

# AE 2020: Low Speed Aerodynamics

## I. Introductory Remarks

Read chapter 1 of Fundamentals of  
Aerodynamics by John D. Anderson

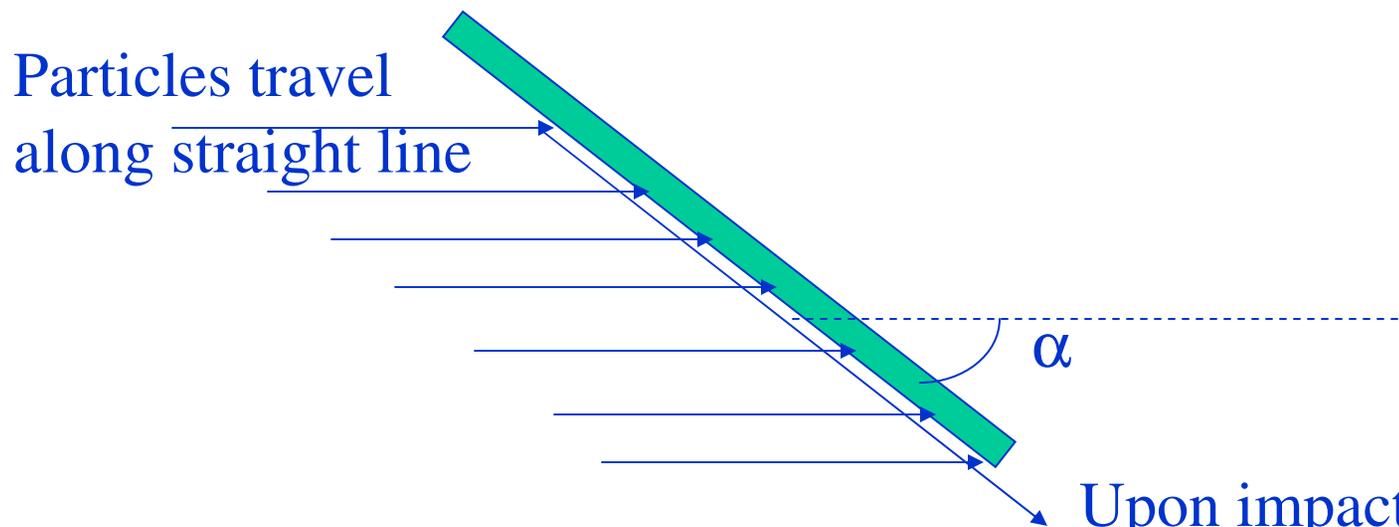
# Text Book

- Anderson, Fundamentals of Aerodynamics, 4th Edition, McGraw-Hill, Inc.
- This text book will be useful in other courses as well.
- Supplementary typed notes are available at: <http://www.ae.gatech.edu/~lsankar/AE2020>
- This web site contains sample homework assignments and exams from the past.
- New homework will be posted from time to time. Watch for deadlines.

# History of Fluid Mechanics

- Compared to many fields (e.g. Electronics), fluid mechanics is an old, well established field.
- It all began with Newton, who tried to apply the theory of solid particle dynamics to fluids in 1687.

# Newton's Theory of Fluid Mechanics

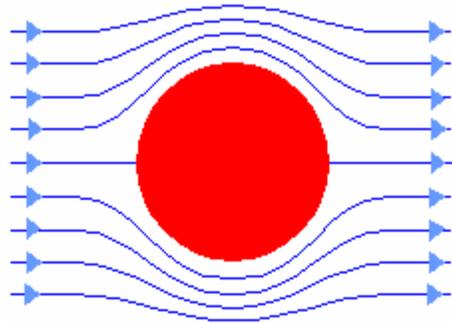


After some algebra, he obtained:

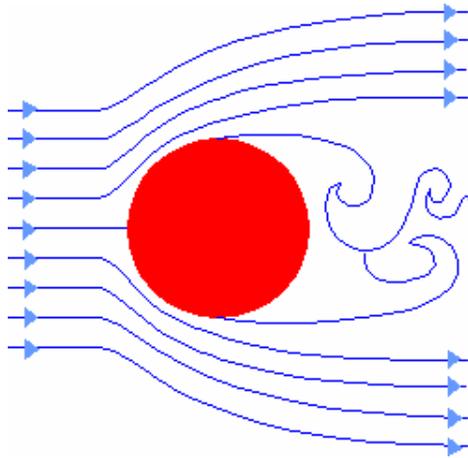
Lift is proportional to  $\sin^2\alpha$ .

Upon impact, the particles lose all the momentum in a direction normal to the surface, and slide off like jello in a tangential direction.

# Why did Newton's model fail?



*Inviscid (ideal) flow*



*Viscous flow*

- He incorrectly assumed that the fluid particles travel on straight lines until they hit the body.
- The body sends signals in the form of acoustic waves to the particles that it is in the way. The particles deflect away from the body.
- Newton also ignored the collisions between particles, which alter their paths.

# History, continued...

- In 1777, d'Alembert, a French engineer tested Newton's theory on plane surfaces immersed in water. He found Newton to be wrong. The lift was proportional to  $\sin\alpha$ , not  $\sin^2\alpha$ !
- In 1781, a Swiss scientist called Euler theoretically showed that the lift force was proportional to  $\sin\alpha$ .
- From such false starts, and half-steps, the field began to grow. It culminated at Kitty Hawk on December 7, 1903.

# Aerodynamics is essential for External Flow Applications

- Airfoil and wing design
- Airfoil and wing analysis
- Analysis and design of compressors, turbines, fans, helicopter rotors, propellers
- Analysis and design of wind turbines
- Analysis and design of automobile, ship and sail shapes

# Aerodynamics is important for internal flow applications as well.

- Design and analysis of channels, ducts, pipes
- Design and analysis of heat transfer devices (heaters, air conditioners, cooling fans, vents...).
- Biomedical Applications - heart pumps, flow through arteries, valves, etc.

# Aerodynamics is a Broad Field

- The scale may vary from a few millimeters to hundreds of meters.
- The speed may vary from a few millimeter/s to hundreds of meters per second.
- A single course can not cover all the aspects of aerodynamics.

# List of Aerodynamics Courses

- AE 2020 - This course. Deals with low speed viscous and non-viscous (or inviscid) flow.
- AE 3450 - Deals with thermodynamics and 1-D compressible flow.
- AE 3021 - Deals with multi-dimensional compressible flow.
- AE 3051 - Fundamentals of aerodynamic measurements in a laboratory setting.

# Units: British or Metric?

- Answer: You need to be familiar with both systems of units.
- You will do numerical examples dealing with both systems of units.
- Across the Atlantic (and the Pacific) the International System of Units is more prevalent.

# Concept of Continuum

- Air is made of particles - molecules.
- We are tempted to treat each of these particles individually, and study its motion as Newton did.
- This approach fails when there are millions of particles to deal with, which randomly collide with each other millions of times per second.
- When we deal with such large number of particles, we can describe their characteristics only in terms of statistical averages.
- In other words, we treat the fluid as a continuous medium, which has certain average properties at any point in space and time.

# Mean Free Path

- Mean free path is defined as the average distance that an air molecule will travel before it collides with another particle.
- In gases, under normal conditions (e.g. low altitudes), the mean free path is very low (of the order of microns) compared to the characteristic dimensions of the vehicle.
  - Knudsen Number = Mean free path/Vehicle Dimension
  - Knudsen number is thus very low at low altitudes
- Concept of continuum works well under such conditions.
- At high altitudes, under rarefied conditions, this concept fails. The molecules must be treated as individual particles.
- See section 1.10.1 in the text for a further discussion of mean free path and the concept of continuum.

# Properties of the Flow

- Density:  $\rho$  (“Rho”)  
Mass of the fluid per unit volume of space as the volume shrinks to zero.
- Velocity:  $\vec{V}$   
Velocity of a fluid particle (i.e. a large collection of molecules treated as a continuum) is a vector. It has three components (u,v,w) in the three directions.
- Temperature T: A measure of the kinetic energy associated with the random motion of the molecules that form the continuous matter.

