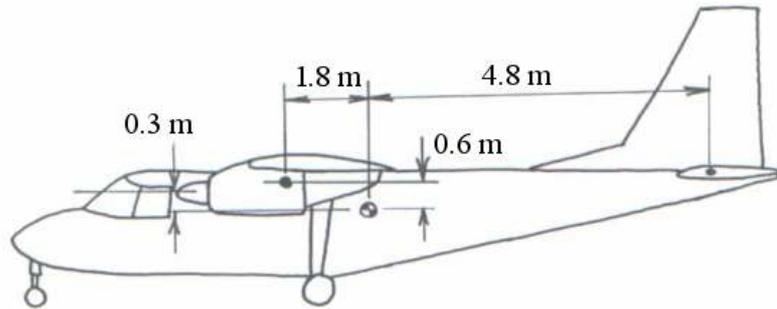
The number of unknowns is usually 4 ( $L_{WB}$ ,  $L_T$ ,  $T$  &  $D$ ) so that an iterative approach is required, allied with the use of the aircraft's drag polar equation ( $C_D = C_{D0} + kC_L^2$ ).

### **Example 6.1**

The dimensions of a Britten-Norman Islander aircraft are shown below in Figure 6.2:

weight = 29 kN; gross wing area = 30 m<sup>2</sup>; wing mean chord = 2 m; wing/body pitching moment coefficient = -0.065; gross tailplane area = 9 m<sup>2</sup>; tailplane mean chord = 1.4 m; tailplane pitching moment coefficient = -0.075; drag polar  $C_D = 0.045 + 0.08 C_L^2$

Obtain the wing and tailplane lift for  $\theta = 10^\circ$  at an equivalent airspeed of 60 m/s.



**Figure 6.2 Britten-Norman Islander dimensions**

**Solution**

Using equations (6.1), in this case:

$$L_{WB} + L_T - W \cos \theta = 0$$

$$T - D - W \sin \theta = 0$$

$$L_{WB}l_w - L_Tl_T - M_o - M_{oT} + D d - T t = 0$$

Using  $M_o = \frac{1}{2} \rho V^2 S \bar{c} C_{M_o}$

$$M_o = 8.6 \text{ kNm} \text{ \& } M_{oT} = 2.08 \text{ kNm}$$

Using data given:

$$L_{WB} + L_T = 28855$$

$$T - D = 5088$$

$$L_T = (1.8 L_{WB} - 0.3 T + 0.6 D - 10683) / 4.8$$

Step 1 – assume  $L_T = 0$ ,  $\therefore L_{WB} = 28855 \text{ N}$

$$C_{L,WB} = L_{WB} / (0.5 \rho_o V_e^2 S) = 0.436$$

$$C_D = 0.045 + 0.08 C_L^2 = 0.0602$$

$$D = (0.5 \rho_o V_e^2 S) C_D = 3983.7 \text{ N}$$

$$\therefore T = 9071.7 \text{ N and } L_T = 8526 \text{ N}$$

Step 2 – now use  $L_T = 8526 \text{ N}$ ,  $\therefore L_{WB} = 20329 \text{ N}$

$$\therefore C_{L,WB} = 0.307, \therefore C_D = 0.0526, \therefore D = 3476.5 \text{ N}$$

$$\therefore T = 8564.5 \text{ N and } L_T = 5297 \text{ N}$$

Step 3 – now use  $L_T = 5297 \text{ N}$ ,  $\therefore L_{WB} = 23558 \text{ N}$

$$\therefore C_{L,WB} = 0.356, \therefore C_D = 0.0551, \therefore D = 3644.9 \text{ N}$$

$$\therefore T = 8732.9 \text{ N and } L_T = 6540 \text{ N}$$

Step 4 – now use  $L_T = 6540 \text{ N}$ ,  $\therefore L_{WB} = 22315 \text{ N}$

$$\therefore C_{L,WB} = 0.337, \therefore C_D = 0.054, \therefore D = 3579 \text{ N}$$

$$\therefore T = 8667 \text{ N and } L_T = 6048 \text{ N}$$

Step 5 – now use  $L_T = 6048 \text{ N}$ ,  $\therefore L_{WB} = 22807 \text{ N}$

$$\therefore C_{L,WB} = 0.345, \therefore C_D = 0.055, \therefore D = 3605.8 \text{ N}$$

$$\therefore T = 8693.8 \text{ N and } L_T = 6234 \text{ N}$$

Step 6 – now use  $L_T = 6234 \text{ N}$ ,  $\therefore L_{WB} = 22621 \text{ N}$

$$\therefore C_{L,WB} = 0.342, \therefore C_D = 0.0544, \therefore D = 3596 \text{ N}$$

$$\therefore T = 8684 \text{ N and } L_T = 6164 \text{ N}$$

Approximate solution is therefore:

$$L_{WB} = \mathbf{22.7 \text{ kN}} \quad \text{and} \quad L_T = \mathbf{6.15 \text{ kN}}$$

## 6.1 Tail Loads

Consider equation (6.1c) with no pitching acceleration: terms due to  $(Dd - Tt)$  and  $M_{oT}$  generally small so may be neglected, hence:

$$L_{WB}l_W - L_Tl_T - M_o = 0 \quad (6.1d)$$

Using equation (6.1a), and substituting:

$$(W - L_T)l_W - L_Tl_T - M_o = 0$$

$$\therefore Wl_W - M_o = L_T(l_T + l_W)$$

$$\text{and} \quad L_T = \frac{Wl_W}{(l_W + l_T)} - \frac{M_o}{(l_W + l_T)} \quad (6.2)$$

### 6.1.1 CG Position Effect

As the aircraft CG moves rearwards,  $l_W$  increases and  $L_T$  gets more positive, thus producing a bigger upload requirement.

### 6.1.2 Speed Effect

$M_o$  increases with  $V^2$ , so  $L_T$  gets more negative as the aircraft speed increases (thus the pilot has to move the control stick more and more forwards as speed increases).