# Properties of the Flow, Continued..

## Pressure

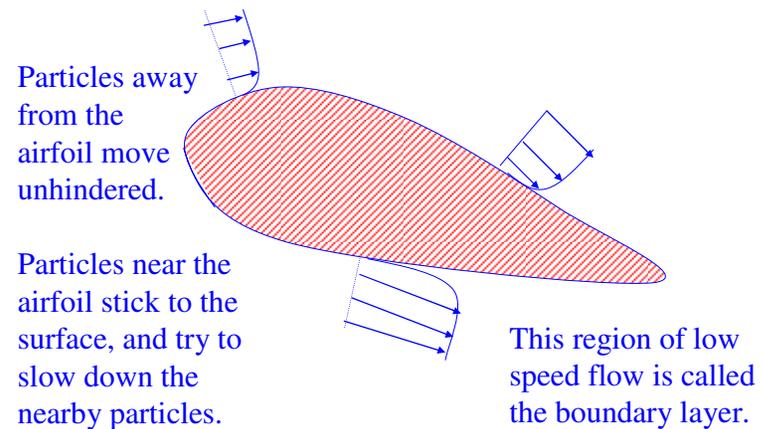
- Pressure is defined as the normal force exerted by the continuous matter on a plane placed in the fluid, per unit area of the plane.
- Pressure always acts normal to the plane.
  - For example, when we say that the atmospheric pressure is 14.7 psi at sea level, we are saying that there a force of 14.7 lbf per square inch acting on any surface exposed to the fluid (e.g. our skin) in a direction normal to the surface.
- Gauge pressure is  $p - p_{\text{atmosphere}}$ 
  - It is the difference between pressure at a point and the atmospheric pressure.
  - Many instruments (gauges) measure the pressure difference.
  - For instance, a pressure gauge measures the difference between the pressure inside the tire and outside the tire.

# Properties of Flow, Continued..

## Viscosity $\mu$

- Air is viscous, sticky. Most fluids are viscous as well.
- When fluid moves past a stationary surface (e.g. over an airfoil) this stickiness causes the fluid to exert a force in the direction of the motion.
- Newton found that the tangential stress associated with this motion (stress = force per unit area) is proportional to viscosity.
- Viscosity  $\mu$  is the property of a fluid, not flow. In other words, we can look it up from a table of fluid properties, without having to compute the flow.
- In liquids, it is caused by intermolecular forces.
  - When a liquid is heated, the molecules move apart, and the intermolecular forces decrease.
  - Thus viscosity of a liquid decreases with temperature.
- In gases, viscosity is associated with exchange of momentum by random collision among molecules.
  - A slow moving molecule collides with a faster moving molecule and slows it down.
  - When this happens over millions of molecules (think continuum) the entire flow slows down.
  - In gases, as temperature increases, the energy of the molecules associated with this random motion increases.
  - Thus viscosity increases in gases with temperature.

### Skin Friction



A tug of war results - airfoil is dragged back with the flow.

# Properties of Flow, continued

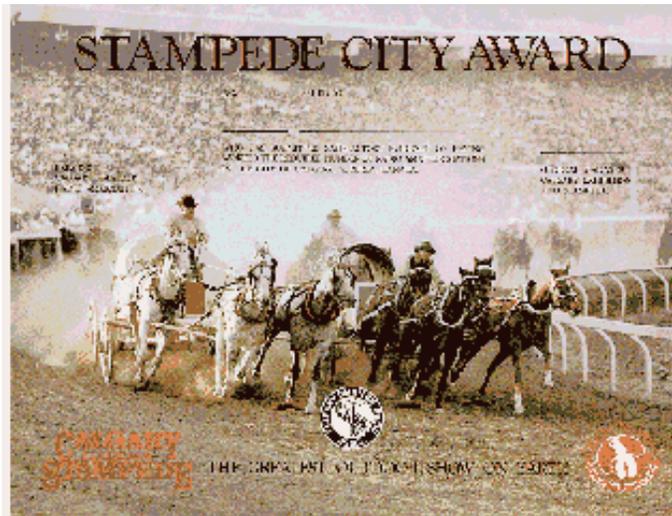
## Speed of Sound $a$ and Mach Number

- In AE 3450, you will learn that the speed of sound is proportional to the square root of temperature.
- $a = \text{square root of } (\gamma RT)$  where
  - $\gamma = \text{Ratio of specific heats, } 7/5 \text{ for air}$
  - $R = \text{gas constant}$
  - $T = \text{temperature in Rankine or Kelvin}$
- $\text{Mach number} = \text{Flow speed} / \text{speed of sound}$
- Speed is a scalar, velocity is a vector.
- The velocity has three components  $(u, v, w)$  along  $x, y,$  and  $z$  directions.
- Flow speed is thus the magnitude of the velocity vector.

# Incompressible Flow

- Air is a compressible fluid.
- Its density WILL change if temperature changes, or if some external force is applied.
  - Example: A child squeezing a balloon
- A flow is said to be incompressible if there are no changes in density attributable to (or caused by) the velocity or speed of the flow.
- Theory and observations in wind tunnels suggest that most flows may be treated as incompressible (i.e. constant density) until the Mach number is sufficiently high ( $>0.4$  or so.)

# What has flow speed got to do with compressibility?

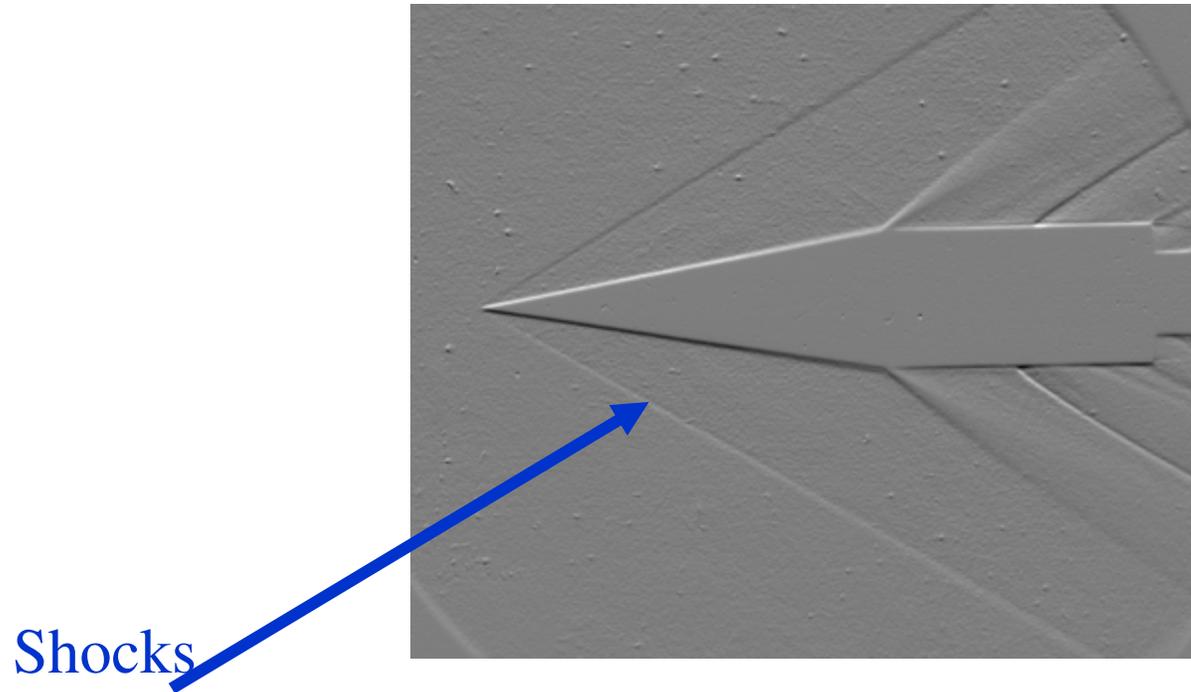


Fluid particles send out signals in the form of acoustic waves to the surrounding fluid, indicating their motion.

If there is sufficient time for the sound waves to travel before the fluid particle arrives, the fluid particles downstream will “hear” the message and clear out.

Otherwise, there will be a crush (compression), or even a stampede (shock wave).

Shocks form when the acoustic waves generated by the air particles in front of the body can not outrun the body.



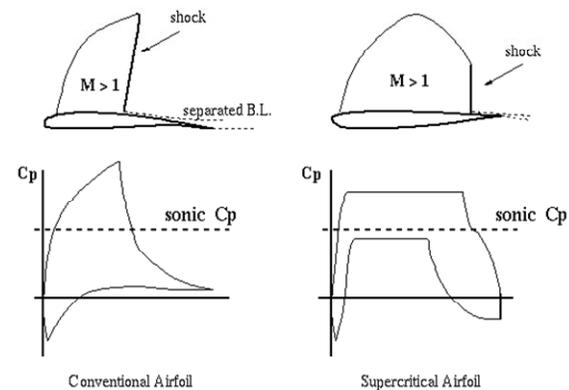
Shocks

You will study compressible flows in AE3450 and 3021.