### 6.1.3 Speed for Zero Tail Load

Using equation (6.2):

$$L_T = 0 \text{ when } \frac{Wl_W}{(l_W + l_T)} = \frac{M_o}{(l_W + l_T)} \quad \text{or when } W l_w = M_o$$

$$\text{Using } M_o = \frac{1}{2} \rho V^2 S \bar{c} C_{M_o}$$

$$\text{Therefore, } L_T = 0 \text{ when } V = \sqrt{\frac{2Wl_W}{\rho S_w \bar{c}_w C_{M_o}}} \quad (6.3)$$

#### 6.1.4 Effects of Speed and Normal Load Factor

Consider steady flight with a normal acceleration and a zero pitching acceleration and that assume (T t – D d) and MoT are negligible; using equations (6.1a) to (6.1c):

$$L_{WB} + L_T = W (1 + a_n/g) = n W$$

$$T - D = 0$$

$$L_{WB} l_W - L_T l_T - M_o = 0$$

Using  $L_{WB} + L_T = n W - L_T$  and substituting:

$$(n W - L_T) l_W - L_T l_T - M_o = 0$$

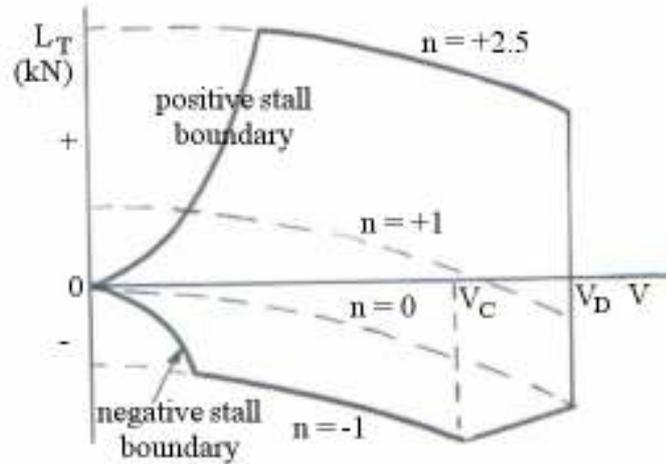
$$\text{Also } M_o = \frac{1}{2} \rho V^2 S_w \bar{c}_w C_{M_o}$$

$$\therefore (n W l_W) - L_T l_W - L_T l_T - \frac{1}{2} \rho V^2 S_w \bar{c}_w C_{M_o} = 0$$

$$\therefore L_T = \left( \frac{Wl_W}{l_W + l_T} \right) n - \left( \frac{\frac{1}{2} \rho S_w \bar{c}_w C_{M_o}}{l_W + l_T} \right) V^2 = K_1 n - K_2 V^2 \quad (6.4)$$

Figure 6.3 shows a plot of tail load versus airspeed for a typical large civilian transport aircraft ( $n_1 = +2.5$ ,  $n_3 = -1$ ), by using eq. (6.4) and with the flight boundaries superimposed. This shows that:

- Maximum  $L_T$  is obtained when  $n$  is large,  $V$  is small,  $K_1$  is large (i.e.  $l_W$  is large and CG is well aft).
- Minimum  $L_T$  is obtained when  $K_1 n$  is small,  $V$  is large and with CG well forward.



**Figure 6.3 Typical tail-load versus airspeed plot**

**Example 6.2**

A civilian aircraft has the following characteristics.

$$W = 63 \text{ kN}; l_w = 0.8 \text{ m}; l_T = 7 \text{ m}; S_w = 40 \text{ m}^2; c = 2.1 \text{ m}; C_{M0} = -0.07; V_D = 120 \text{ m/s (EAS)}; V_C = 90 \text{ m/s (EAS)}; C_{L,max} = 1.35$$

Construct a graph of tail load versus aircraft speed for values of n equal to +2.5, +1.0 and -1.0. Superimpose onto this graph the manoeuvre n-V diagram. Comment on the manoeuvres likely to produce maximum upwards and downwards tail loads.

**Solution**

Using eq. (6.4):

$$L_T = \left( \frac{63000 \times 0.8}{0.8 + 7} \right) n - \left( \frac{\frac{1}{2} \times 1.225 \times 40 \times 2.1 \times 0.07}{0.8 + 7} \right) V_e^2 = 6461.5n - 0.462V_e^2$$

|            | $V_e = 0 \text{ m/s}$ | $V_e = 50 \text{ m/s}$ | $V_e = 90 \text{ m/s}$ | $V_e = 120 \text{ m/s}$ |
|------------|-----------------------|------------------------|------------------------|-------------------------|
| $n = +2.5$ | 16154                 | 15000                  | 12412                  | 9501                    |
| $n = +1$   | 6462                  | 5307                   | 2719                   | -191                    |
| $n = 0$    | 0                     | -1155                  | -3742                  | -6653                   |
| $n = -1.0$ | -6462                 | -7617                  | -10214                 | -13114                  |

For stall curves, use:

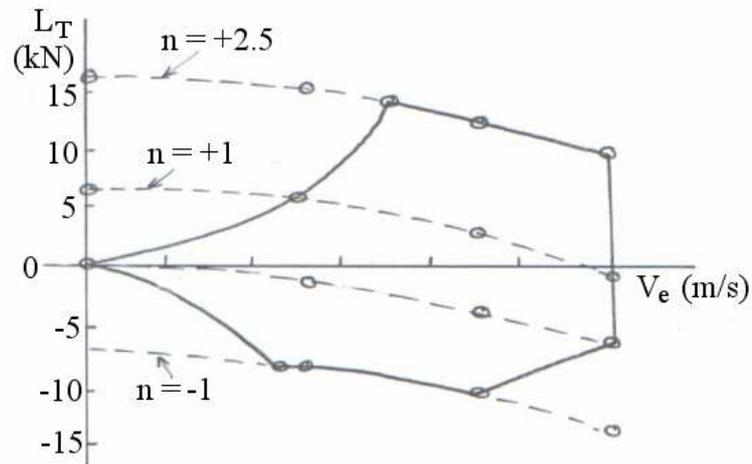
$$n = L / W = \frac{1}{2} \rho_0 V_e^2 S C_{L,max} / W$$

$$\therefore V_e = \sqrt{[n W / (\frac{1}{2} \rho_o S C_{L,max})]} = 43.64 \sqrt{n}$$