# Mach Number Regime

- Read 1.10.3 and 1.10.4 in text.
- A flow is subsonic if  $M < 1$
- In low subsonic flows ( $M < 0.3$  or so) there are no appreciable differences in density attributable to the flow velocity. We call such flows incompressible flows. This course exclusively deals with such flows.
- Above  $M > 0.3$ , but below  $M < 1$ , the flow is called subsonic.
- A flow is transonic if there are large regions of subsonic and supersonic flow, both..
  - The flow over a 757 wing has regions where  $M < 1$  and regions where  $M > 1$ .
- A flow is considered supersonic if  $M > 1$  over most of the flow region.
- We will learn more about subsonic, transonic, and supersonic flows in AE 3450 and AE 3021.

## Conventional vs. Supercritical Airfoils



# Properties of Flow: Summary

- The most important properties of a flow at any point  $(x,y,z)$  at any time  $t$ , in any fluid application are:  $\rho$ ,  $p$ ,  $T$ , and velocity  $V$ . Viscosity  $\mu(T)$  is a function of temperature, and alters the flow properties.
- Fluid Mechanics and Aerodynamics give us the tools we need for predicting these properties.

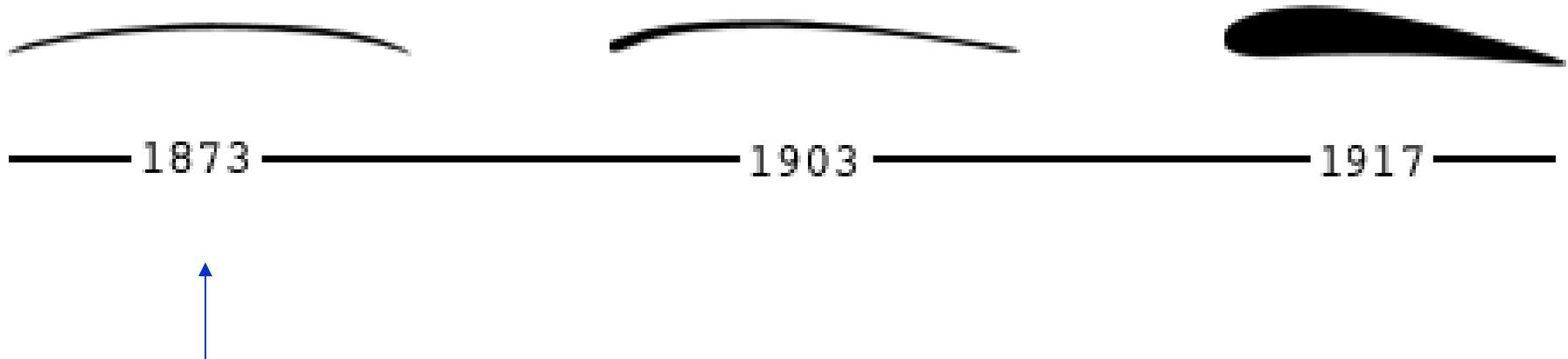
# Topics To be Studied

- Airfoil Nomenclature
- Lift and Drag forces
- Lift, Drag and Pressure Coefficients

# Uses of Airfoils

- Wings
- Propellers and Turbofans
- Helicopter Rotors
- Compressors and Turbines
- Hydrofoils (wing-like devices which can lift up a boat above waterline)
- Wind Turbines

# Evolution of Airfoils

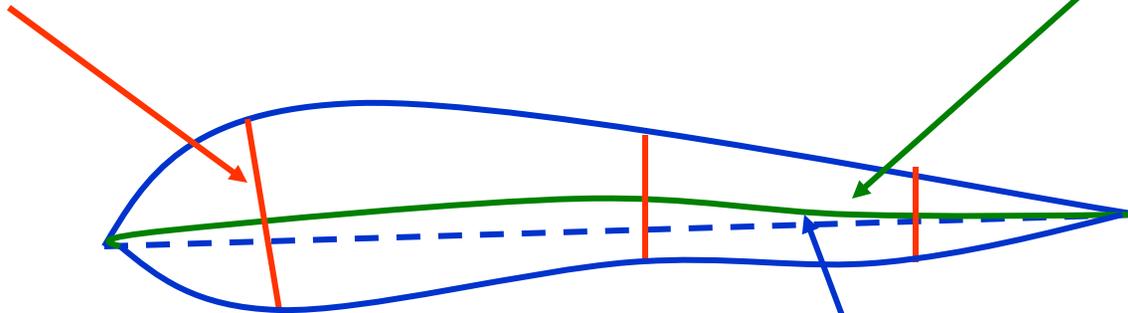


Early Designs - Designers mistakenly believed that these airfoils with sharp leading edges will have low drag. In practice, they stalled quickly, and generated considerable drag.

# Airfoil

Equal amounts of thickness is added to camber in a direction normal to the camber line.

Camber Line

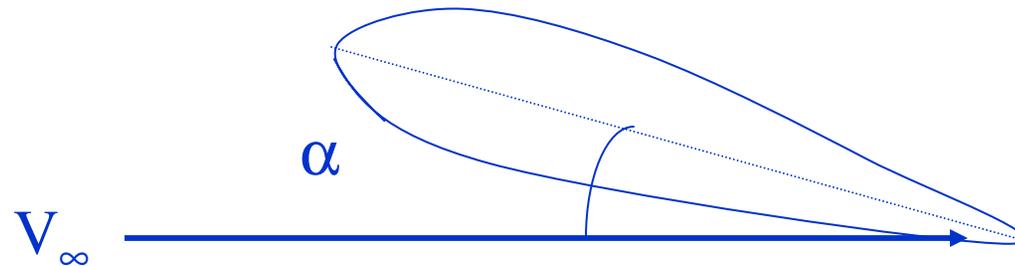


Chord Line

# An Airfoil is Defined as a superposition of

- Chord Line
- Camber line drawn with respect to the chord line.
- Thickness Distribution which is added to the camber line, normal to the camber line.
- Symmetric airfoils have no camber.

# Angle of Attack



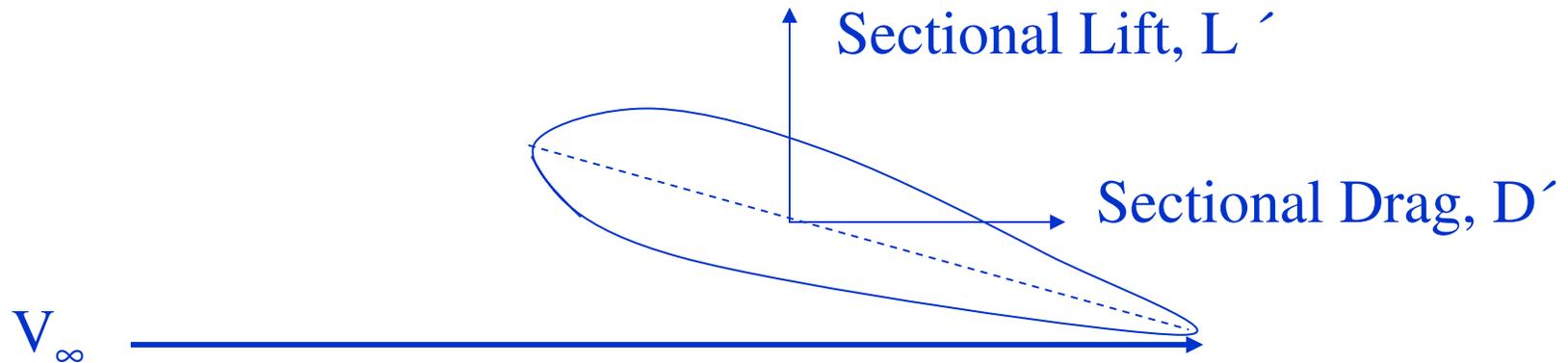
Angle of attack is defined as the angle between the freestream and the chord line. It is given the symbol  $\alpha$ .

Because modern wings have a built-in twist distribution, the angle of attack will change from root to tip.

The root will, in general, have a high angle of attack.

The tip will, in general, have a low (or even a negative)  $\alpha$ .

# Lift and Drag Forces acting on a Wing Section



The component of aerodynamic forces normal to the freestream, per unit length of span (e.g. per foot of wing span), is called the sectional lift force, and is given the symbol  $L'$ .

The component of aerodynamic forces along the freestream, per unit length of span (e.g. per foot of wing span), is called the sectional drag force, and is given the symbol  $D'$ .

# Sectional Lift and Drag Coefficients

- The sectional lift coefficient  $C_l$  is defined

as: 
$$C_l = \frac{L'}{\frac{1}{2} \rho V_\infty^2 c}$$

- Here  $c$  is the airfoil chord, i.e. distance between the leading edge and trailing edge, measured along the chordline.