For  $n = +2.5$ ,  $V_e$  (stall) = 69.0 m/s

For  $n = +1$ ,  $V_e$  (stall) = 43.6 m/s

For  $n = -1$ ,  $V_e$  (stall) = 43.6 m/s (assume)



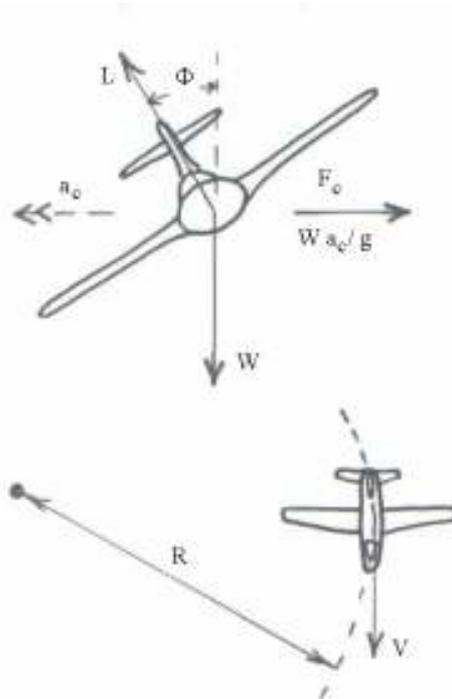
**Figure 6.3 Tail-load versus airspeed plot for Example 6.2**

Here, the maximum tail upload is with a high  $n$  value when at low speed (e.g. pulling out of dive). The maximum download is with a high  $-n$  value when at high speed (e.g. when initiating a sudden dive at high speed).

## 7. Asymmetric Manoeuvre Loads

### 7.1 Banked Turns

Consider an aircraft banked at an angle of  $\phi$ , flying at velocity  $V$  around a circular path of radius  $R$ . It maintains a constant altitude in flight with no side-slip (i.e. it is a *co-ordinated turn*). In this case,  $a_c$  is the *centripetal* acceleration and the inertial reaction is the *centrifugal* force.



**Figure 7.1 Banked turn free body diagram**

$$a_c = V^2 / R \text{ and } F_c = (W/g) a_c = W V^2 / (R g)$$

Resolving forces in the horizontal & vertical directions:

$$L \sin\phi = W V^2 / (R g) \quad (7.1a)$$

$$L \cos\phi = W \quad (7.1b)$$

$$\text{Dividing (7.1a) by (7.1b) gives: } \tan \phi = V^2 / (R g) \quad (7.2)$$

i.e. there is a single correct bank angle for co-ordinated turn at a given speed & radius.

$$\text{Also: } n = V^2 \operatorname{cosec}\phi / (R g) \quad (7.3a)$$

$$\text{And } n = \sec\phi \quad (7.3b)$$

From the force triangle in Figure 7.1 and Pythagora's Theorem:

$$L^2 = W^2 + [W V^2 / (g R)]^2$$

$$\therefore [W V^2 / (g R)] = \sqrt{(L^2 - W^2)}$$

Dividing through by  $W$  gives:

$$V^2 / (Rg) = \sqrt{(L^2 - W^2) / W^2} = \sqrt{(n^2 - 1)} \quad (7.4)$$

For turn radius (R):  $R = V^2 / g\sqrt{n^2 - 1}$  (7.5a)

For turn rate ( $\omega$ ):  $\omega = V / R = g\sqrt{n^2 - 1} / V$  (7.5b)

### **Example 7.1**

An aircraft weighs 200 kN and flies at a constant speed of 300 m/s, while making a coordinated banked turn of 5 km radius. Calculate the bank angle, increased lift requirement above the steady, level flight case and normal load factor.

### **Solution**

Using eq. (7.2):  $\tan \phi = V^2 / (R g)$

$$\therefore \tan \phi = 300^2 / (9.81 \times 5000) = 1.835 \text{ and } \phi = \mathbf{61.41^\circ}$$

Using eq. (7.1b):  $L = W \sec \phi = 200 / \cos 61.41^\circ = 417.94 \text{ kN}$

$\therefore$  increased lift = **217.94 kN**

And  $n = L / W = 417.94 / 200 = \mathbf{2.09}$

### **Example 7.2**

While flying at 130 km/hr (EAS) at a 3 km altitude ( $\sqrt{\sigma} = 0.861$ ), an aircraft weighing 9 kN makes a banked turn through  $90^\circ$  in 15 s, maintaining the same incidence and altitude throughout. The wing loading is  $100 \text{ N/m}^2$  and the lift/drag ratio is 10 at this speed. Calculate the normal load factor, bank angle, radius of turn and power required.

### **Solution**

$V_e = 130 \text{ km/hr} = 36.11 \text{ m/s}$ ,  $V_t = 36.11 / \sqrt{\sigma} = 41.94 \text{ m/s}$

$\omega = 2 \pi / 60 = 0.105 \text{ rad/s}$

Using eq. (7.5b):  $\sqrt{(n^2 - 1)} = \omega V / g = 0.449$ ,  $\therefore n = \mathbf{1.096}$

$\therefore L = 1.096 W = 1.096 \times 9 = 9.856 \text{ kN}$

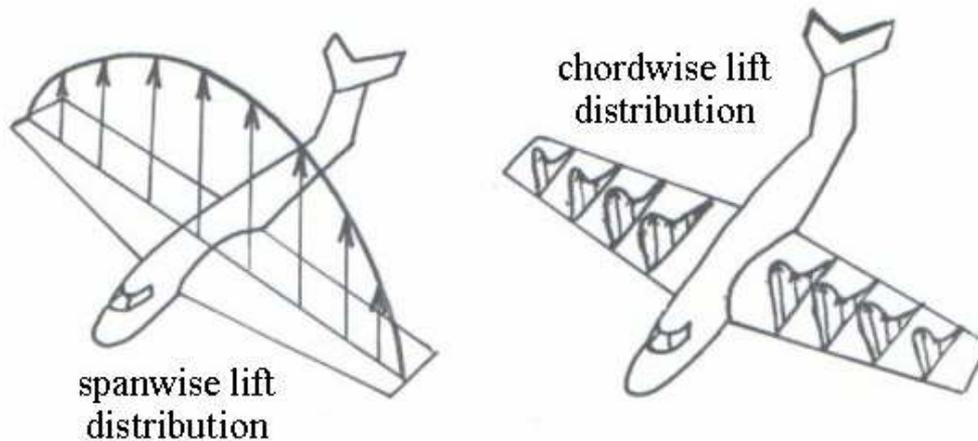
Using eq. (7.1b):  $= \cos^{-1} (W/L) = \mathbf{24.16^\circ}$

Using eq. (7.5a):  $R = V^2 / [g\sqrt{(n^2 - 1)}] = \mathbf{400 \text{ m}}$

Power = drag x velocity =  $(L / 10) \times V = \mathbf{41.34 \text{ kW}}$

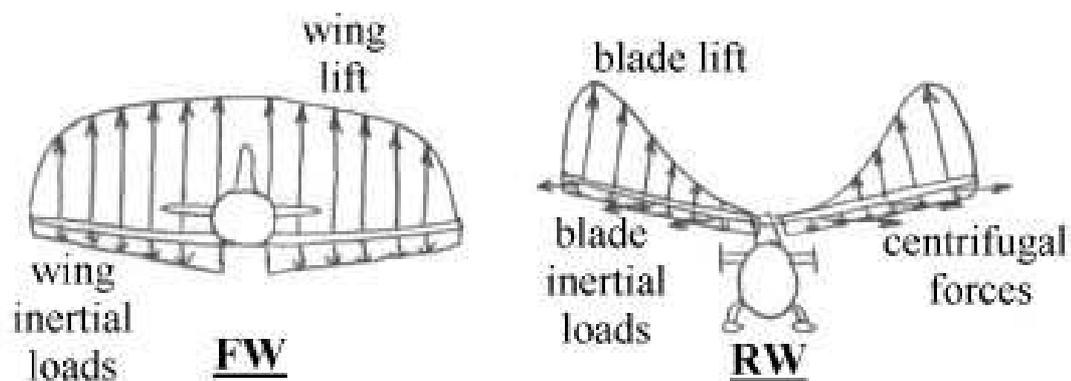
## 8. Subsonic Wing Aerodynamic Loading

Accurate predictions involve complicated processes, e.g. through the use of wind tunnel experiments, aerodynamic panel methods and CFD programs. For preliminary design work, however, classical lifting-line theory (e.g. using *vortex lifting line* and *Kutta-Joukowski* theories) is valid. The span-wise distribution may be obtained using *Schrenk's approximation method*.



**Figure 8.1 Typical chord-wise and span-wise lift distributions**

There are obviously substantial aerodynamic loading differences between FW and RW aircraft, as illustrated in Figure 8.2.



**Figure 8.2 Aerodynamic loading differences between FW and RW aircraft**

In particular, for FW aircraft there are only two main forces (due to inertial loads & lift).

As a consequence, the wing bends upwards and there is a high shear force and bending moment at the wing root. For RW aircraft there are three main forces (inertia, lift & centrifugal loads). On a conventionally articulated head design there will be no bending moment possible through the flapping hinges. It is therefore more difficult to damage a RW blade than a FW a/c wing under extreme manoeuvre conditions.

### 8.1 Schrenk's Approximation Method

This method is commonly used to determine overall span-wise lift distribution, especially at the preliminary design stage for low sweep and moderate to high aspect ratio wings on FW aircraft.

The method states that the resultant load distribution is an arithmetic mean of:

- A load distribution representing the actual planform shape
- An elliptical distribution of the same span and area

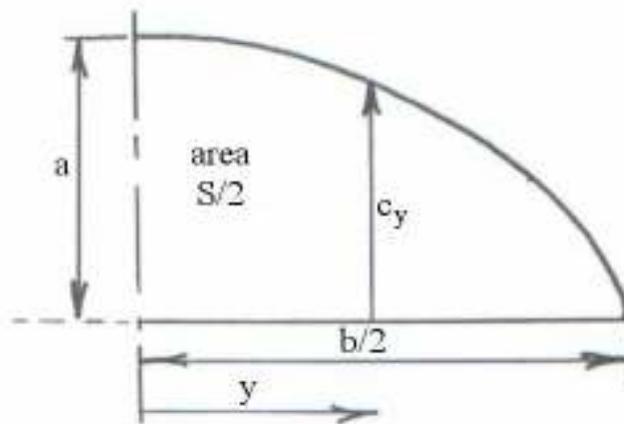
An elliptical distribution is presented in Figure 8.3, here the semi-span wing area = area of elliptic quadrant =  $S/2$ .

$$\text{Area } S/2 = \frac{1}{4} [(\pi/4)(2a)(b)] \quad \therefore a = 4S / (\pi b)$$

$$\text{For an ellipse, } \frac{y^2}{(b/2)^2} + \frac{c_y^2}{a^2} = 1 \quad \therefore c_y = \frac{4S}{\pi b} \sqrt{1 - \left(\frac{2y}{b}\right)^2}$$

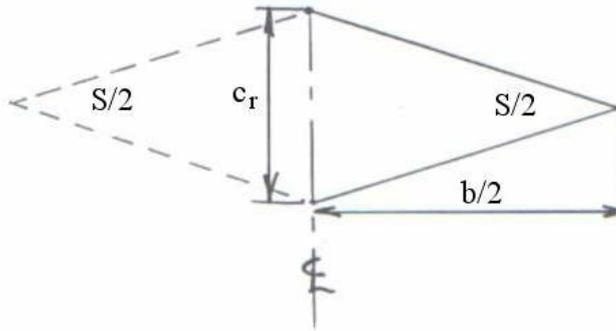
To convert into a load distribution, put  $w_y$  (N/m) in place of  $c_y$  and put  $L$  (N) in place of  $S$ .