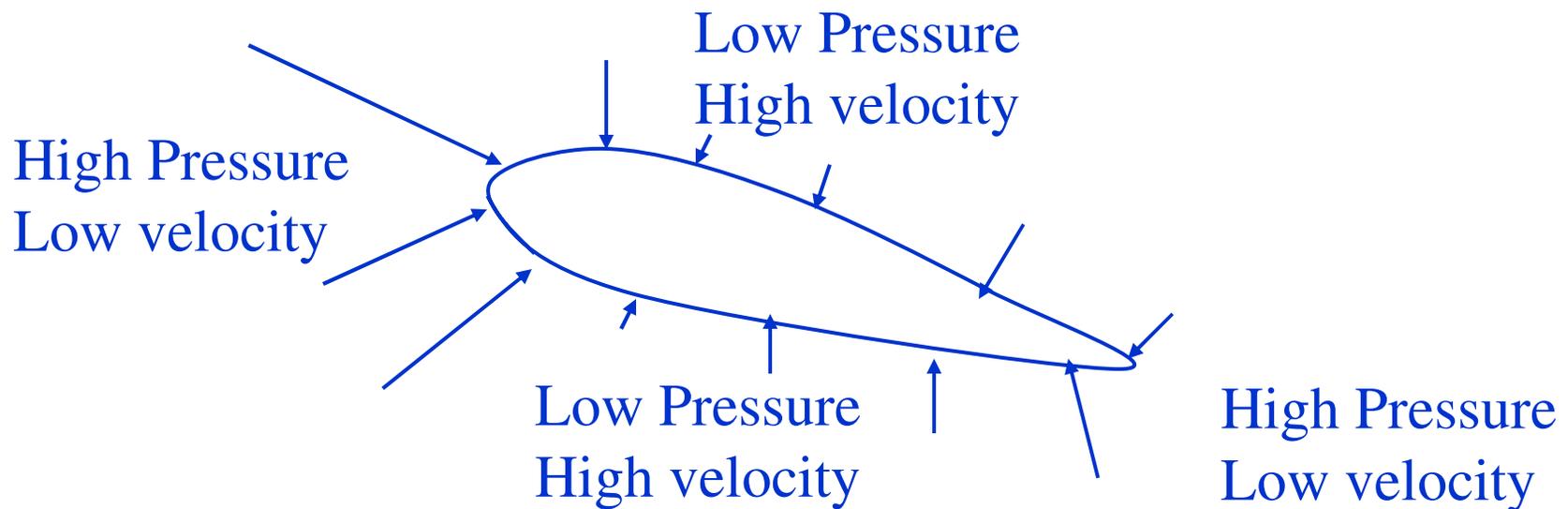
- The sectional drag force coefficient  $C_d$  is

likewise defined as: 
$$C_d = \frac{D'}{\frac{1}{2} \rho V_\infty^2 c}$$

## Note that...

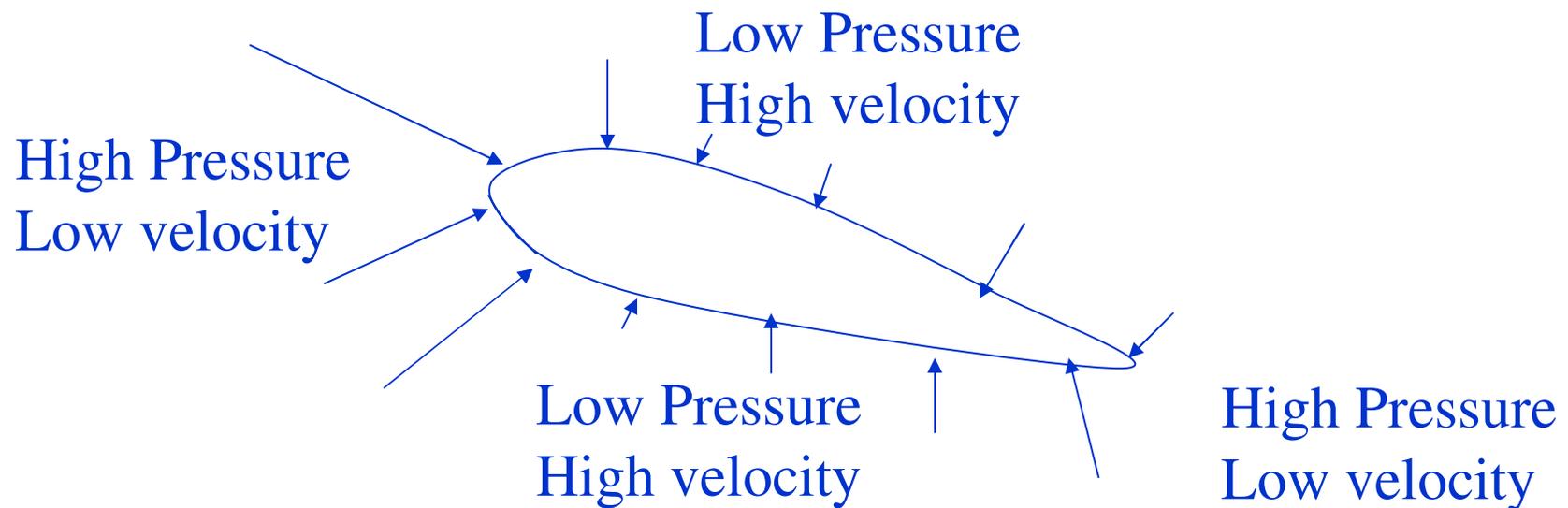
- When we are taking about an entire wing we use  $L$ ,  $D$ ,  $C_L$  and  $C_D$  to denote the forces and coefficients.
- When we are dealing with just a section of the wing, we call the forces acting on that section (per unit span)  $L'$  and  $D'$ , and the coefficients  $C_l$  and  $C_d$

# Pressure Forces acting on the Airfoil



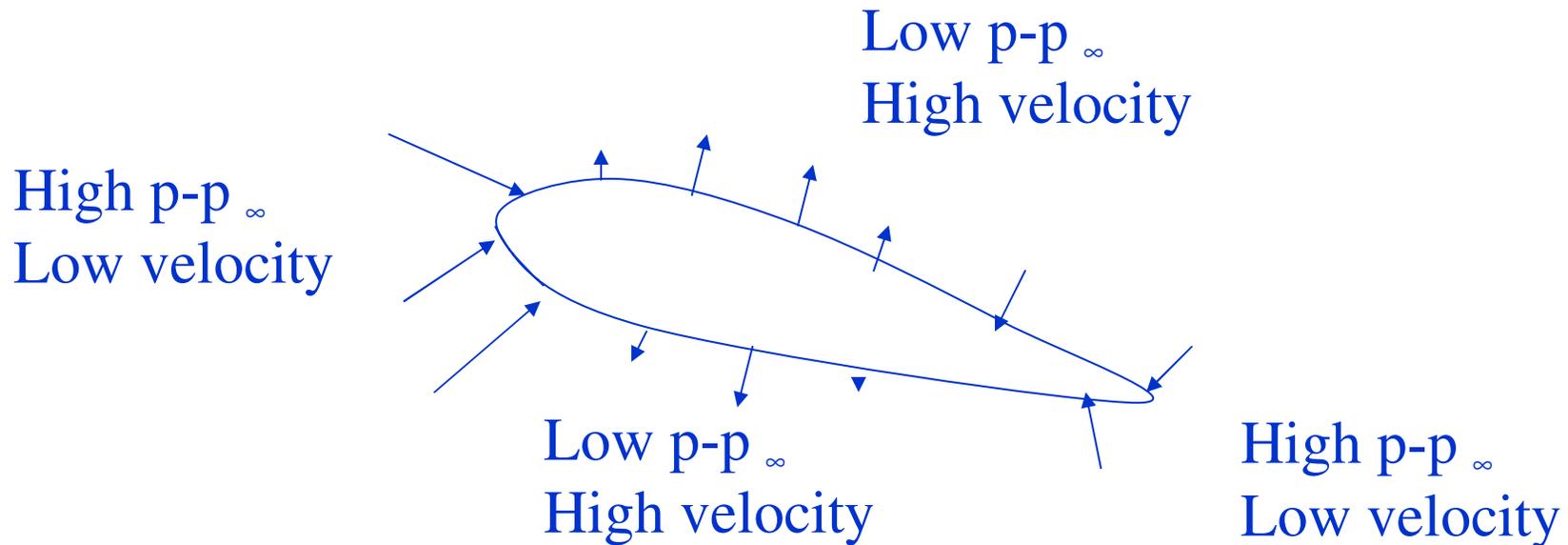
Bernoulli's equation says where pressure is high, velocity will be low and vice versa.

# Pressure Forces acting on the Airfoil



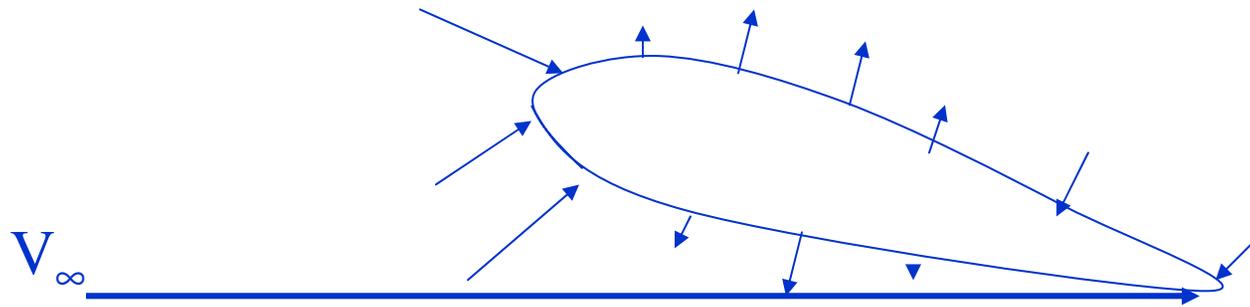
Bernoulli's equation says where pressure is high, velocity will be low and vice versa.

Subtract off atmospheric Pressure  $p_\infty$  everywhere.  
Resulting Pressure Forces acting on the Airfoil



The quantity  $p-p_\infty$  is called the gauge pressure. It will be negative over portions of the airfoil, especially the upper surface. This is because velocity there is high and the pressures can fall below atmospheric pressure.

# Relationship between $L'$ and $p$



$L'$  = Force normal to the wind direction

= Forces acting on the lower side - Force on upper side

$$= \int_{\text{Leading Edge}}^{\text{Trailing Edge}} p_{\text{lower side}} dx - \int_{\text{Leading Edge}}^{\text{Trailing Edge}} p_{\text{upper side}} dx$$

$$= \int_{\text{Leading Edge}}^{\text{Trailing Edge}} (p_{\text{lower side}} - p_{\text{upper side}}) dx$$

# Relationship between $L'$ and $p$ (Continued)

$$\begin{aligned}
 L' &= \int_{\text{Leading Edge}}^{\text{Trailing Edge}} (p_{\text{lower side}} - p_{\text{upper side}}) dx \\
 &= \int_{\text{Leading Edge}}^{\text{Trailing Edge}} ([p_{\text{lower side}} - p_{\infty}] - [p_{\text{upper side}} - p_{\infty}]) dx
 \end{aligned}$$

Divide left and right sides by  $\frac{1}{2} \rho V_{\infty}^2 c$

We get:

$$\frac{L'}{\frac{1}{2} \rho V_{\infty}^2 c} = \int_{\text{Leading Edge}}^{\text{Trailing Edge}} \left( \frac{p_{\text{lower}} - p_{\infty}}{\frac{1}{2} \rho V_{\infty}^2} - \frac{p_{\text{upper}} - p_{\infty}}{\frac{1}{2} \rho V_{\infty}^2} \right) d \frac{x}{c}$$

# Pressure Coefficient $C_p$

From the previous slide,

$$\frac{L'}{\frac{1}{2} \rho V_\infty^2 c} = \int_{\text{Leading Edge}}^{\text{Trailing Edge}} \left( \frac{p_{\text{lower}} - p_\infty}{\frac{1}{2} \rho V_\infty^2} - \frac{p_{\text{upper}} - p_\infty}{\frac{1}{2} \rho V_\infty^2} \right) d \frac{x}{c}$$

The left side was previously defined as the sectional lift coefficient  $C_l$ .

The pressure coefficient is defined as:

$$C_p = \frac{p - p_\infty}{\frac{1}{2} \rho V_\infty^2}$$

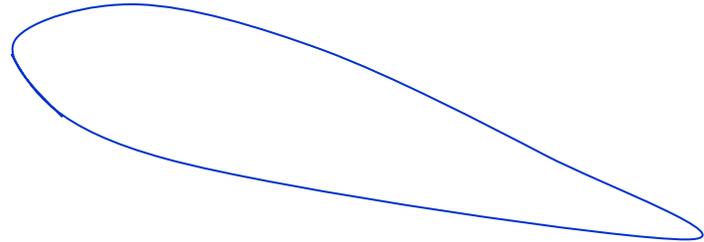
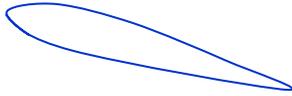
Thus,

$$C_l = \int_{\text{Leading edge}}^{\text{Trailing edge}} (C_{p,\text{lower}} - C_{p,\text{upper}}) d \frac{x}{c}$$

# Why use $C_l$ , $C_p$ etc.?

- Why do we use “abstract” quantities such as  $C_l$  and  $C_p$ ?
- Why not directly use physically meaningful quantities such as Lift force, lift per unit span , pressure etc.?

# The Importance of Non-Dimensional Forms



Consider two geometrically similar airfoils.

One is small, used in a wind tunnel.

The other is large, used on an actual wing.

These will operate in different environments - density, velocity.

This is because high altitude conditions are not easily reproduced in wind tunnels.

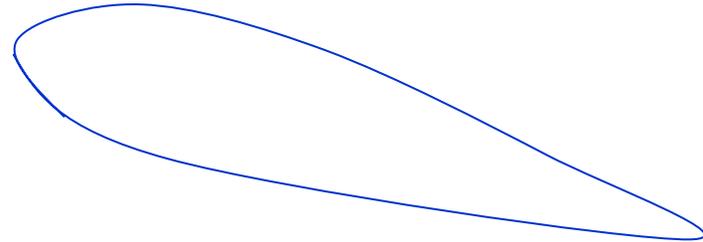
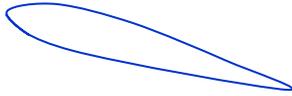
They will therefore have different Lift forces and pressure fields.

They will have identical  $C_l$ ,  $C_d$  and  $C_p$

- if they are geometrically alike

- operate at identical angle of attack, Mach number and Reynolds number

# The Importance of Non-Dimensional Forms



In other words,

a small airfoil , tested in a wind tunnel.

And a large airfoil, used on an actual wing

will have identical non-dimensional coefficients  $C_l$  ,  $C_d$  and  $C_p$

- if they are geometrically alike

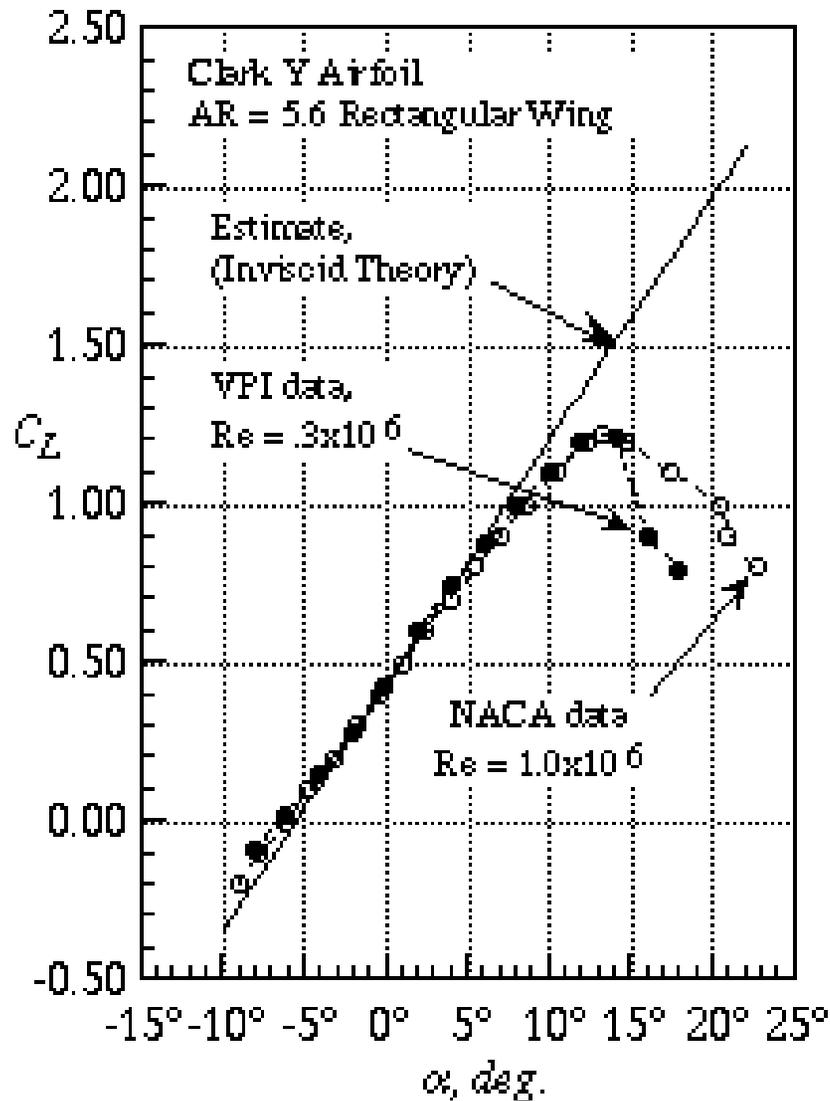
- operate at identical angle of attack, Mach number and Reynolds number.