$$\therefore w_y = \frac{4L}{\pi b} \sqrt{1 - \left(\frac{2y}{b}\right)^2} \quad (8.1)$$



**Figure 7.3 Elliptical distribution**

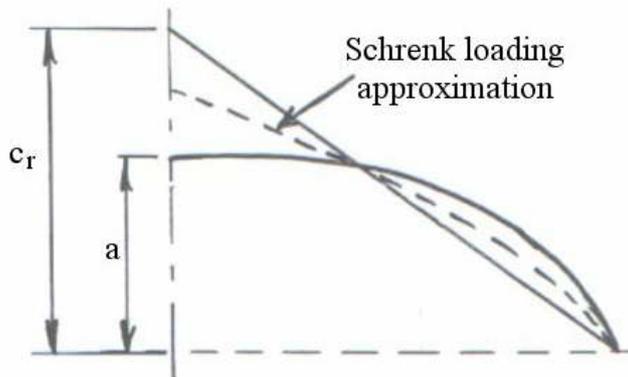
### **Example 8.1 – Triangular or Delta Wing**



**Figure 8.4 Triangular wing**

Area of wing (semi-span) =  $S/2 = \frac{1}{2} \times c_r \times b/2 = (c_r b) / 4$

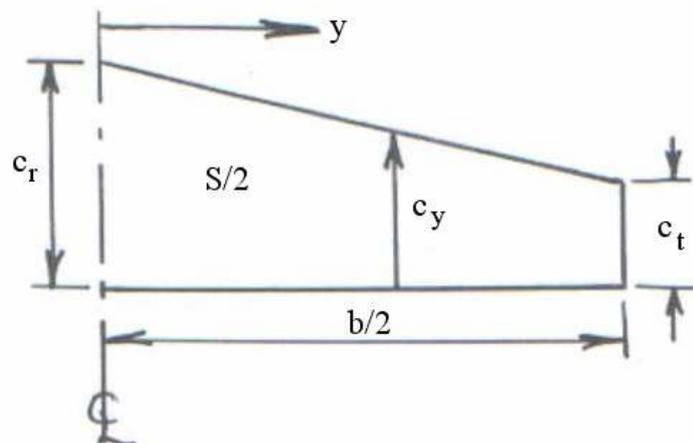
$$\therefore c_r = 2 S / b$$



**Figure 8.5 Triangular wing – Schrenk distribution**

For the semi-ellipse,  $a = 4 S / (\pi b)$ , so the overall distribution is the average of the elliptical and triangular distributions.

**Example 8.2 – Trapezoidal Wing**



**Figure 8.6 Trapezoidal wing**

Let taper ratio =  $\lambda = c_t / c_r$

$$\begin{aligned} \text{Area} = S/2 &= [(c_r + c_t) / 2](b / 2) = (c_r/c_r + c_t/c_r)(b c_r / 4) \\ &= (1 + \lambda) b c_r / 4 \end{aligned}$$

$$\therefore c_r = 2 S / [(1 + \lambda) b]$$

$$\text{Now } c_y = c_r - [y / (b / 2)](c_r - c_t) = c_r [1 + (2 y / b)(\lambda - 1)]$$

$$\text{i.e. } c_y = 2 S / [(1 + \lambda) b] \cdot [1 + (2 y / b)(\lambda - 1)]$$

$$\text{or } w_y = \frac{2L}{(1 + \lambda)b} \left[ 1 + \left( \frac{2y}{b} \right) (\lambda - 1) \right] \quad (8.2)$$

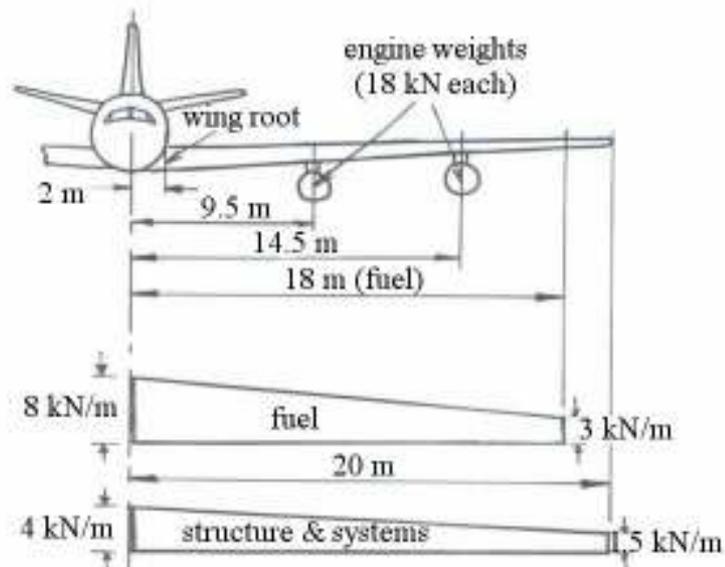
### **Example 8.3 – Aircraft – Boeing 707**

An approximation of the wing span-wise aerodynamic loading is given by eq. (8.2):

$$w_y = \frac{2L}{(1 + \lambda)b} \left[ 1 + \left( \frac{2y}{b} \right) (\lambda - 1) \right]$$

where  $L$  = total wing lift = 750 kN,  $b$  = wing span = 40 m,  $y$  = outboard distance from a/c centre-line

- (a) Calculate and plot the aerodynamic loading at the span-wise stations  $y = 0, 5, 9.5, 14.5, 18$  & 20 m, (b) Calculate and plot the total weight distribution, (c) construct the SF and BM diagrams, (d) calculate the SF & BM values at wing root.