This allows designers (and engineers) to build and test small scale models, and extrapolate qualitative features, but also quantitative information, from a small scale model to a full size configuration.

## Tables 5.1 from White

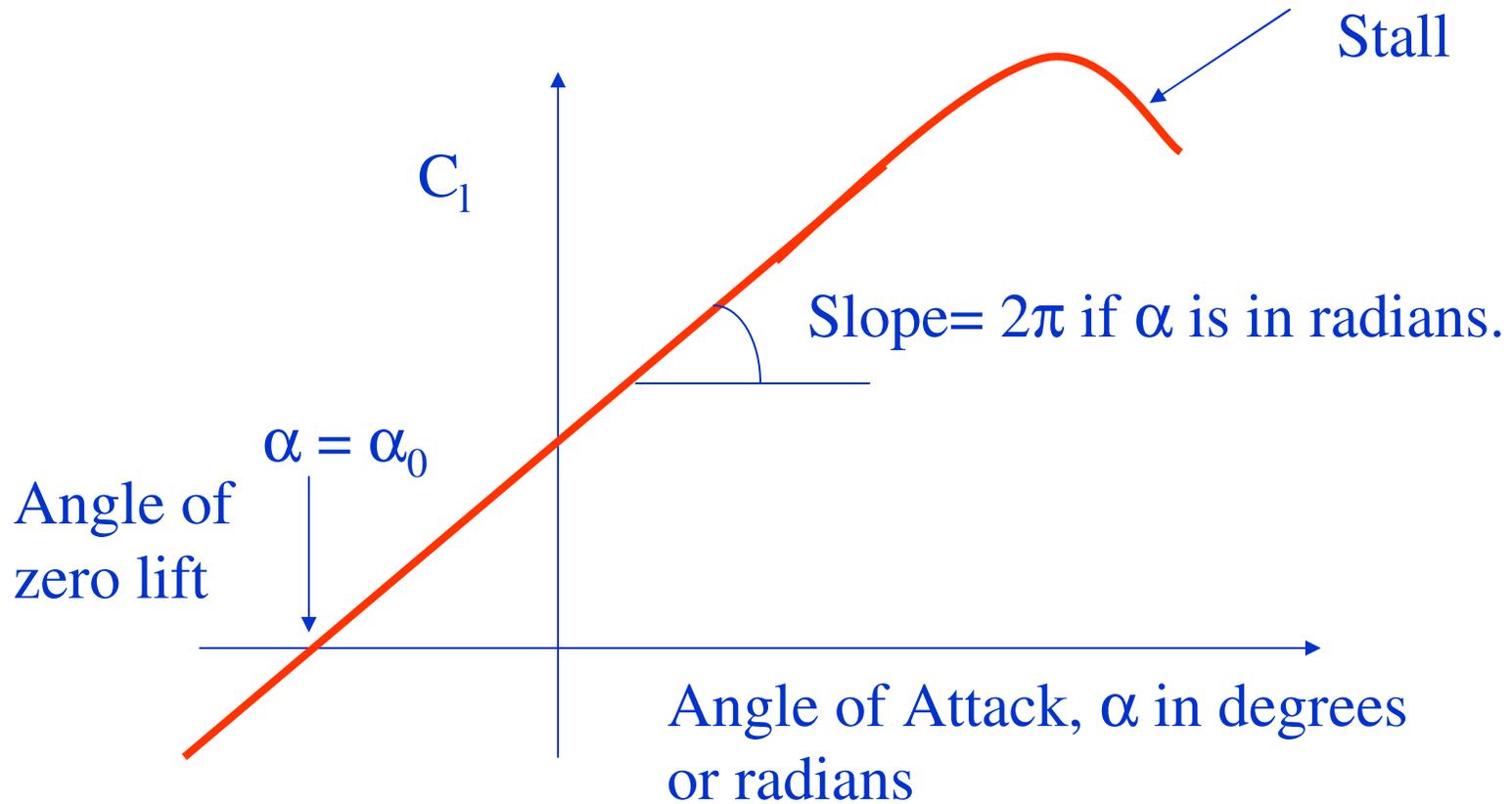
Parameter	Definition	Qualitative ratio of effects	Importance
Reynolds number	$Re = \frac{\rho UL}{\mu}$	$\frac{\text{Inertia}}{\text{Viscosity}}$	Always
Mach number	$Ma = \frac{U}{a}$	$\frac{\text{Flow speed}}{\text{Sound speed}}$	Compressible flow
Froude number	$Fr = \frac{U^2}{gL}$	$\frac{\text{Inertia}}{\text{Gravity}}$	Free-surface flow
Weber number	$We = \frac{\rho U^2 L}{\gamma}$	$\frac{\text{Inertia}}{\text{Surface tension}}$	Free-surface flow
Cavitation number (Euler number)	$Ca = \frac{p - p_v}{\rho U^2}$	$\frac{\text{Pressure}}{\text{Inertia}}$	Cavitation
Prandtl number	$Pr = \frac{\mu c_p}{k}$	$\frac{\text{Dissipation}}{\text{Conduction}}$	Heat convection
Eckert number	$Ec = \frac{U^2}{c_p T_0}$	$\frac{\text{Kinetic energy}}{\text{Enthalpy}}$	Dissipation
Specific-heat ratio	$k = \frac{c_p}{c_v}$	$\frac{\text{Enthalpy}}{\text{Internal energy}}$	Compressible flow
Strouhal number	$St = \frac{\omega L}{U}$	$\frac{\text{Oscillation}}{\text{Mean speed}}$	Oscillating flow
Roughness ratio	$\frac{\epsilon}{L}$	$\frac{\text{Wall roughness}}{\text{Body length}}$	Turbulent, rough walls
Grashof number	$Gr = \frac{\beta \Delta T g L^3 \rho^2}{\mu^2}$	$\frac{\text{Buoyancy}}{\text{Viscosity}}$	Natural convection
Temperature ratio	$\frac{T_w}{T_0}$	$\frac{\text{Wall temperature}}{\text{Stream temperature}}$	Heat transfer
Pressure coefficient	$C_p = \frac{p - p_\infty}{\frac{1}{2} \rho U^2}$	$\frac{\text{Static pressure}}{\text{Dynamic pressure}}$	Aerodynamics, hydrodynamics
Lift coefficient	$C_L = \frac{L}{\frac{1}{2} \rho U^2 A}$	$\frac{\text{Lift force}}{\text{Dynamic force}}$	Aerodynamics, hydrodynamics
Drag coefficient	$C_D = \frac{D}{\frac{1}{2} \rho U^2 A}$	$\frac{\text{Drag force}}{\text{Dynamic force}}$	Aerodynamics, hydrodynamics

Once  $C_l$ ,  $C_d$  etc. are found, they can be plotted for use in all applications - model or full size aircraft

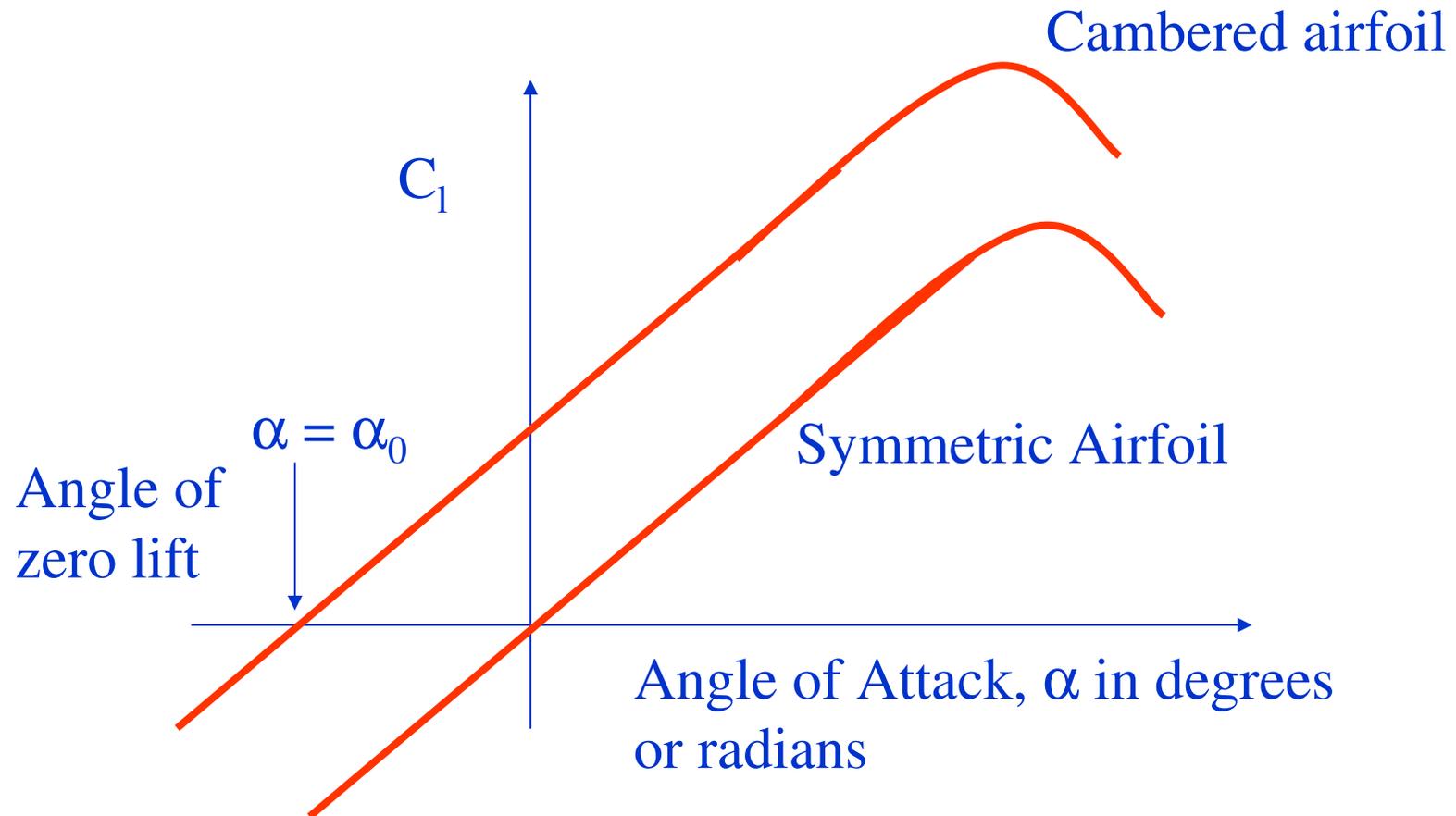


- The geometry must be similar (i.e. scaled) between applications.
- The Reynolds number must be the same for the model and full scale.
- The Mach number must be the same for the model and full scale.
- Then, the behavior of non-dimensional quantities  $C_p$ ,  $C_L$ ,  $C_D$ , etc will also be the same.

# Characteristics of $C_l$ vs. $\alpha$



# The angle of zero lift depends on the camber of the airfoil



# Mathematical Model for $C_l$ vs. $\alpha$ at low angles of attack

Incompressible Flow:  $C_l = 2\pi(\alpha - \alpha_0)$

Compressible Flow:  $C_l = \frac{2\pi}{\sqrt{1 - M_\infty^2}} (\alpha - \alpha_0) = \frac{C_{l, \text{incompressible}}}{\sqrt{1 - M_\infty^2}}$

If we know how an airfoil behaves in low speed, incompressible flow, we can easily estimate how the lift will be altered in high speed flight.

This relation works until the Mach number over the airfoil exceeds unity, and shocks form on the airfoil.

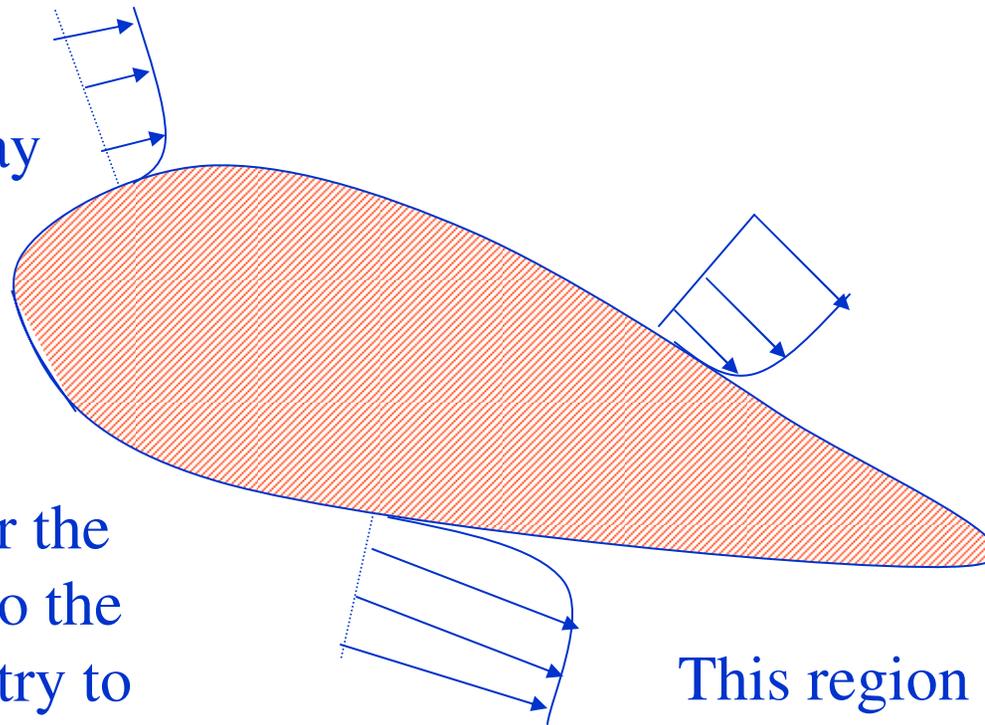
# Drag is caused by

- Skin Friction - the air molecules try to drag the airfoil with them. This effect is due to viscosity.
- Pressure Drag - The flow separates near the trailing edge, due to the shape of the body. This causes low pressures near the trailing edge compared to the leading edge. The pressure forces push the airfoil back.
- Wave Drag: Shock waves form over the airfoil, converting momentum of the flow into heat. The resulting rate of change of momentum causes drag.

# Skin Friction

Particles away from the airfoil move unhindered.

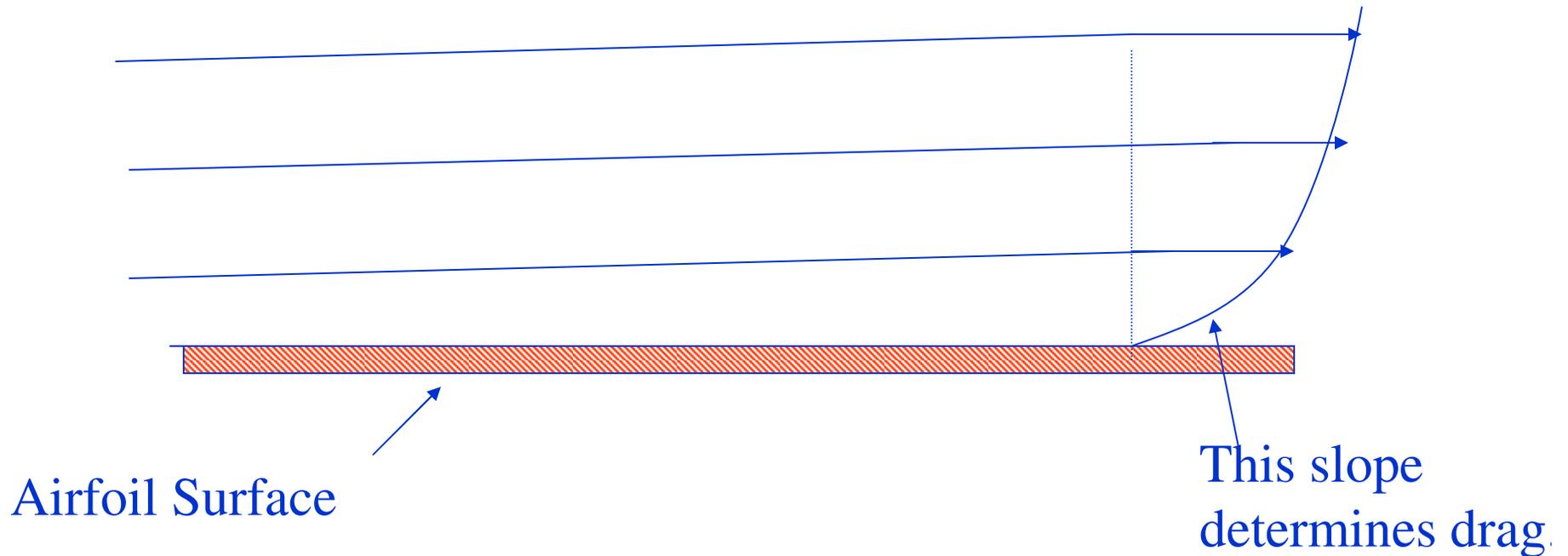
Particles near the airfoil stick to the surface, and try to slow down the nearby particles.