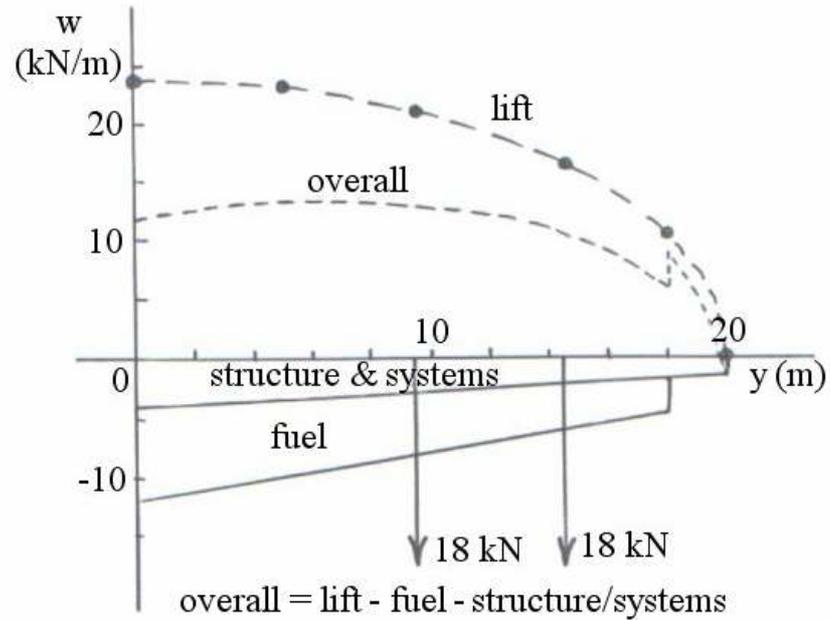


**Figure 8.7 Boeing 707 dimensions**

### **Solution**

- (a) For aerodynamic loading:

|          |       |       |       |       |       |    |
|----------|-------|-------|-------|-------|-------|----|
| y (m)    | 0     | 5     | 9.5   | 14.5  | 18    | 20 |
| w (kN/m) | 23.87 | 23.12 | 21.01 | 16.44 | 10.41 | 0  |

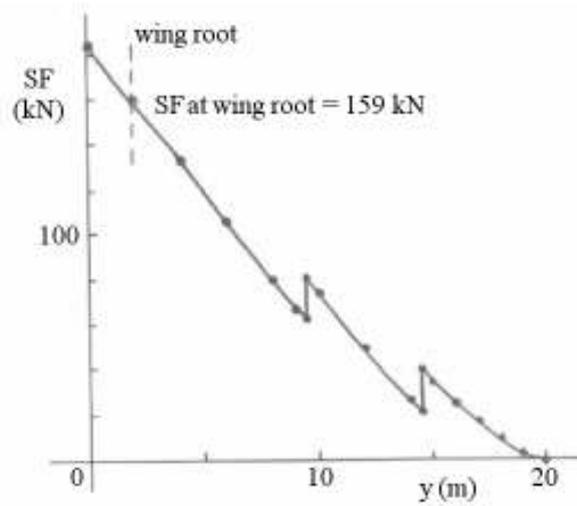


**Figure 8.8 Boeing 707 load distributions**

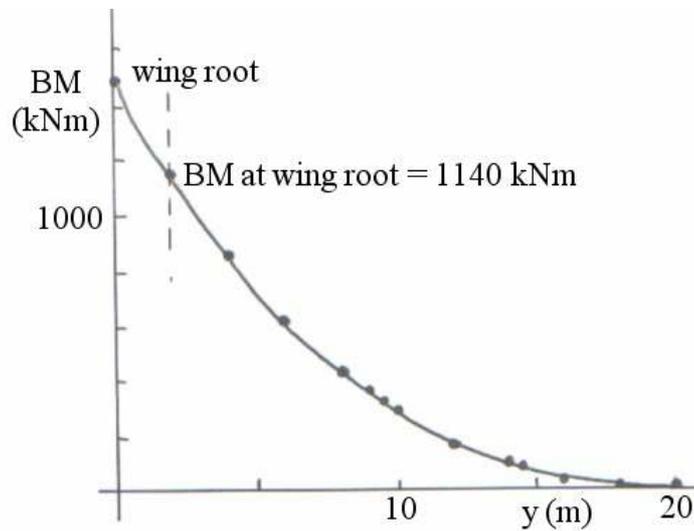
(b, c & d)

| y (m) | $\Delta SF$ (kN) | SF (kN) | $\Delta BM$ (kNm) | BM (kNm) |
|-------|------------------|---------|-------------------|----------|
| 20    | 0                | 0       | 0                 | 0        |
| 19    | 1.6              | 1.6     | -                 | -        |
| 18    | 6.5              | 8.1     | 3                 | 3        |
| 17    | 6.5              | 14.6    | -                 | -        |
| 16    | 8                | 22.6    | 32                | 35       |
| 15    | 9.3              | 31.9    | -                 | -        |
| 14.5  | 5                | 36.9    | 46.5              | 81.5     |
| 14.5  | -18              | 18.9    | -                 | -        |
| 14    | 5.4              | 24.3    | 11.5              | 93       |
| 12    | 22.8             | 47.1    | 72                | 165      |
| 10    | 25.2             | 72.3    | 122               | 287      |
| 9.5   | 6.4              | -18     | 60.7              | -        |
| 9.5   | -18              | 60.7    | -                 | -        |
| 9     | 6.4              | 67.1    | 31.5              | 356.5    |

|   |      |       |     |        |
|---|------|-------|-----|--------|
| 8 | 13   | 80.1  | 73  | 429.5  |
| 6 | 26.4 | 106.5 | 184 | 613.5  |
| 4 | 26.4 | 132.9 | 240 | 853.5  |
| 2 | 26   | 158.9 | 292 | 1145.5 |
| 0 | 25.4 | 184.3 | 340 | 1485.5 |