This region of low speed flow is called the boundary layer.

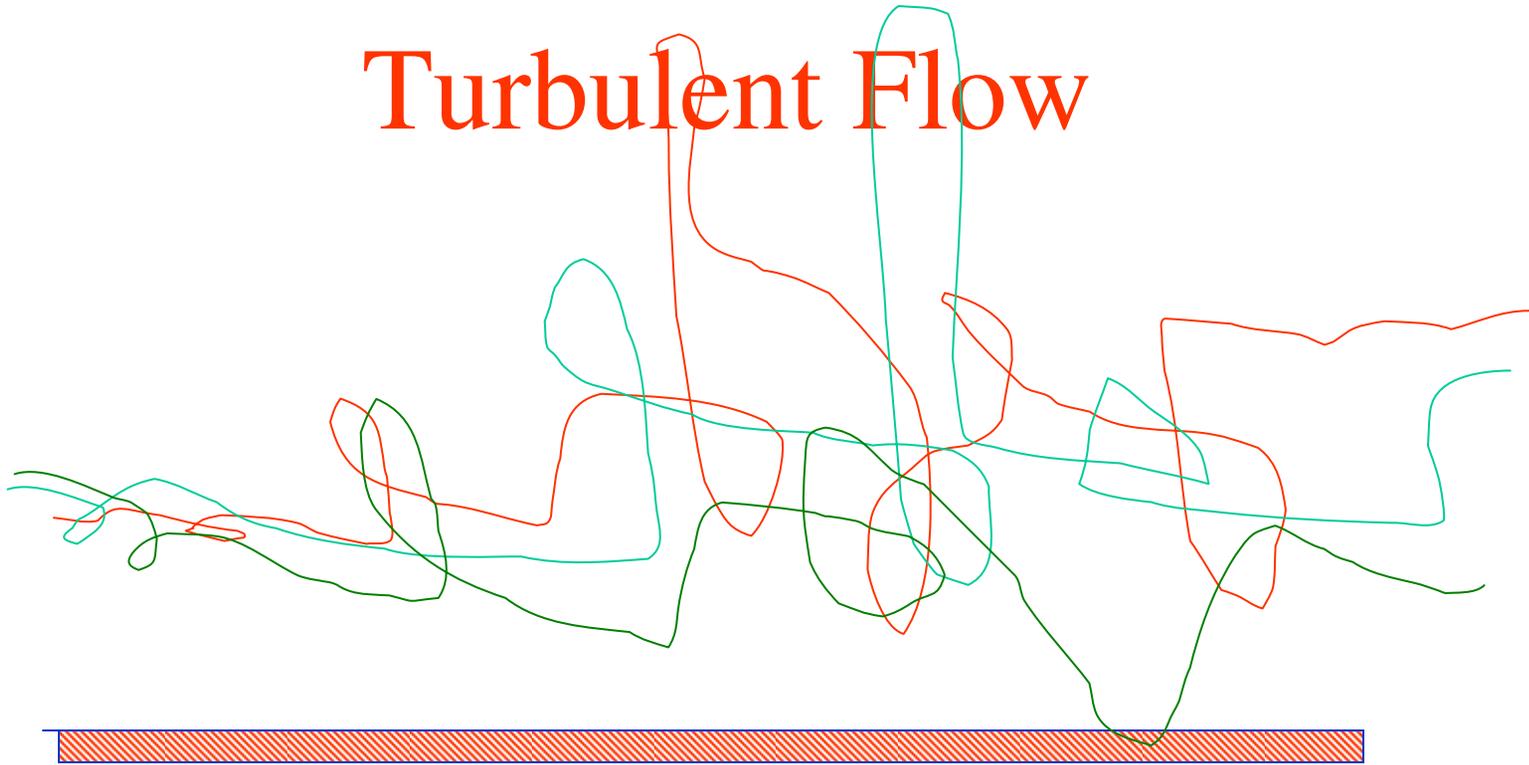
A tug of war results - airfoil is dragged back with the flow.

# Laminar Flow



Streamlines move in an orderly fashion - layer by layer.  
The mixing between layers is due to molecular motion.  
Laminar mixing takes place very slowly.  
Drag per unit area is proportional to the slope of the velocity profile at the wall.  
In laminar flow, drag is small.

# Turbulent Flow

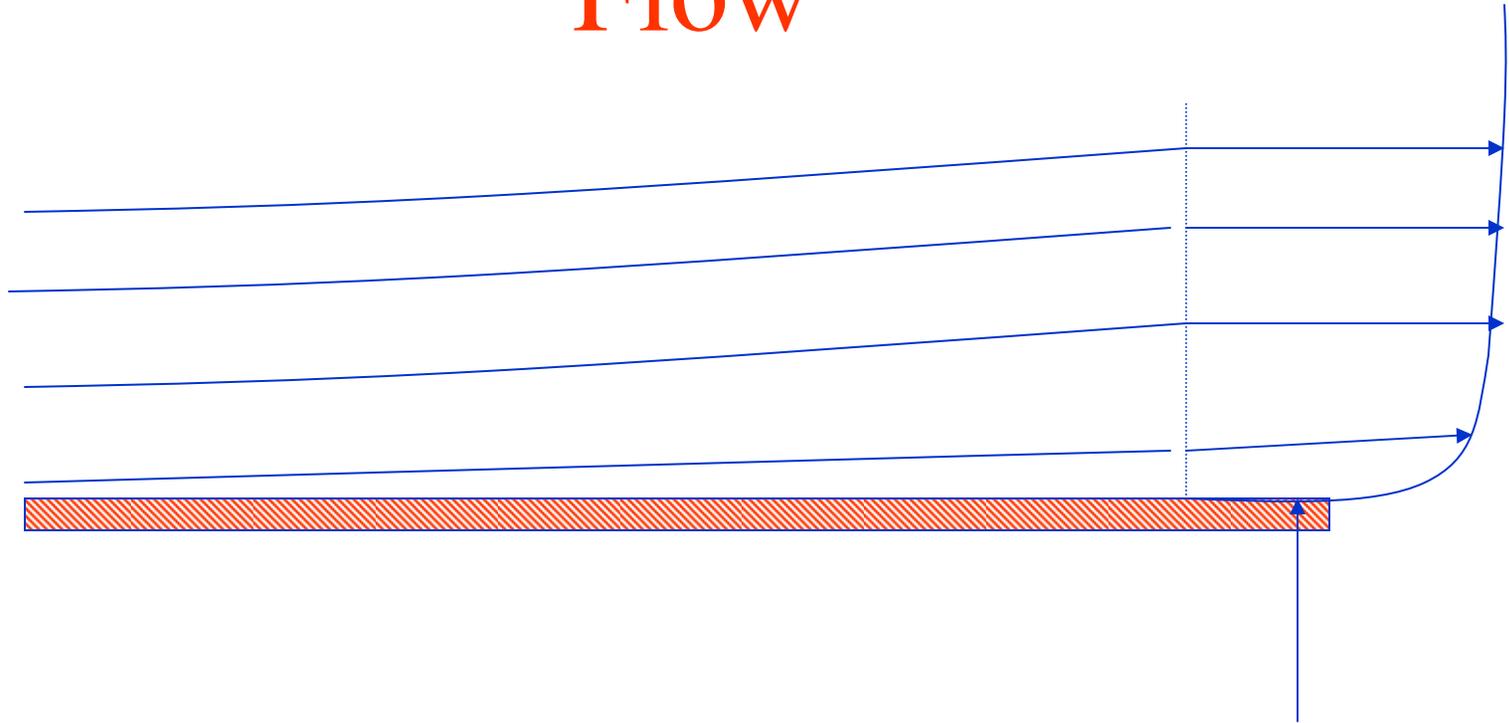


Airfoil Surface

Turbulent flow is highly unsteady, three-dimensional, and chaotic. It can still be viewed in a time-averaged manner.

For example, at each point in the flow, we can measure velocities once every millisecond to collect 1000 samples and average it.

# “Time-Averaged” Turbulent Flow



Velocity varies rapidly  
near the wall due to increased  
mixing.  
The slope is higher. Drag is higher.

## In summary...

- Laminar flows have a low drag.
- Turbulent flows have a high drag.
- Read section 1.11 to learn more about viscous effects.