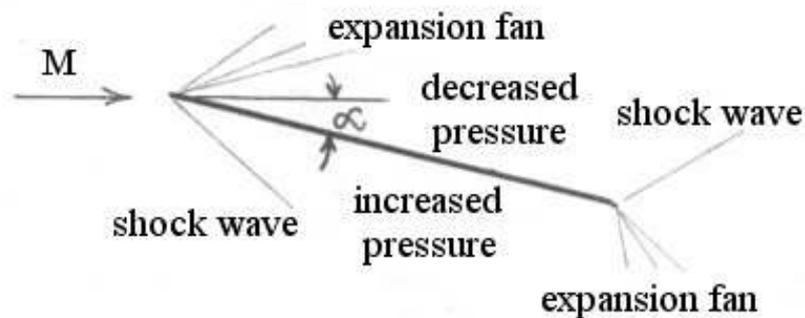
**Figure 8.9a Boeing 707 SF diagram**



**Figure 8.9a Boeing 707 BM diagram**

## 9. Supersonic Wing Aerodynamic Loads

### 9.1 Supersonic Flat Plate



**Figure 9.1 Supersonic flat plate**

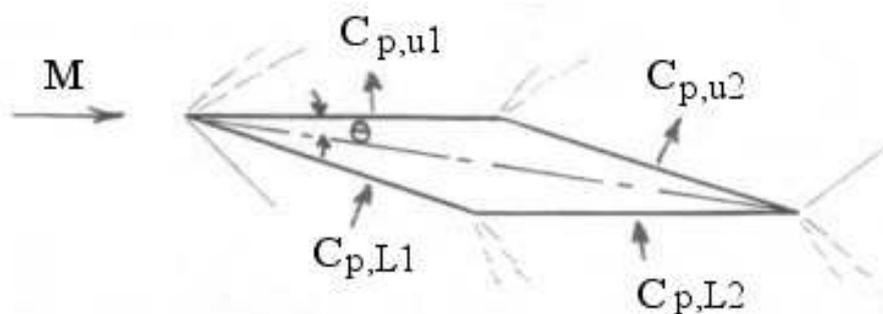
From *Ackeret theory*:

$$\text{On upper surface, } C_p = -2 \alpha / \sqrt{M^2 - 1}$$

$$\text{On lower surface, } C_p = 2 \alpha / \sqrt{M^2 - 1}$$

$$\text{Overall } C_N = 4 \alpha / \sqrt{M^2 - 1} \quad (9.1)$$

### 9.2 Supersonic Symmetrical Double Wedge



**Figure 9.2 Supersonic symmetrical double wedge**

From *Ackeret theory*:

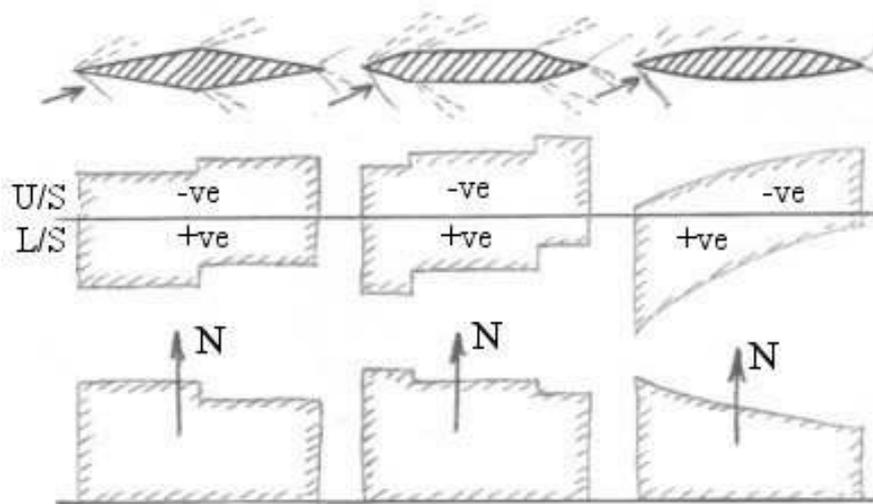
$$\text{Upper surface } C_p = -2 (\alpha - \theta) / \sqrt{M^2 - 1} \text{ \& } -2 (\alpha + \theta) / \sqrt{M^2 - 1}$$

$$\text{Lower surface } C_p = 2 (\alpha + \theta) / \sqrt{M^2 - 1} \text{ \& } 2 (\alpha - \theta) / \sqrt{M^2 - 1}$$

$$\text{Overall } C_N = 4 \alpha / \sqrt{M^2 - 1} \text{ (same as for flat plate)} \quad (9.2)$$

### 9.3 General 2-D Supersonic Aerofoil Sections

The pressure and load distributions for the main supersonic wing sections used in practice are given in Figure 8.3.



**Figure 9.3 Pressure & load distributions for general 2-d supersonic sections**

Note that in all cases:

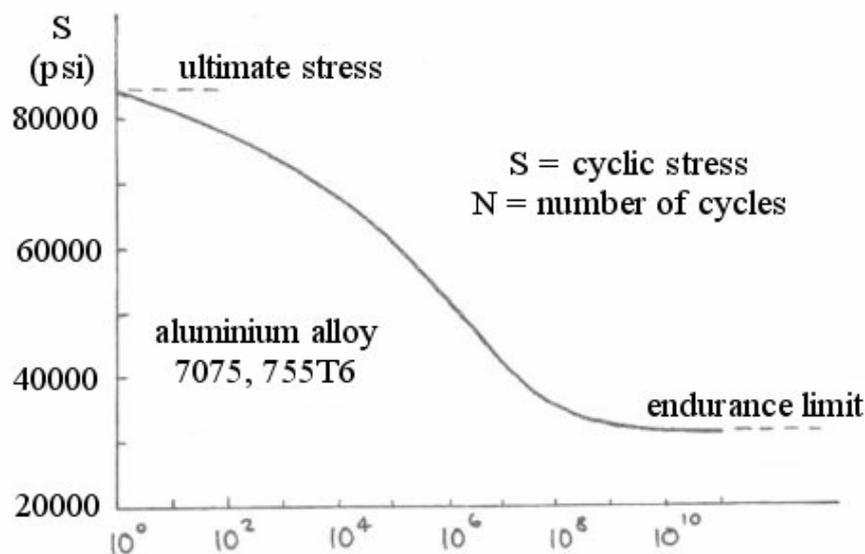
- The centre of Pressure is always slightly forward of the section's mid-point.
- $C_N = 4 \alpha / \sqrt{M^2 - 1}$

## **10. Metal Fatigue on Rotary Wing Aircraft**

Most RW aircraft have parts that must be scrapped after a certain number of flight hours, e.g. typically the main rotor blades at 1200 hrs, tail rotor blades at 1800 hrs, main transmission at 2300 hrs, etc. These relatively severe limitations primarily due to *metal fatigue* due to applied variable load spectrum.

A repeated application of the load somewhere between the *ultimate* and the *endurance* limit levels will eventually damage the bonds between the material grains and thus cause cracks to form. Any further repetition of the variable load will cause the crack to propagate further, thus weakening structure and eventually causing failure.

The S-N diagram of a material or component is used to design against metal fatigue by the setting of appropriate component life-time limits. These diagrams are constructed from tests performed on individual material samples or on individual components, especially if of complex shape. Experimental scatter on the diagrams is reduced by increasing number of specimens tested, though this has the drawback of increasing the cost.



**Figure 10.1 Typical aluminium alloy S-N diagram**

To perform a fatigue life calculation, the structural engineer also requires the *loading spectrum*, comprising:

- The proportion of the helicopter life spent performing certain manoeuvres (*mission profile*).
- The magnitudes of the loads during those manoeuvres.

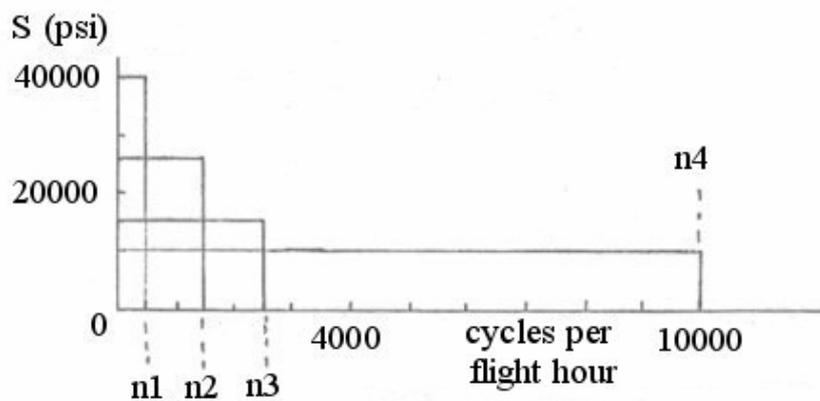
The mission profile loads may be provided by the engineer, customer and/or certification authority, e.g. typically:

- Warm-up (4%), hover (8%), level flight at transitional speed (6%), climb at take-off power (8%), level flight at 60% cruise speed (20%), level flight at cruise speed (25%), level flight at maximum speed (9%), tight turn at cruise speed (15%), approach to landing (5%).

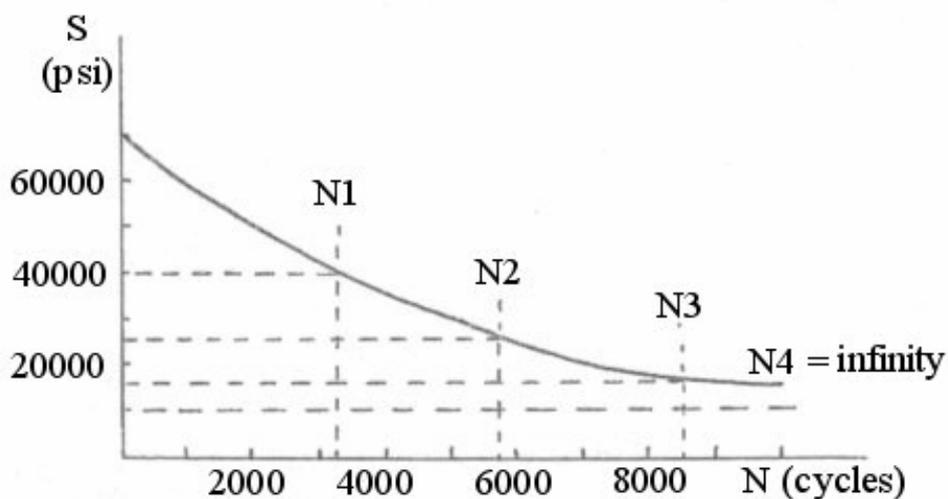
The load magnitudes will be obtained from a variety of sources, such as previous operational experience, flight tests, wind tunnel models and computer simulation studies.

**Example 10.1**

Calculate the component fatigue life based upon following loading spectrum (Figure 10.1) and S-N diagrams (Figure 10.3):



**Figure 10.2 Loading spectrum for Example 10.1**



**Figure 10.3 S-N diagram for Example 10.1**

**Solution**

From the S-N diagram:

At  $S = 40,000$  psi,  $n_1 = 400$  cycles/hr,  $N_1 = 3200$ ,  $n_1/N_1 = 0.125$  /hr

At  $S = 25,000$  psi,  $n_2 = 1500$  cycles/hr,  $N_2 = 5750$ ,  $n_2/N_2 = 0.261$  /hr

At  $S = 15,000$  psi,  $n_3 = 2500$  cycles/hr,  $N_3 = 8550$ ,  $n_3/N_3 = 0.292$  /hr

At  $S = 10,000$  psi,  $n_4 = 10000$  cycles/hr,  $N_4 = \infty$ ,  $n_4/N_4 = 0$  /hr

$$\therefore \Sigma(n/N) = 0.678 \text{ /hr}$$

And fatigue life =  $1 / 0.678 = \mathbf{1.47 \text{ hrs}}$

This is clearly a very low value and would need to be improved significantly. Some possible methods for increasing component fatigue include:

- Make the component bigger & stronger (but then also heavier).
- Eliminate stress concentration areas where cracks start.
- Change the material to one with better fatigue characteristics.
- Shot peening (this process locks compressive stress into surface & makes cracks less likely to start).
- Test more specimens if the S-N curve has significant scatter.

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